# CS 184, Fall 1992
# MT Solutions
# Professor Brian A. Barsky

## Problem #1

A) There are two ways to solve this problem - the easy way and the hard way. Let's solve it the easy way first. Think o

```
       __     __
       | a b c |
    T= | d e f |
       | g h i |
       --     --
```

We are given 3 initial points (A,B,C) and 3 final points (P,Q,R). The equations relating them are:
P=AT & Q=BT & R=CT
which are the following three equations:

```
                          __     __                                __     __
__      __   __      __   | a b c |    __      __   __      __   | a b c |    __
| 3 3 1 | = | 0 0 1 |   | d e f |  &  | 6 7 1 | = | 3 0 1 |   | d e f |  &  | 4 9 1
--      --   --      --   | g h i |    --      --   --      --   | g h i |    --
                          --     --                                --     --
```

Since these are simultaneous equations, we can rewrite them as:

```
__ __    __ __ __     __     __   __     __    __    __ __     __
| P |    | A | | a b c |    | 3 3 1 |    | 0 0 1 | | a b c |
| Q | =  | B | | d e f | or | 6 7 1 | =  | 3 0 1 | | d e f |
| R |    | C | | g h i |    | 4 9 1 |    | 3 1 1 | | g h i |
-- --    -- -- --     --    --    --    --    -- --     --
```

```
                                                                        __
This can be solved two ways. The first is to pre-multiply each side by | A B C
                                                                        --
                      -1                                  -1
    __     __    __ __  __ __        __     __   __     __  __     __    __
    | a b c |    | A |  | P |        | a b c |   | 0 0 1 |  | 3 3 1 |    | 1 4/3
T = | d e f | =  | B |  | Q | or T = | d e f | = | 3 0 1 |  | 6 7 1 | = |-2  2
    | g h i |    | C |  | R |        | g h i |   | 3 1 1 |  | 4 9 1 |    | 3  3
    --     --    -- --  -- --        --     --   --     --  --     --    --
```

The second is to explicitly solve for each of the variables in the unknown T using 9 equations and 9 unknowns from:

```
__       __    __       __ __       __    __                       __
```

$$\begin{bmatrix} 3 & 3 & 1 \\ 6 & 7 & 1 \\ 4 & 9 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} g & h & i \\ 3a+g & 3b+h & 3c+i \\ 3a+d+g & 3b+e+h & 3c+f+i \end{bmatrix}$$

This provides us with 9 equations and 9 unknowns. Immediately *g, h* and *i* are known from the top row:
$g=3$ & $h=3$ & $i=1$
Solving for *a, b* and *c* in the second row gives us:
$3a + g = 6 \longrightarrow 3a + 3 = 6 \longrightarrow 3a = 3 \longrightarrow a = 1$
$3b + h = 7 \longrightarrow 3b + 3 = 7 \longrightarrow 3b = 4 \longrightarrow b = 4/3$
$3c + i = 1 \longrightarrow 3c + 1 = 1 \longrightarrow 3c = 0 \longrightarrow c = 0$
Finally, solving for *d, e* and *f* (the remaining unknowns) from the third row gives us:
$3a + d + g = 4 \longrightarrow 3(1) + d + 3 = 4 \longrightarrow d = -2$
$3b + e + h = 9 \longrightarrow 3(4/3) + e + 3 = 9 \longrightarrow e = 2$
$3c + + f + i = 1 \longrightarrow 3(0) + f + 1 = 1 \longrightarrow f = 0$
Which gives the resulting matrix T:

$$T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 4/3 & 0 \\ -2 & 2 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

The hard way to solve this problem is to think of it as a composition of Scale, Shear, Rotate and Translation.

$$T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \mathrm{Shear}(sh_x, sh_y) * \mathrm{Scale}(sc_x, sc_y) * \mathrm{Rotate}(theta) * \mathrm{Translate}(t_x, t_y) *$$

Where these transformation matrices are the familiar:

$$\mathrm{Shear}(sh_x, sh_y) = \begin{bmatrix} 1 & sh_y & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

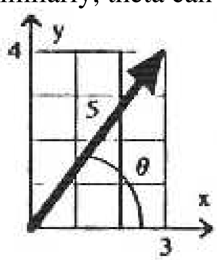
$$\mathrm{Scale}(sc_x, sc_y) = \begin{bmatrix} 1 & sc_y & 0 \\ sc_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Rotate}(theta) = \begin{bmatrix} \cos(theta) & \sin(theta) & 0 \\ -\sin(theta) & \cos(theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Translate}(Tx, Ty) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx & Ty & 1 \end{bmatrix}$$

Neither shear, scale or rotate change the position of the origin. Therefore, $h=i=t_x=t_y=3$, and Translate($t_x$,$t_y$) is:

```
                      --        --    --      --
                      | 1  0  0 |    | 1 0 0 |
Translate(Tx,Ty) =    | 0  1  0 | =  | 0 1 0 |
                      | Tx Ty 1 |    | 3 3 1 |
                      --        --    --      --
```

Similarly, theta can be found from the angle of rotation of the vector AB to PQ. This angle is theta= $\tan^{-1}(4/3)$=53.13 c

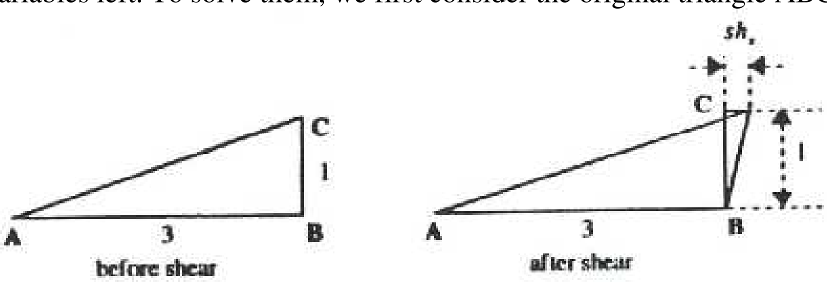



$$\tan(\theta) = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \sin(\theta) = \frac{4}{5} \Rightarrow \cos(\theta) = \frac{3}{5}$$

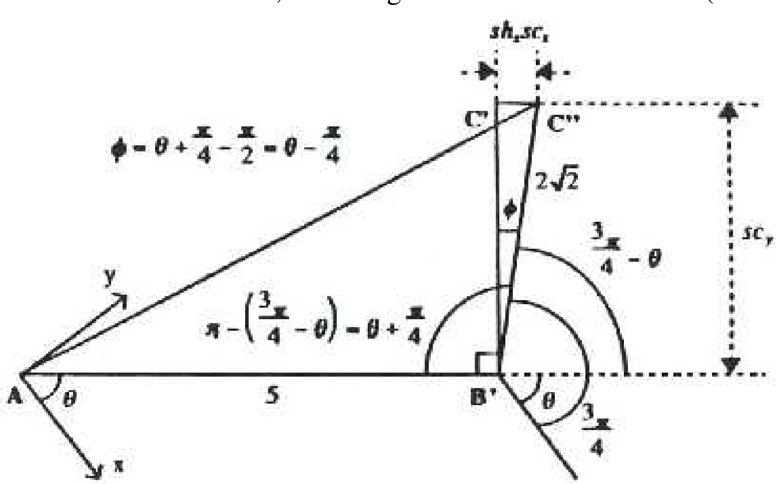
Our rotation matrix is now trivial:

```
                  --                           --    --              --
                  |  cos(theta) sin(theta) 0 |    |  3/5 4/5 0 |
Rotate(theta) =   | -sin(theta) cos(theta) 0 | =  | -4/5 3/5 0 |
                  |      0           0     1 |    |   0   0  1 |
                  --                           --    --              --
```

Now, the only remaining variables are $sh_x$,$sh_y$,$sc_x$ and $sc_y$. To make the problem easier, let's consider no shear in the y makes our scaling easier, since the vector AB does not get sheared at all, and thus $sc_x$ is simply the length of PQ over A variables left. To solve them, we first consider the original triangle ABC and the triangle after the shear in the x directi




After the shear *and* scale, the triangle is deformed as follows (the diagram *is* to scale):



The small triangle (B', C', C'') allows us to solve for $sh_x$ and for $sc_y$. Here is the algebra.
cos(a-b) = cos(a)cos(b) + sin(a)sin(b) and sin(a-b) = sin(a)cos(b) - cos(a)sin(b)
phi = theta + pi/4 - pi/2 = theta - pi/4

sin(phi) = cos(theta-pi/4) = cos(theta)cos(pi/4) + sin(theta)sin(pi/4) = (3/5)(sqrt(2)/2) + (4/5)(sqrt(2)/2)
= (7sqrt(2)/10) = $sc_y$/(2sqrt(2)) = 7sqrt(2)/10 --> $sc_y$ = 28/10 = 14/5
sin(phi) = sin(theta-pi/4) = sin(theta)cos(pi/4) - cos(theta)sin(pi/4) = (4/5)(sqrt(2)/2) - (3/5)(sqrt(2)/2)
= sqrt(2)/10 = $sh_x sc_x$/(2sqrt(2)) = sqrt(2)/10 --> $sh_x$ = 4/(10(5/3)) = 6/25

Now that we've solved for all the variables, we can write the overall transformation matrix T:

```
    __      __  __          __  __          __  __      __     __          __
    |  1   0 0 | | 5/3  0   0 | | 3/5 4/5 0 | | 1 0 0 |    | 1 4/3 0 |
T = | 6/25 1 0 | |  0 14/5  0 | |-4/5 3/5 0 | | 0 1 0 | =  |-2   2   0 |
    |  0   0 1 | |  0   0   1 | |  0   0  1 | | 3 3 1 |    | 3   3   1 |
    --      --  --          --  --          --  --      --     --          --
```

If we swap the order of multiplication of the Scale and Shear matrix, $sh_x$ changes to 1/9. This can be proved similarly.

B) Yes, the inverse *can* be found. It is simply $T^{-1}$, which exists because the determinant of T is non-zero.

## Problem #2

A)For each of the 18 regions labeled a-r in figure 2, fill in the chart below with the words "IN" or "OUT" which repres
rule (odd/even vs. non-zero winding) would conclude about that region.

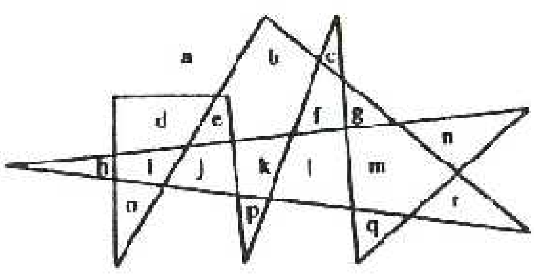
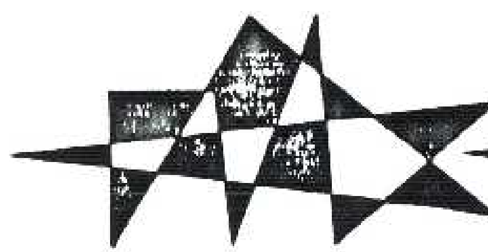
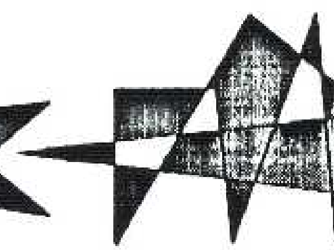


Figure 2

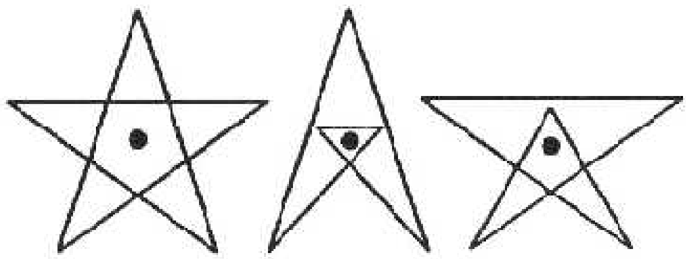

Odd/Even rule


Non-zero Windin

| Label | Odd/Even | Non-zero Winding |
|---|---|---|
| a | OUT | OUT |
| b | IN | IN |
| c | IN | IN |
| d | IN | IN |
| e | OUT | OUT |
| f | OUT | OUT |
| g | IN | IN |
| h | IN | IN |
| i | OUT | OUT |
| j | IN | IN |
| k | OUT | IN |

| | | |
|---|---|---|
| l | IN | IN |
| m | OUT | IN |
| n | IN | IN |
| o | IN | IN |
| p | IN | IN |
| q | IN | IN |
| r | IN | IN |

B)What is the minimum number of edges a polygon would need so that the non-zero winding rule and odd/even rule h region of a polygon? Draw it, highlight the region which is labeled differently and tell which rule labeled it in and whi

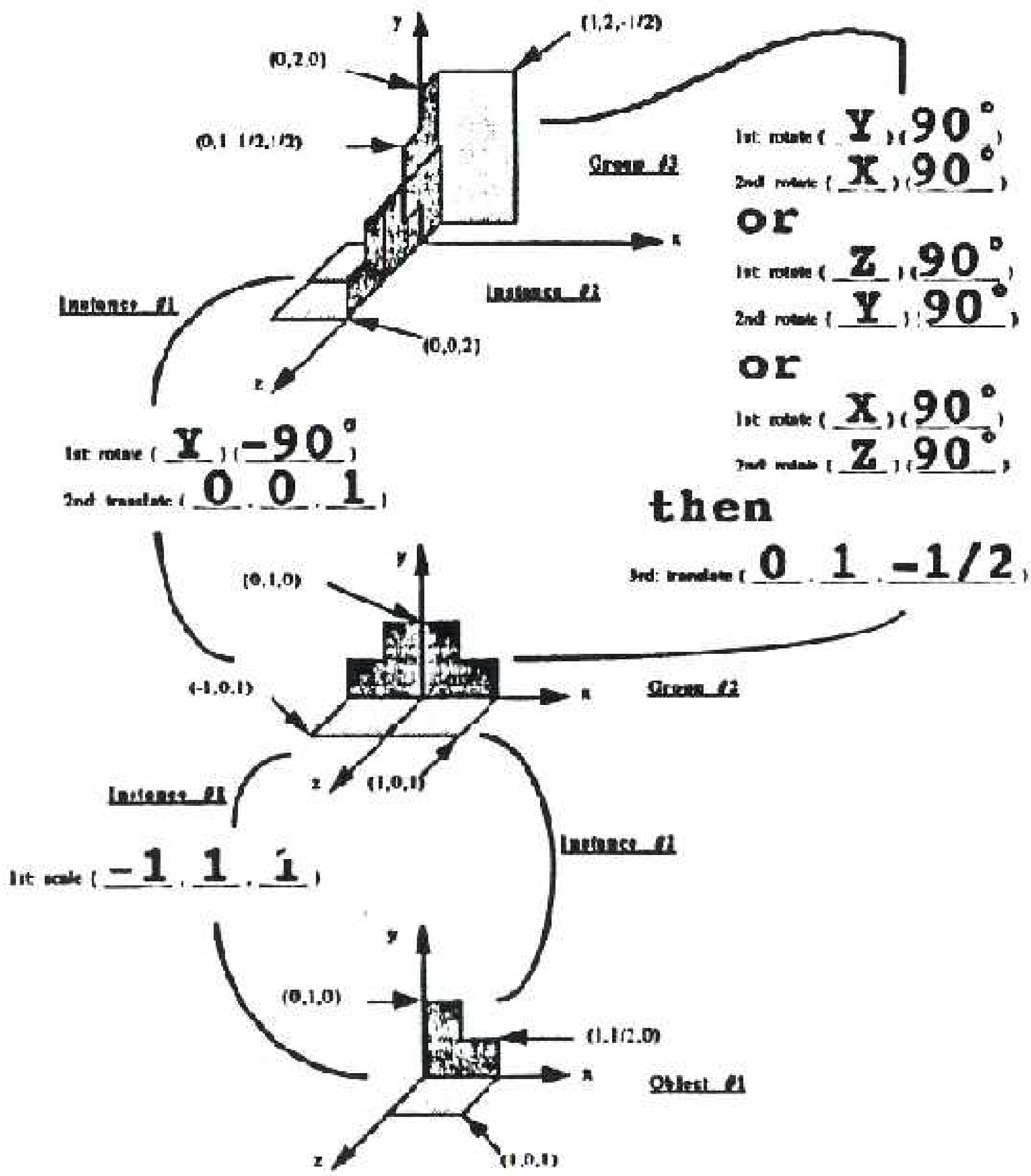


The answer is 5. These are 3 possible answers. The dot indicates the region that is labeled IN by the non-zero winding

## Problem #3

A) The following diagram represents a hierarchical object description which might be found in an SDL file. Fill in the transformation statements so that object #1 is instanced correctly in group #2 and group #2 is instanced correctly in gro

The transformation statement format is: rotate (axis) (degrees), translate (tx,ty,tz) and scale (sx,sy,sz). Note that the axe system, aas in SDL and GL, therefore use the right hand convention for rotations.




B) The composite object shown in part (A) could be represented alternatively as a list of vertices and faces in group #3 transformations. List some advantages and disadvantages of both the hierarchical and non-hierarchical modeling schem rendering, animation, storage and anything else you can think of.

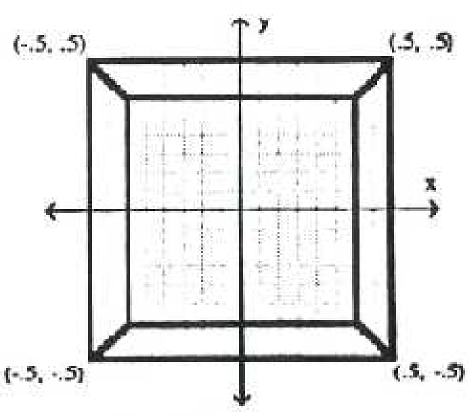
| | Hierarchical | Non-Hie |
|---|---|---|
| **Rendering** | Disadvantages: When rendering the hierarchical structure the main disadvantage is that every vertex needs to be transformed before it can be rendered. This is time consuming. It is also time consuming to traverse the tree and maintain the current transformation matrix by a series of multiplication, pushes and pops. | Advantages: It is easy to render the sce through the list of objects and faces. Th travers. ALso, the vertices of the object no time is spent transforming them. |
| **Animation** | Advantage: It is very easy to animate composite objects, and parts of composite objects. If you have constructed your hierarchy correctly a whole object or part of an object can be moved by changing just the matrix at the appropriate node in the hierarchy and re-rendering the | Disadvantage: To transform an object i vertices and recalculate them all accord You could argue that this is not a disad modeling scheme needs to compute thi But it is certainly more difficult to tran |

| | | |
|---|---|---|
| | hierarchy. | the arm of a robot you would need to k the arm joint and compute a complicate whereas in the hierarchical case all that couple of entries in a rotation matrix sc |
| **Storage** | Advantage: Hierarchical modeling is good if you have one object (which maybe quite complex) that appears a number of times in the scene (with possible transformations). In this case the object's list of vertices (which may be large) can be stored only once, and each instance of the object can be represented by a small node in the tree. | Disadvantage: There is no object instan which had rows of identical objects you each object, rather than one general obj transformations. |
| **Other** | Advantage: Having a hierarchical modeling system lets you take a more "object oriented" approach to representing your models. One of the advantages to this approach is the ability to inherit surface and object characteristics from hierarchical parents.<br><br>Disadvantage: For simple scenes with no repetition of objects it may take a lot of "unecessary" space to store the tree nodes and transformation matrics. It is also more complicated to manage the tree structure than a simple list of vertices. | Advantage: You don't have to bother m hierarchy. |

## Problem #4

A) The outer square remains the same as in the parallel projection since the front face of the cube lies exactly on the pl the inner square (which is the projection of the back face of the cube) needs to be calculated. We use similar triangles t

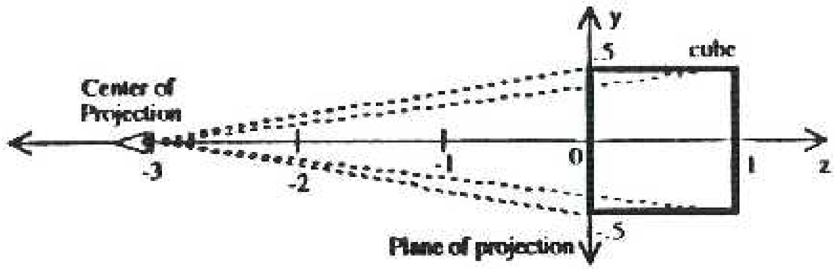
Note that the grid line on the graph is equal to 1/16.
3/8 = 6/16, so the back face projects to the +/-6 grid line.



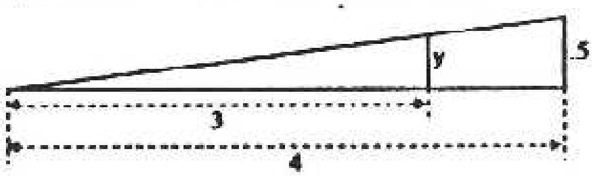

The projection for part (A) above

B) The coordinates proved below are
{ (-.5,-.5,0),(-.5,.5,0),(.5,.5,0),(.5,-.5,0),(-1.5,-1.5,1),(-1.5,1.5,1),(1.5,1.5,1), (1.5,-1.5,1) }

If the system was viewed from x=+infinity, it would look like this:



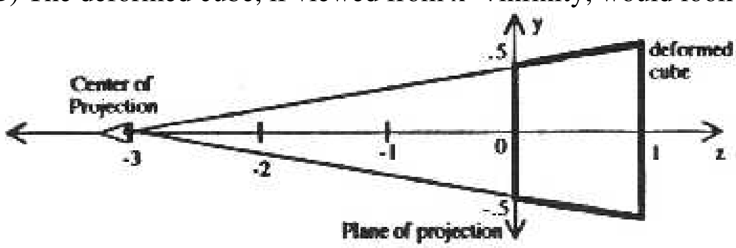

To find the intersection of the projector with the xy plane, we use similar triangles.



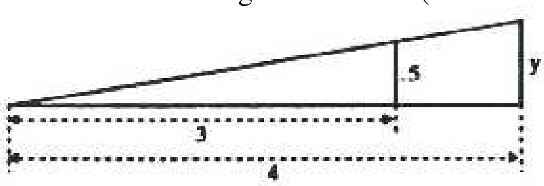

Then calculate the value for y (which will be the same as the value for x since the cube is symmetrical) as follows:

height/length = y/3 = .5/4 --> 4y=3(.5)=3/2 -->y=3/8

B) The deformed cube, if viewed from x=+infinity, would look like this:




and have similar triangles as follows (note the front face doesn't change so its coordinates remain constant):




we use the same method to find the value for y (which is the same as the value of x due to symmetry):

height/length = .5/3 = y/4 --> 3y=4(.5)=2 -->y=3/2

The modified back-face coordinates are:
{(-1.5,-1.5,1),(-1.5,1.5,1),(1.5,1.5,1),(1.5,-1.5,1)}

---