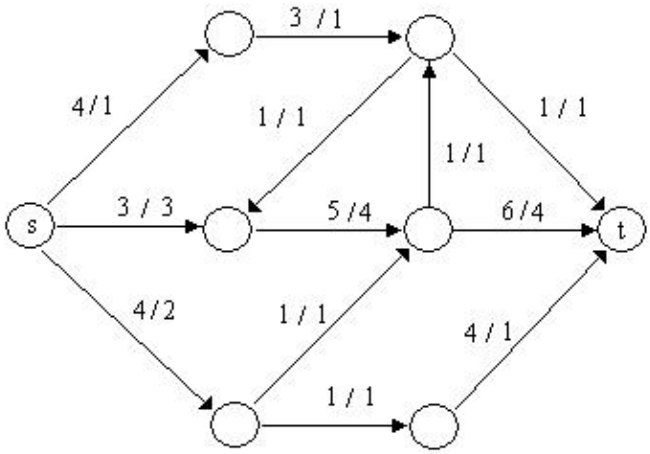
CS 170 Spring 1996
Midterm #3
Professor David Wolfe

You may not use a calculator for this exam. You may use a Manber and a few pages of notes which you have folded into your book before you begin. You may not use any other text or notebook.
You will lose credit if you are unclear or if you ramble. So please proofread your solutions and cross out anything that is irrelevant to the solution of the problem. Again, rather than erase part of your answer, it is to your advantage to cross it out and leave it legible in case it is correct.
If you are having trouble understanding a question, don't hesitate to ask.

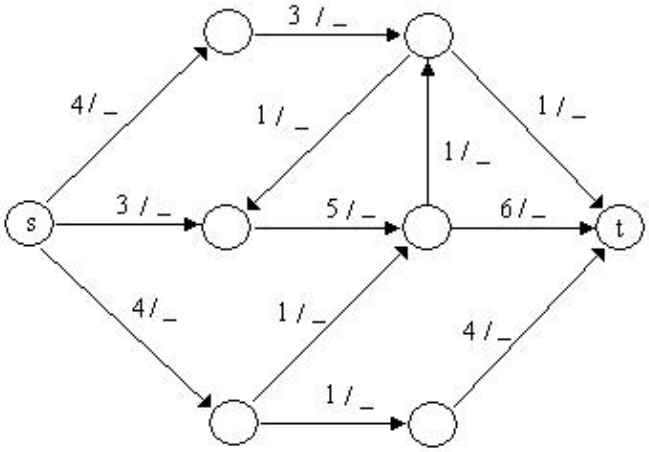
Problem #1 (20 points)
Problem #1a

Indicate an augmenting path in the following network by darkening the edges used along the path.



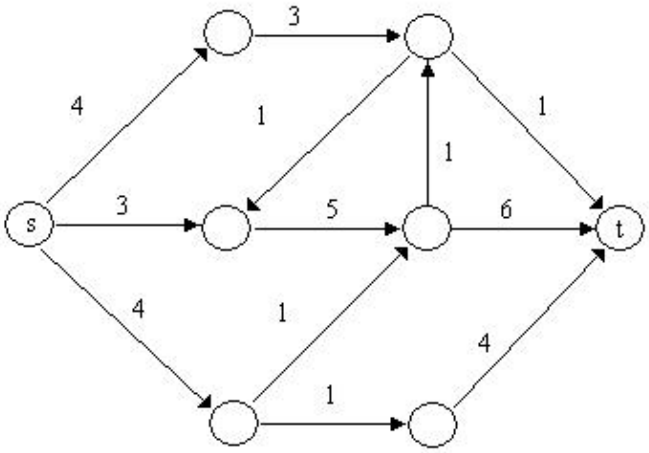
Problem #1b

Indicate below a maximum flow in the same network shown below by writing in those flow values that differ from part (a)



Problem #1c

Prove your flow is maximum by drawing a minimum cut in the graph below. (Naturally, the capacity of the cut should be identical to the max-flow you found above.)



Problem #2 (30 points)

Problem #2a

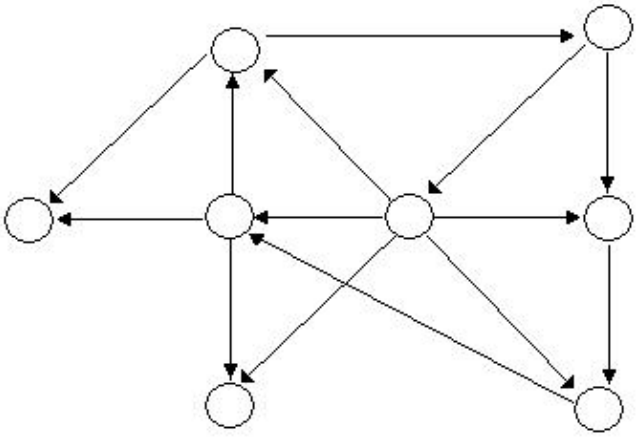
Compute 7^129 mod 20. You may not use a calculator.

Problem #2b

Determine x and y such that $30x + 23y = \gcd(30,23)$. You may not use a calculator.

Problem #2c

Add one edge to the following graph so that it has as few strongly connected components as possible.



Problem #3 (30 points)

Consider the problem of finding a perfect matching in a tree if one exists. Although your goal is to find a fast algorithm, you can receive 80% of the points for finding any polynomial time algorithm. Clarity is important, so I recommend solving part (d) separately so you don't lose points on parts a-c.

Problem #3a

Give a polynomial algorithm to find a perfect match in a tree if one exists.

Problem #3b

Analyze the running time of your algorithm as a function of V.

Problem #3c

Be sure to explain why your algorithm is correct; i.e. show that your algorithm gives a perfect matching if and only if one exists.

Problem #3d

(20%) Give an $O(V)$ or $O(V \log V)$ algorithm.

Problem #4 (20 points)

Let G be a connected undirected graph. Consider the problem of finding a spanning tree of G of minimum height. The height of a tree is measured as the maximum distance from its root to a leaf. (Your algorithm should choose both a root and a tree.)

Problem #4a

Give any polynomial time algorithm to solve the problem.

Problem #4b

Analyze the running time of your algorithm in terms of E and V.

Problem #4c

Be sure to explain why your algorithm is correct if not obvious.
