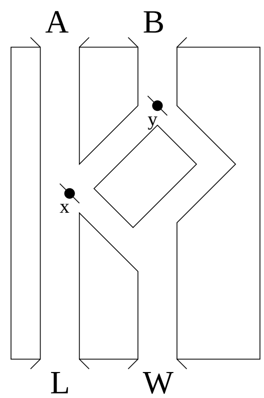
CS172, Spring 2001
Midterm 1

This midterm is open-book. There are 3 problems, 8 true/false questions, and 3 short-answer questions. You have 80 minutes; the number of points assigned to each problem reflects the number of minutes expected to be spent on it, so there's a total of 80 points. We are not looking for rigidly formal construction proofs unless otherwise specified, but you should give sufficient detail to prove that you can make the necessary construction based on constructions presented in class and/or in the textbook. Good luck!

1. a. (15 pts) By some odd twist of fate, you somehow land a job at a marketing division of a toy company. Your first task on the job is to run a "preliminary feasibility report" on a particular toy that they are planning to produce. The toy consists of a box with two slots on top, A and B, into which you can drop balls, two slots at the bottom, L and W, which the balls come out of, and the following mechanism inside:



The levers x and y inside are designed so that every time a ball hits one, the ball slides in the current direction of the lever, but then the lever spins by 90°, forcing the next ball that hits it to move down the other path. So, if the machine is in the above configuration, and you drop a ball in the A slot twice, the first will come out of W, and the second will come out of L. The levers always start out in the state shown above. A sequence of balls that results in the last ball coming out the W slot is considered "Winning."

You play with the toy for a finite amount of time, and then notice that it's rather boring. You must now prove to your superiors that this toy is indeed boring. For the purposes of this problem, we define *boring* to mean "equivalent to a particular DFA." So, demonstrate a DFA that accepts the language of all strings over the alphabet $\{A, B\}$ that correspond to sequences of balls that, starting from the above initial configuration, cause the last ball to come out of the W slot.

b. (5 pts) Your manager, impressed by the above construction, asks you to prepare an 1-line executive summary of the "report" for the CEO, G.R. Epman. And what better way to do that than by regular expressions? Write out a regular expression equivalent to the above DFA.

2. (15 pts) Prove that the following language over the alphabet $\{a, b, c\}$ is not context-free:
$A = \{w | w$ contains an equal number of a's, b's, and c's $\}$

3. (15 pts) Given a language L over $\{0, 1\}$, prove that if both L and $\overline{L}$ (the complement of L) are enumerable, then L is decidable.

4. (15 pts) True/False:

T	F	If the language L is regular, so is any subset of L.
T	F	The regular expression $(0 \cup 1 \cup \emptyset) \circ (\epsilon \circ \emptyset)$ defines the empty language.
T	F	There exists an integer N such that the language P_N of all prime factors of N expressed in unary, is not regular.
T	F	If a PDA reaches an accepting state as it processes an input string, it accepts that string.
T	F	A Turing machine may never write a space to its tape.
T	F	Over a given alphabet Σ, there is only a finite number of languages accepted by a 1-state NFA.
T	F	Any CFL over an alphabet Σ is accepted by a PDA which has Σ as both the input alphabet and the stack alphabet.
T	F	If the DFA definition is modified to allow infinite input alphabets, then any language L over alphabet Σ is recognized by a DFA accepting $w \in L$ as a single symbol over infinite alphabet Σ^*.

5. Short answers:

a. (5 pts) Give an NFA recognizing the language described by the following regular expression: $1((0(0 \cup 1)1)^*0^*)^*$.

b. (5 pts) Give the parse tree for the string *bbacadd* under the following grammar (start variable S):

$$\begin{aligned} S &\rightarrow Ta|bTaT|UU \\ T &\rightarrow aU|Tba|b \\ U &\rightarrow TcT|d \end{aligned}$$

c. (5 pts) Sketch a PDA that accepts the language generated by the grammar consisting of **just** the rules for T and U above (in (b)), with T as the start variable; you can use Sipser's shorthand for "transitions" pushing multiple symbols on the stack in a single step. The input alphabet is $\{a, b, c, d\}$.