Cs170 - Exam 2

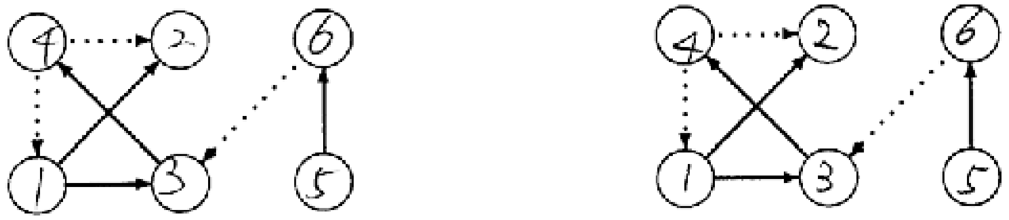
March 13, 1996

Prof. David Wolfe

1. (40 points)

For each of the following questions, short-answer questions, no explanation is necessary. No partial credit is possible, so recheck your answer. (Note: I avoid trick questions.)

(a) (25%) Depth-first search is run on the following graph, G, obtaining the DFS tree shown by the solid edges. (The dotted edges indicate non-tree edges in G.) Indicate the preorder number (or, as Manber calls it, the DFS-number) for each vertex by writing the numbers

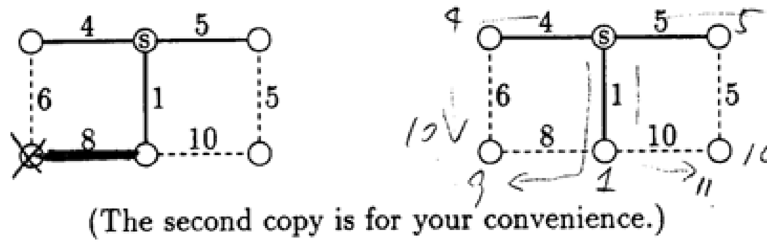


(The second copy is for your convenience.)

[figure 1.a]

(b) (25%) Dijkstras algorithm is being run to find shortest paths from source s in the following undirected graph. The graph is shown in dashed edges, and the tree so far is solid. Darken the one dashed edge which would next be added to the tree:

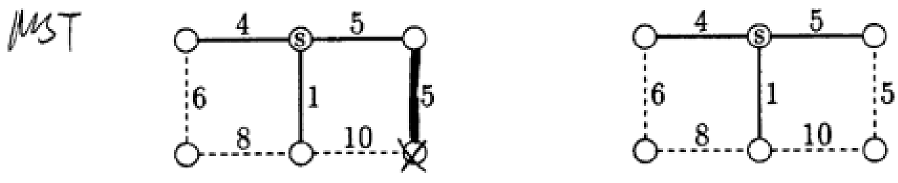
SSSP



(The second copy is for your convenience.)

[figure 1.b]

(c) (25%) Prims algorithm is being run to find the minimum spanning tree on the same graph, growing the tree from vertex s. Darken the one dashed edge which will next be added to the tree:



(The second copy is for your convenience.)

[figure 1.c]

(d) (25%) The following idea is used for an algorithm which checks if the product of two polynomials is correct:

Input: Three polynomials of degree at most d, p1(x), p2(x) and p3(x).

Problem: Determine if p1(x)*p2(x)=p2(x) for all real values of x.

Algorithm sketch: plug in a random value for x0 from {1, , 2d}, and evaluate the three polynomials at x0. Output equal if and only if p1(x0)p2(x0)=p3(x0).

Is the algorithm Monte-Carlo, Las-Vegas or neither? ____________

2. (20 points)

Give a linear-time algorithm to find all the longest paths from a single source s in a directed acyclic unweighted graph. (Hint: I can think of two algorithms. Once uses topological sort, the other uses a shortest path algorithm.)

3. (20 points)

A ski rental agency has m pairs of skis, where the height of the jth pair of skis is s_j, There are n<m skiers who wish to rent skis, where the height of the ith skier is h_i. Each skier wants a pair of skis whose height matches her own height as closely as possible. The goal is to assign the skis to the skiers so that the sum of the absolute differences of the heights of each skier and her skis is minimized.

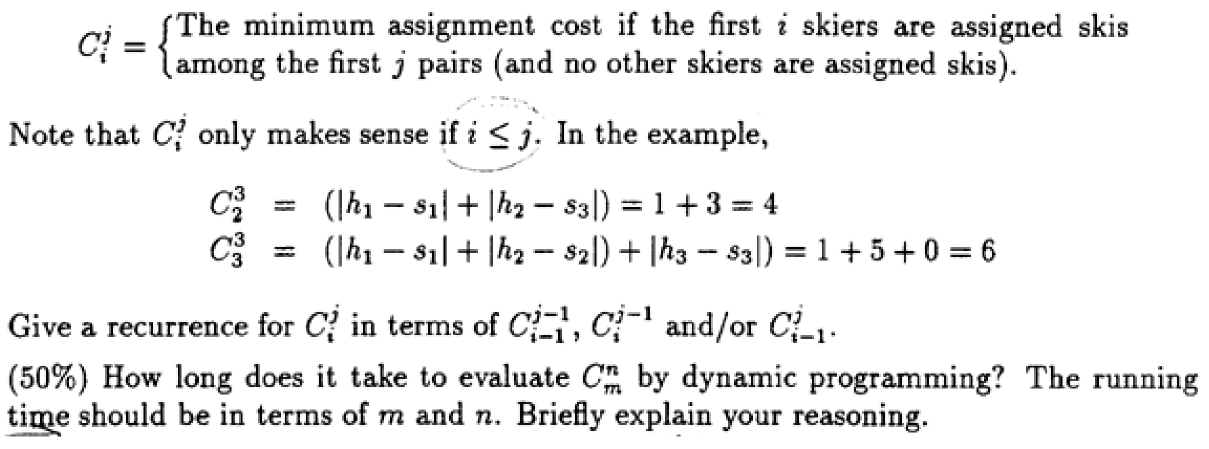
In the following example, the algorithm should output the optimum value 5:

Input		Optimum matching	
Skiers	Skis	Match	Cost
h1=60	s1=59	h1 with s1	|60-59|=1
h2=67	s2=62	h2 with s3	|67-70|=3
h3=70	s3=70	h3 with s4	|70-71|=1
	s4=71		
Output: Total cost of optimal			5

(a) (0% - DO NO ANSWER) Prove that in the optimal assignment of skier to skis, the taller skier always gets the taller pair of skis. In other words, prove that if skier A is taller than skier B then in the optimal assignment, skier As skis are taller than skier Bs skies.

[You dont have to do this part, assume its been done]

(b) (50%) Assume the height of the skis and skiers are sorted as in the example. Define



$$C_i^j = \begin{cases} \text{The minimum assignment cost if the first } i \text{ skiers are assigned skis} \\ \text{among the first } j \text{ pairs (and no other skiers are assigned skis).} \end{cases}$$

Note that C_i^j only makes sense if $i \leq j$. In the example,

$$\begin{aligned} C_2^3 &= (|h_1 - s_1| + |h_2 - s_3|) = 1 + 3 = 4 \\ C_3^3 &= (|h_1 - s_1| + |h_2 - s_2|) + |h_3 - s_3|) = 1 + 5 + 0 = 6 \end{aligned}$$

Give a recurrence for C_i^j in terms of C_{i-1}^{j-1}, C_i^{j-1} and/or C_{i-1}^j.

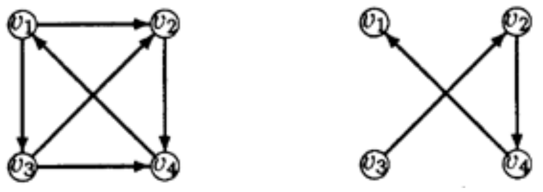
(c) (50%) How long does it take to evaluate C_m^n by dynamic programming? The running time should be in terms of m and n. Briefly explain your reasoning.

[figure 3]

4. (20 points)

A tournament is directed graph with exactly one dge between every pair of vertices. In other words, to get a tournament, take a complete undirected graph and direct each edge. Show that every tournament has a Hamilton path. (recall that a Hamilton path is a path which includes each vertex exactly once. It will not be a cycle.)

For example, for the tournament on four vertices shown on the left, I have drawn a possible Hamilton path on the right:



[figure 4]

Big hint: one way to being a proof is:

Let v be any vertex in tournament G. Partition the vertices of G into three sets, {v}, S and T, where S is the set of vertices in G which point to v, and T is the set of vertices which v points to.