**UNIVERSITY OF CALIFORNIA, BERKELEY**
**College of Engineering**
**Department of Electrical Engineering and Computer Sciences**

EE 105: Microelectronic Devices and Circuits — Fall 2007

# MIDTERM EXAMINATION #1

Time allotted: 80 minutes

**NAME:** SOLUTIONS ______________ ______________ ______________
(print) Last First Signature

**STUDENT ID#:** ______________

**INSTRUCTIONS:**

1. **Use the values of physical constants provided below.**
2. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
3. **Clearly mark (underline or box) your answers.**
4. **Specify the units on answers whenever appropriate.**

**PHYSICAL CONSTANTS**

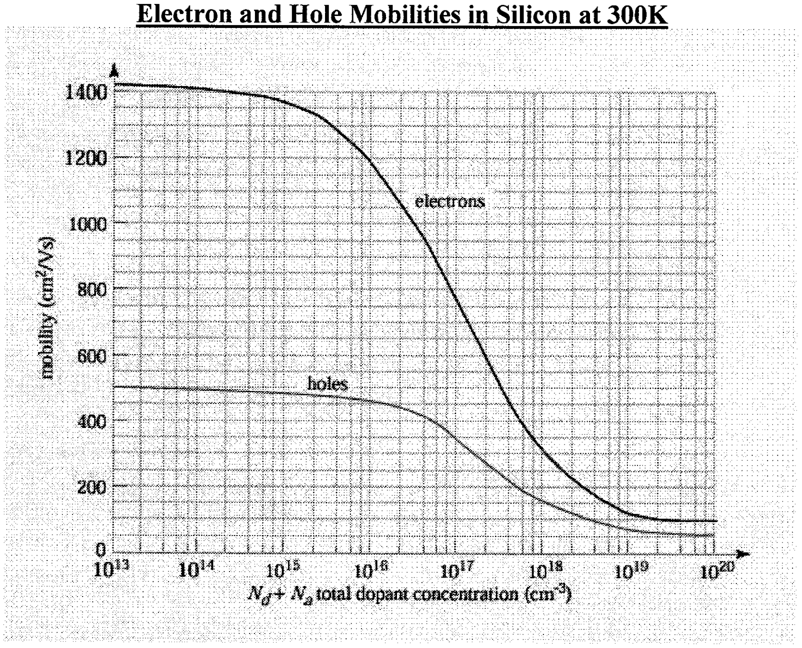
| Description | Symbol | Value |
|---|---|---|
| Electronic charge | $q$ | $1.6\times10^{-19}$ C |
| Boltzmann's constant | $k$ | $8.62\times10^{-5}$ eV/K |
| Thermal voltage at 300K | $V_T = kT/q$ | 0.026 V |

Note that $V_T \ln(10) = 0.060$ V at $T$=300K

**PROPERTIES OF SILICON AT 300K**

| Description | Symbol | Value |
|---|---|---|
| Band gap energy | $E_G$ | 1.12 eV |
| Intrinsic carrier concentration | $n_i$ | $10^{10}$ cm$^{-3}$ |
| Dielectric permittivity | $\varepsilon_{Si}$ | $1.0\times10^{-12}$ F/cm |

**Electron and Hole Mobilities in Silicon at 300K**




| | | |
|---|---|---|
| **SCORE:** | 1 ________ | / 25 |
| | 2 ________ | / 25 |
| | 3 ________ | / 30 |
| **Total:** | ________ | / 80 |

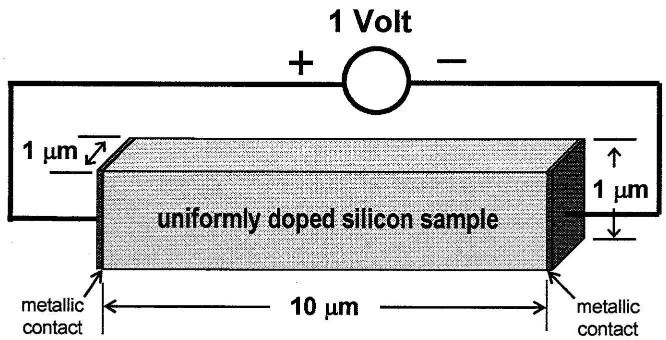
Fall 2007

# EE105 MIDTERM #1 SOLUTIONS

Prof. Liu

**Problem 1 [25 points]: Semiconductor Basics**

**a)** Consider a Si sample of length 10 μm and cross-sectional area 1μm$^2$, uniformly doped with $10^{18}$ cm$^{-3}$ arsenic, maintained at $T$ = 300K. 1 Volt is applied across its length, as shown below:




**i)** What are the electron and hole concentrations, $n$ and $p$, in this sample? **[4 pts]**

Arsenic is a donor in silicon ⇒ $N_D = 10^{18}\,cm^{-3}$. $N_A = 0$

Since $N_D > N_A$, this sample is n-type.

$n = N_D - N_A = \boxed{10^{18}\,cm^{-3}}$

$p = n_i^2/n = 10^{20}\,cm^{-6} / 10^{18}\,cm^{-3} = \boxed{100\,cm^{-3}}$

**ii)** Estimate the resistance of this sample. **[5 pts]** From plot on Page 1, $\mu_n \simeq 300\,cm^2/V\cdot s$

resistivity $\rho = \frac{1}{q\mu_n n + q\mu_p p} \simeq \frac{1}{q\mu_n n}$ since $n \gg p$

$\rho = \frac{1}{(1.6\times10^{-19}C)(300\,cm^2/V\cdot s)(10^{18}\,cm^{-3})} \cong \underline{0.02\ \Omega\text{-}cm}$

resistance $R = \rho\frac{L}{A} = (0.02\ \Omega\text{-}cm)\left(\frac{10\times10^{-4}\,cm}{1\times10^{-8}\,cm^2}\right) = \boxed{2000\,\Omega}$

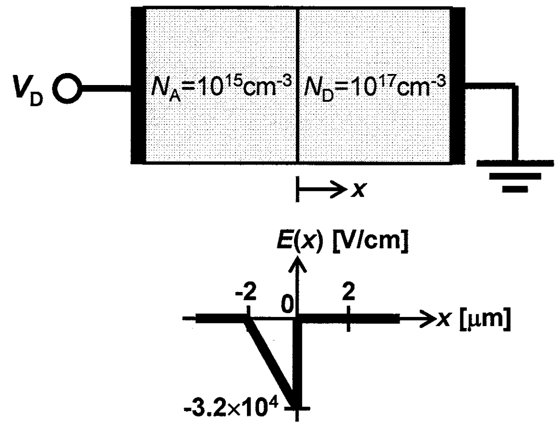
**iii)** Qualitatively (no calculations required), how would the resistance of this sample change if it were to be **additionally doped with 2×10$^{18}$ cm$^{-3}$ boron**? Explain briefly. **[4 pts]**

Boron is an acceptor in silicon ⇒ $N_A = 2\times10^{18}\,cm^{-3}$

The sample would be converted to p-type material with hole concentration $p = N_A - N_D = 10^{18}\,cm^{-3}$ (same majority-carrier concentration as before).

Since the hole mobility is lower than the electron mobility, <u>the resistance of the sample would increase.</u>

**b)** Consider a Si pn junction diode, maintained at $T = 300\text{K}$, with a structure and $E$-field distribution as shown.




**i)** Calculate the built-in potential, $V_0$. **[4 pts]**

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026 \ln\left(\frac{10^{15} \cdot 10^{17}}{10^{20}}\right) = 0.026 \ln\left(10^{12}\right)$$

$$= 12\,(0.026)\ln(10) = 12 \times 0.060 = \boxed{0.72\,V}$$

**ii)** What is the applied voltage, $V_D$? Is this diode forward or (reverse) biased (circle one) ? **[4 pts]**

The total potential dropped across the junction is $-\int E(dx)$

i.e. the area under the $E(x)$ distribution times $(-1)$.

From the plot of $E(x)$, this area is $\frac{1}{2}(2\times10^{-4}\text{cm})(-3.2\times10^{4}\,\text{V/cm})$

$= -3.2\,V$.

The total potential dropped across the junction is also $V_0 - V_D$:

$$V_0 - V_D = 3.2V \quad \Rightarrow \quad V_D = V_0 - 3.2V = 0.72V - 3.2V = \boxed{-2.48V}$$

**iii)** Calculate the areal junction capacitance. **[4 pts]**

$$C_{dep} = \frac{\varepsilon_{Si}}{W_{dep}} = \frac{10^{-12}\,\text{F/cm}}{2\times10^{-4}\,\text{cm}} = \boxed{5\times10^{-9}\,\text{F/cm}^2}$$

**Problem 2 [25 points]: Bipolar Junction Transistor**

**a)** **i)** What is the Early effect (*i.e.* how is it manifested in the *I-V* characteristic of a BJT)? **[2 pts]**

The Early effect is an increase in $I_C$ with increasing $|V_{CE}|$, for a fixed $|V_{BE}|$.

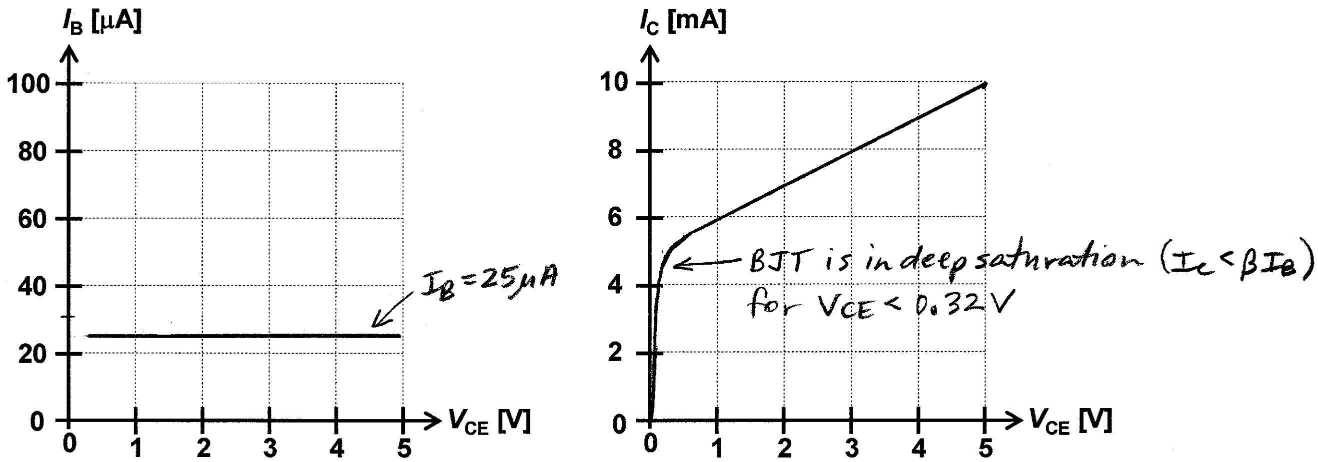
**ii)** Why is the Early effect undesirable, for BJT amplifier applications? **[2 pts]**

The Early effect degrades small-signal voltage gain and decreases output resistance, each of which are undesirable for amplifier applications.

**b)** Indicate in the table below (by checking the appropriate box) how the BJT parameters would change, if the **emitter doping were to be increased** (*e.g.* by 2×). Provide **qualitative reasoning** for your answers. **[9 pts]**

| BJT Parameter | Parameter will: increase | decrease | not change significantly | Brief Justification (No equations or formulas!) |
|---|---|---|---|---|
| Reverse saturation current, $I_S$ | | | ✓ | The concentration of minority carriers in the quasi-neutral base (which affects the carrier concentration gradient and hence $I_C$) is not dependent on $N_E$. |
| Common-emitter DC current gain, $\beta$ | ✓ | | | The concentration of minority carriers at the edge of the depletion region in the emitter would be decreased. ⇒ minority carrier injection into the emitter (hence $I_B$) is decreased. |
| Early Voltage, $V_A$ | | | ✓ | Since $N_B << N_E$ typically, the width of the emitter junction depletion region is determined primarily by $N_B$. A modest increase in $N_E$ will not affect the depletion width (hence the quasi-neutral base width) significantly. |

c) **i)** Accurately sketch on the plots below the $I_B$-$V_{CE}$ and $I_C$-$V_{CE}$ characteristics (for $0V < V_{CE} < 5V$) of an NPN BJT operating at $T = 300K$ with $\boldsymbol{I_S = 5\times10^{-15}}$ **A**, $\boldsymbol{\beta = 200}$, and $\boldsymbol{V_A = 5V}$, biased at $\boldsymbol{V_{BE} = 0.72V}$. **[6 pts]**
Note that $e^{0.72/0.026} \cong 10^{12}$.




$$I_C = I_S e^{V_{BE}/V_T}\left(1+\frac{V_{CE}}{V_A}\right) = (5\times10^{-15}A)\,e^{0.72/0.026}\left(1+\frac{V_{CE}}{5V}\right)$$

$$= (5\times10^{-15})(10^{12})\left(1+\frac{V_{CE}}{5V}\right) = \boxed{5\times10^{-3}A\left(1+\frac{V_{CE}}{5V}\right)}$$

For $V_{CE} \simeq 0V$, $I_C = 5\,mA$ $\qquad I_B = I_C/\beta = 5mA/200 = \underline{25\mu A}$

For $V_{CE} = 5V$, $I_C = 10\,mA$

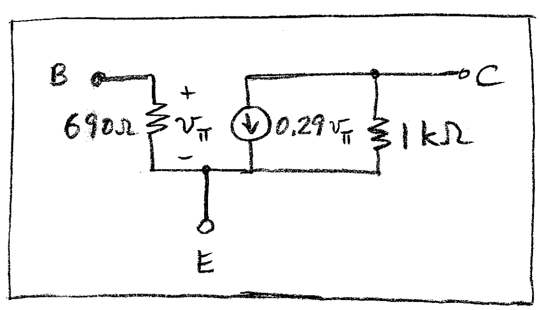
**ii)** Draw the small-signal model for this BJT, biased at $V_{BE} = 0.72V$ and $V_{CE} = 2.5V$. **[6 pts]**
Indicate numerical values for the small-signal parameters, and label the transistor terminals.
(Note: $r_o \equiv V_A/I_{C,nominal}$ where $I_{C,nominal}$ is the collector current for $V_{CE} << V_A$)

At $V_{CE} = 2.5V$, $I_C = 5\,mA\left(1+\frac{2.5V}{5V}\right) = \underline{7.5\,mA}$

$$g_m = \frac{I_C}{V_T} = \frac{7.5\times10^{-3}A}{0.026\,V} \cong 0.29\,S$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{0.29\,S} \cong 690\,\Omega$$

$$r_o = \frac{V_A}{I_{C,nominal}} = \frac{5V}{5\,mA} = 1000\,\Omega$$



**Problem 3 [30 points]: BJT Amplifiers**

**a)** Consider the BJT amplifier stage shown below, operating at $T = 300$K with a bias current $I_C = 0.1$**mA**.
Assume $I_S = 1\times10^{-16}$**A**, $\beta = 100$, $V_A =$ ~~5V~~ $\infty$

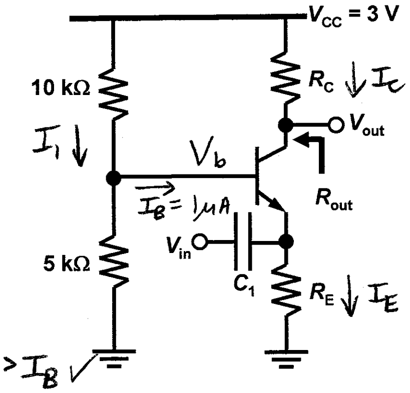
If $I_B << I_1$,

then $V_b = \frac{5}{10+5}(3V)$

$V_b = 1V$

Check assumption:

$I_1 = \frac{3V - 1V}{10k\Omega} = 200\mu A >> I_B$ ✓




$V_{CE} = V_{CC} - I_C R_C - I_E R_E$
$\simeq V_{CC} - I_C(R_C + R_E)$

**i)** What is the value of $R_E$? **[~~5~~ 6 pts]**

$I_C = I_S e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.026 \ln\left(\frac{10^{-4}}{10^{-16}}\right)$

$= 0.026 \ln(10^{12}) = 12 \times 0.026 \ln(10) = 0.72V$

$I_E = I_C + \frac{I_C}{\beta} \simeq I_C = 0.1mA$

$V_b = V_{BE} + I_E R_E \Rightarrow R_E = \frac{V_b - V_{BE}}{I_E} = \frac{1V - 0.72V}{0.1\times10^{-3}A} = 2.8\ k\Omega$

**ii)** For what range of $R_C$ values is the BJT operating in the active mode? **[4 pts]**

For the BJT to be in active mode, $V_{out} \geq V_b$:

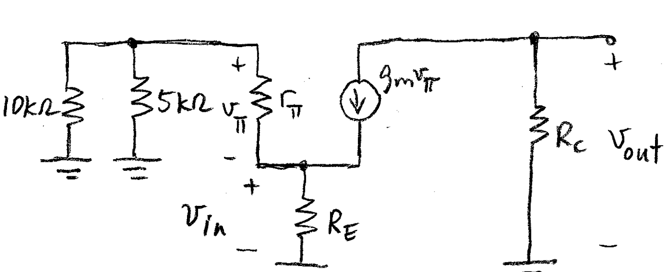
$V_{CC} - I_C R_C \geq V_b \Rightarrow R_C \leq \frac{V_{CC} - V_b}{I_C} = \frac{3V - 1V}{10^{-4}A} = 20k\Omega$

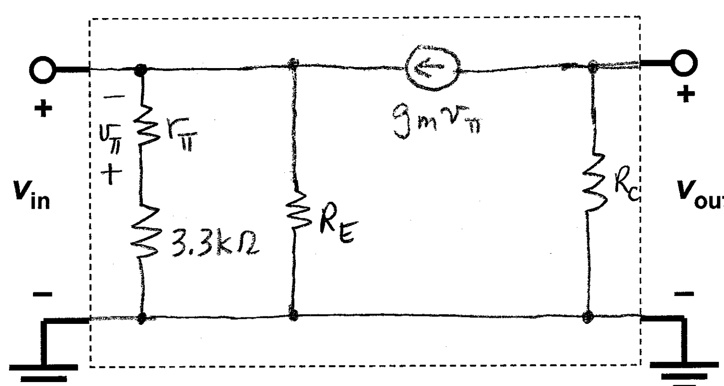
$R_C \leq 20\ k\Omega$

**iii)** Draw (in the box provided) the **most simplified circuit** that can be used for AC analysis to determine $A_v$, for $R_C$ = **10 kΩ.** $C_1$ is large, so that its impedance is negligible at the small-signal frequency of interest. Label ~~Indicate numerical values for~~ the various circuit elements, but **DO NOT SOLVE FOR $A_v$. [6 pts]**

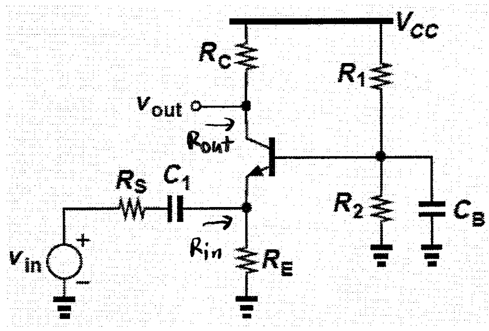
For AC analysis:

- Short $V_{cc}$ to GND
- Short $C_1$



**b)** Consider the circuit below:



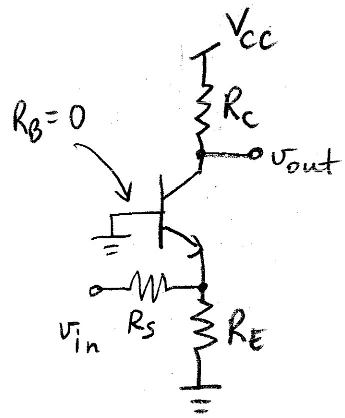

**i)** Is this a common emitter, common base, or emitter follower circuit? Justify your answer. **[2 pts]**

- Input signal is applied to the emitter.
- Output signal is taken from the collector.

$\Rightarrow$ common base topology

**ii)** Derive expressions for the voltage gain ($A_v$), input resistance ($R_{in}$), and output resistance ($R_{out}$). **[~~10~~ 8 pts]** You may assume that the capacitors $C_1$ and $C_B$ are large, so that their impedances are negligible at the small-signal frequency of interest. You may also neglect the Early effect (*i.e.* assume $V_A = \infty$).

With the capacitors shorted, the circuit becomes:




$$A_v = \frac{R_C}{\frac{1}{g_m} + R_S \| R_E} \cdot \frac{R_E}{R_S + R_E}$$

$$R_{in} = \frac{1}{g_m} \| R_E$$

$$R_{out} = R_C$$

**iii)** Describe ~~one of~~ the design trade-offs involved, when selecting the value of $R_C$. **[~~3~~ 4 pts]**

To achieve high voltage gain, $R_C$ should be large – but then a large $R_C$ results in larger $R_{out}$ (which is undesirable for an amplifier) and reduced headroom (limiting the output voltage swing).