Score summary (leave blank):	
P1:	_____
P2:	_____
P3:	_____
P4:	_____
P5:	_____
P6:	_____
Total:	_____

Name: **Solution**

SID: ______________________________

Discussion section (day / time):

____

Lab section (day / time):

____

UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

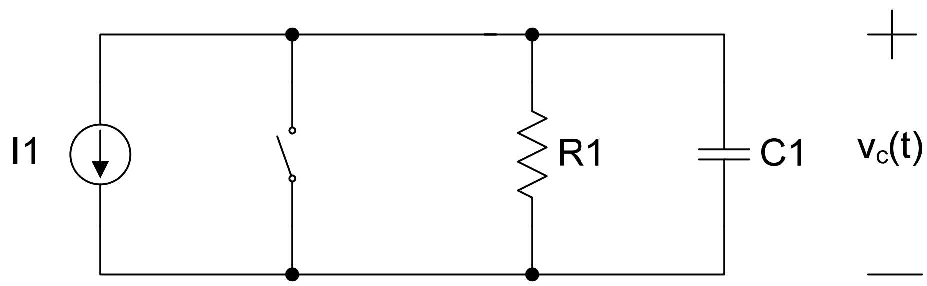
B. E. BOSER

Midterm 2
Solution

EECS 42/100
FALL 2006

- *One 8.5" x 5.5" sheet with equations, two-sided.*
  (originals only, handwritten; no computer output or photocopies, equations only, no problem solutions).
- *No electronic devices (store calculators and phones in hallway).*
- *Copy your answers into marked boxes on exam sheets.*
- *Simplify numerical and algebraic results as much as possible.*
  Up to 10 points penalty for results that are not reasonably simplified.
- *Mark your name and SID at the top of the exam and all extra sheets.*
- *Be kind to the graders and write legibly. No credit for illegible results.*
- *No credit for multiple differing answers for same problem.*

Problem 1 [17 points]



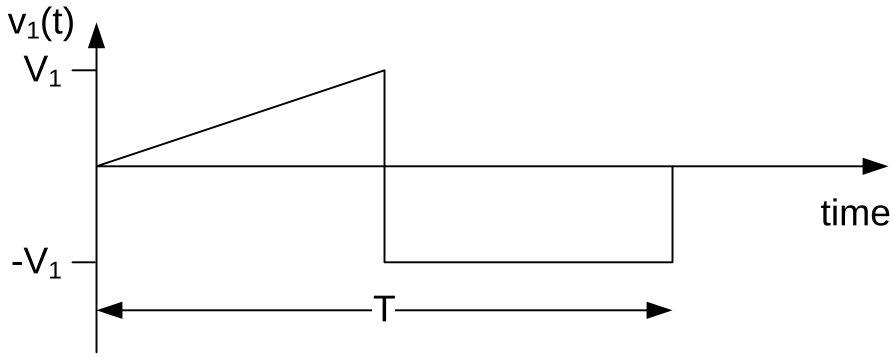
In the circuit shown above, the switch *opens* at t=0. Find an algebraic for $v_c(t)$, $t>0$. Assume that the circuit has reached steady state at t=0 (i.e. the node voltages are not changing just before the switch is closed).
Given: I1, R1, C1.

$$v_c(t) = -I_1 R_1 \left(1 - e^{-t/R_1 C_1}\right)$$

Partial credit:

- Correct exponent: 5pts
- Correct at t=0: 5pts
- Correct at t→infinity: 5pts
- Sign error: -3pts (each sign)

Problem 2 [17 points]



The voltage $v_1(t)$ in the above circuit is periodic with period T. Find an algebraic expression for the average power dissipated in R_1.
Given: V_1, T, R_1.

$$P = \frac{V_1^2}{R_1}\left(\frac{1}{6} + \frac{1}{2}\right) = \frac{2}{3}\frac{V_1^2}{R_1}$$

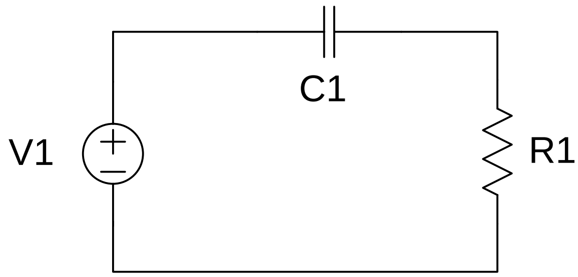
$$P_1 = \frac{V_1^2}{R_1} \cdot \frac{1}{T} \cdot \int_0^{T_2} \left(\frac{t}{T_2}\right)^2 dt = \frac{V_1^2}{R_1} \cdot \frac{1}{T} \cdot \frac{T_2^3}{3 \cdot T_2^2} = \frac{V_1^2}{R_1} \cdot \frac{1}{6}$$

$$P_2 = \frac{V_1^2}{R_1} \cdot \frac{1}{2}$$

Partial credit:

- P1 correct: 12 pts
- P2 correct: 5pts
- Penalty for 2x more power: -3pts

Problem 3 [17 points]



V1 is a sinusoidal source with frequency ω and amplitude V1. Find algebraic expressions for the actual (i.e. the power dissipated by the circuit) and reactive power delivered by the source.
Given: V1, ω, R1, C1.

$$P = \frac{V_1^2}{2} \cdot R \cdot \frac{(\omega \cdot C)^2}{1 + (\omega \cdot R \cdot C)^2}$$

$$Q = -\frac{V_1^2}{2} \cdot \frac{1}{\omega \cdot C} \cdot \frac{(\omega \cdot C)^2}{1 + (\omega \cdot R \cdot C)^2} = -\frac{V_1^2}{2} \cdot \frac{\omega \cdot C}{1 + (\omega \cdot R \cdot C)^2}$$

$$Z(j \cdot \omega) = R + \frac{1}{j \cdot \omega \cdot C} = \frac{1 + j \cdot \omega \cdot R \cdot C}{j \cdot \omega \cdot C} = \frac{j \cdot \omega \cdot C - \omega^2 \cdot R \cdot C^2}{-\omega^2 \cdot C^2} = R - \frac{j}{\omega \cdot C} = R + \frac{1}{j \cdot \omega \cdot C}$$

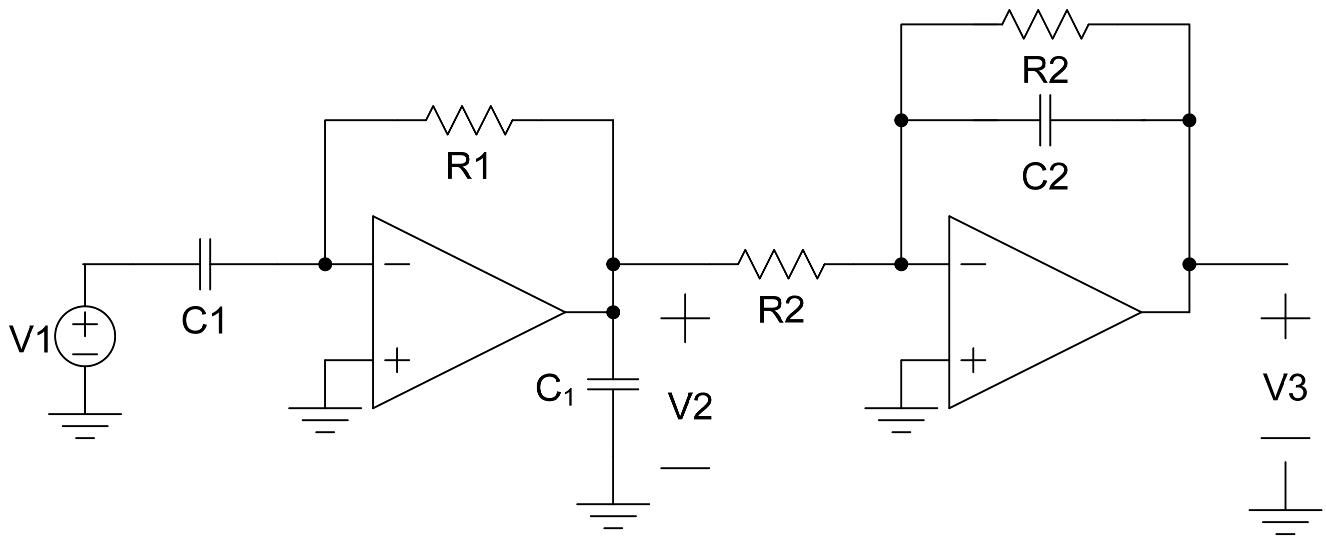
$$(|Z(j \cdot \omega)|)^2 = \frac{1 + (\omega \cdot R \cdot C)^2}{(\omega \cdot C)^2}$$

$$P = V_{rms} \cdot I_{rms} \cdot \cos(\Phi) = V_{rms} \cdot \frac{V_{rms}}{|Z|} \cdot \frac{R}{|Z|} = \frac{V_1^2}{2} \cdot \frac{R}{(|Z|)^2}$$

Partial credit: (min penalty for any error: 2pts)

- P correct: 9pts
- Q correct: 9pts
- |Z|^2 correct: 6pts
- Correct eq for P, Q: 6pts

Problem 4 [17 points]



Derive an algebraic expression for $H(j\omega) = \frac{V_3(j\omega)}{V_1(j\omega)}$.

Given: R1, C1, R2, C2 (note: there are two capacitors with value C1 and two resistors with value R2).

$$H(j \cdot \omega) = \frac{j \cdot \omega \cdot R_1 \cdot C_1}{1 + j \cdot \omega \cdot R_2 \cdot C_2}$$

$i_1 = v_1 \cdot j \cdot \omega \cdot C$

$v_2 = -R_1 \cdot i_1 = -v_1 \cdot j \cdot \omega \cdot R_1 \cdot C_1$

$i_2 = \frac{v_2}{R_2}$

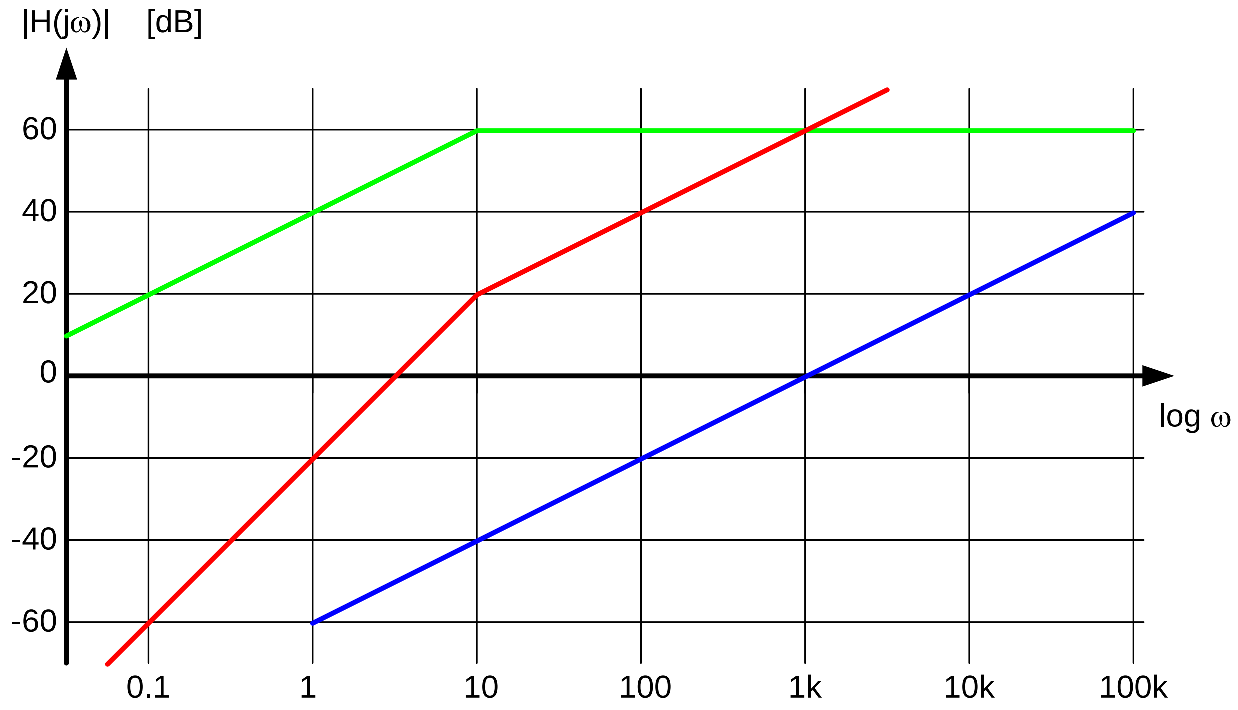
$v_3 = -\frac{R_2}{1 + j \cdot \omega \cdot R_2 \cdot C_2} \cdot \frac{1}{R_2} \cdot v_2$

$$H(j \cdot \omega) = \frac{v_3}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} = -(j \cdot \omega \cdot R_1 \cdot C_1) \cdot \left(-\frac{1}{1 + j \cdot \omega \cdot R_2 \cdot C_2}\right)$$

Partial credit:

- V2/V1 correct: 8pts
- V3/V2 correct: 8pts
- Sign error: -4pts
- Left as double fraction: -3pts
- Nominator error: -8pts
- Denominator error: -8pts

Problem 5 [17 points]
In the graph paper provided below draw the Bode plot for $H_1(j\omega)$ in green, $H_1(j\omega)$ in blue, and $H_3(j\omega) = H_1(j\omega) \times H_2(j\omega)$ in red.
Use piece-wise linear approximations of the magnitude (do NOT "round" the corners).
Note: neatness counts!
Given: $H_1(j\omega) = \dfrac{100 j\omega}{1 + \dfrac{j\omega}{10}}$, $H_2(j\omega) = \dfrac{j\omega}{1000}$



Partial credit: (max 15 pts for a solution that is only partially correct)

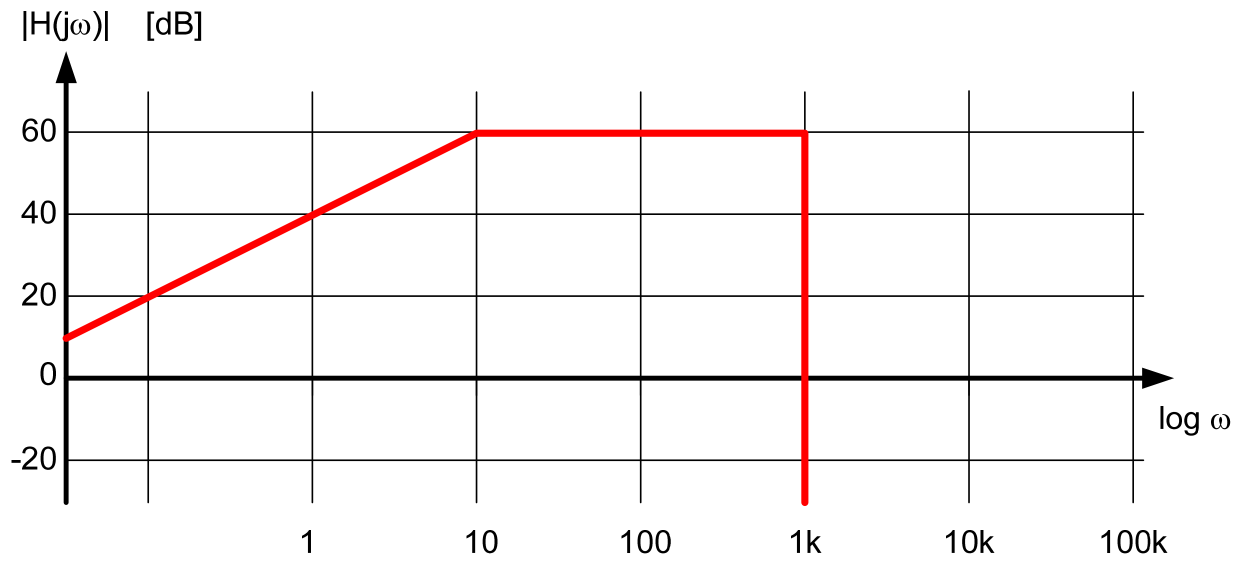
- H1 correct: 8pts
  - **a.** Breakpoint wrong: -5pts
  - **b.** Slope wrong: -4pts (each slope)
  - **c.** Gain error: -4pts (i.e. shifted vertically)
- H2 correct: 8pts
  - **a.** Slope wrong: -4pts
  - **b.** Gain error: -4pts (i.e. shifted)
- H3 = H1+H2: 4pts
- Penalty for H3 != H1+H2: -5pts

Problem 6 [17 points]

An electronic filter has the magnitude response shown below (beyond 1krad/s the magnitude response of the filter is zero). The filter is fed the following input:

$$V_{in}(j\omega) = 0.5\cos(t) + 2\sin(100t) + 3\cos(10{,}000t)$$

Write an expression for the output voltage $V_{out}(j\omega)$ of the filter (assume that the filter does not change the phase of the input).



$$V_{out}(j\omega) = 50\cos(t) + 2000\sin(100t)$$

Partial credit:

- Amplitude error: -6pts (each amplitude)
- Frequency error: -6pts (each frequency)
- Magnitude at 10k not zero: -6pts