Midterm 2 for CS 170

This exam contains four problems for a total of 150 points. Good luck!

Problem 1. [Greedy] (30 points)

Give the pseudo-code for a Greedy algorithm that solves the following optimization problem. Justify briefly why your algorithm finds the optimum solution. What is the asymptotic running time of your algorithm in terms of n?

There are n gas stations $S_1, \ldots, S_n$ along I-80 from San Francisco to New York. On a full tank of gas your car goes for D miles. Gas station S_1 is in San Francisco, each gas station S_i, for $2 \leq i \leq n$, is $d_i \leq D$ miles after the previous gas station S_{i-1}, and gas station S_n is in New York. What is the minimum number of gas stops you must make when driving from San Francisco to New York?

Problem 2. [Dynamic Programming] (45 points)

Give the pseudo-code for a dynamic-programming algorithm that solves the following optimization problem. What is the recursive relationship between the optimal solutions for subproblems that your algorithm exploits? What is the asymptotic running time of your algorithm in terms of n?

A paragraph consisting of the n words $W_1, \ldots, W_n$ (in this order) is to be broken into lines without breaking any word. The length of a line is L (possibly fractional) inches. The length of each word W_i is $w_i \leq L$ inches. The minimum length of the blank space between adjacent words on a line is u inches. If a line contains more than one word, then all blank spaces between words are "stretched" equally so that the last word of the line ends at the length of the line. If the stretched blank spaces of a line have length $x \geq u$, then the penalty *of the line is $x - u$. In particular, if a line contains the $k \geq 2$ words $W_i, W_{i+1}, \ldots, W_{i+k-1}$, then the penalty of the line is $((L - w_i - w_{i+1} - \cdots - w_{i+k-1})/(k-1)) - u$. In the special case that a line contains only a single word W_i, the penalty of the line is $L - w_i$. The cost of the paragraph is the sum of all line penalties. What is the minimum cost of the paragraph?*

Problem 3. [Linear Programming] (30 points)

Formulate the following optimization problem as a linear program. Explain what each variable means.

The m generators $G_1, \ldots, G_m$ supply the n cities $C_1, \ldots, C_n$ with power. Each generator G_i can produce at most g_i megawatts per day. Each city C_j consumes c_j megawatts per day. The cost of producing 1 megawatt at generator G_i and shipping it to city C_j is p_{ij} dollars. Each city is legally obligated to purchase at most half of its power from any one generator. What is the minimum total daily cost of power for the cities?

Problem 4. [Network Flow] (45 points)

Formulate the following optimization problem as a max-flow problem. What is the asymptotic running time of the Edmonds-Karp algorithm on your network in terms of m, n, and k?

There are m houses $H_1, \ldots, H_m$ for sale, n persons $B_1, \ldots, B_n$ looking to buy a house, and k real-estate agents $A_1, \ldots, A_k$. Each agent A_i knows a subset $h_i \subseteq \{H_1, \ldots, H_m\}$ of the properties and a subset $b_i \subseteq \{B_1, \ldots, B_n\}$ of the potential buyers. Due to workload constraints, each agent A_i can close at most a_i real-estate transactions. What is the maximum total number of transactions that can be arranged?