CS 172 — Fall 2000	**Prelim 2**
Computability and Complexity	**November 8, 2000**

Problem 1. (50 points) Determine which of the following two languages are context-free:

$$A = \{w \in \{0,1\}^* : w = w^{\mathcal{R}}\}.$$
$$B = \{w \in \{0,1\}^* : w \neq w^{\mathcal{R}}\}.$$

Here, $w^{\mathcal{R}}$ is the word w reversed. Prove your answers.

Problem 2. (100 points)

a. Consider a machine M and a state q of M. The state q is *dead* if for all input words w and all runs r of M on input w, the state q does not occur in r. The state q is *redundant* if $L(M) = L(M \backslash q)$, where $M \backslash q$ is the machine that results from M by removing the state q, as well as all transitions in and out of q.

If q is dead, does it follow that q is redundant?
If q is redundant, does it follow that q is dead?

b. Given a machine M and a state q of M, the *dead-state problem* asks if q is a dead state of M. Given a machine M and a state q of M, the *redundant-state problem* asks if q is a redundant state of M.

Consider the following six problems:

D_{NFA}, D_{PDA}, D_{TM}: the dead-state problems for NFAs, PDAs, and TMs.
R_{NFA}, R_{PDA}, R_{TM}: the redundant-state problems for NFAs, PDAs, and TMs.

For each of these six problems, determine which of the following four statements is true:

S1 The problem is recursive.
S2 The problem is r.e., but not recursive.
S3 The problem is co-r.e., but not recursive.
S4 The problem is neither r.e. nor co-r.e.

Prove your answers. You can use the membership, emptiness, universality, and equivalence problems for NFAs, PDAs, and TMs, and what we learned about them in class.