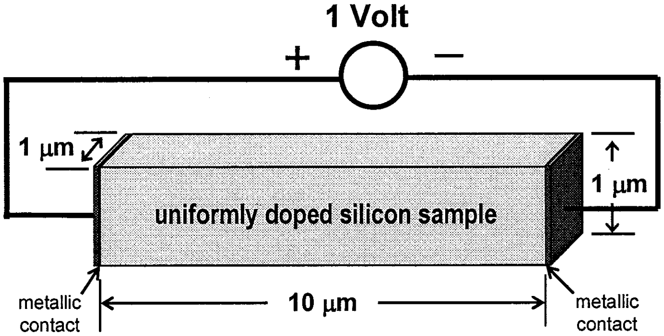
EE105 MIDTERM #1 SOLUTIONS

Fall 2008

Problem 1 [15 points]: Semiconductor Basics
Consider a Si sample of length 10 μm and cross-sectional area 1μm^2, uniformly doped with 10^{16} cm^{-3} boron, maintained at $T = 300$K. 1 Volt is applied across its length, as shown below:

↑ acceptor atom



a) Estimate the resistance of this sample. **[6 pts]**

$N_A = 10^{16}\,cm^{-3}$, $N_D = 0$

Since $N_A > N_D$, this sample is $\underline{\text{p-type}}$; $p = N_A - N_D = 10^{16}\,cm^{-3}$

$n = n_i^2/p = 10^{20}/10^{16} = 10^4\,cm^{-3}$

From plot on Page 1, $\mu_p \cong 450\,cm^2/V{\cdot}s$ and $\mu_n \approx 1200\,cm^2/V{\cdot}s$

$$\rho \cong \frac{1}{q\mu_p p} = \frac{1}{(1.6\times10^{-19})(450)(10^{16})} \cong 1.4\,\Omega\text{-cm}$$

$$R = \rho\frac{L}{A} = 1.4\,\frac{10\times10^{-4}}{(10^{-4})^2} = 1.4\times10^5\,\Omega = \boxed{140\,k\Omega}$$

b) Estimate the electron drift velocity. **[5 pts]**

$$\mathcal{E} = \frac{1V}{10\times10^{-4}cm} = 10^3\,V/cm$$

From plot on Page 1, $\mu_n \approx 1200\,cm^2/V{\cdot}s$

$$v_e = \mu_n\mathcal{E} = 1200\cdot10^3 = \boxed{1.2\times10^6\,cm/s}$$

c) Qualitatively (no calculations required), how would the resistivity of this sample change if it were to be **additionally doped uniformly with 2×10^{16} cm^{-3} phosphorus**? Explain briefly. **[4 pts]**

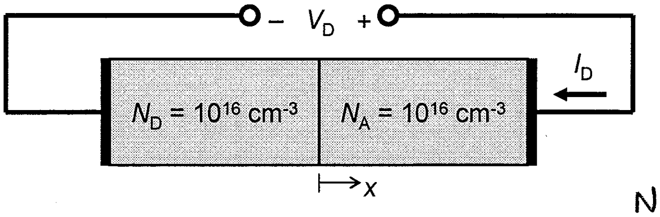
↑ donor atom

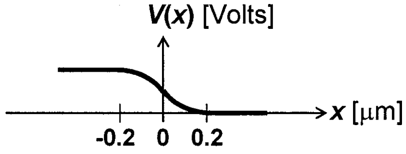
Since $N_D > N_A$, this sample is now n-type, with $n = N_D - N_A = 10^{16}\,cm^{-3}$

From the plot on Page 1, $\mu_n \approx 900\,cm^2/V{\cdot}s$, which is 2X greater than the hole mobility in the uncompensated sample.
Since the majority-carrier concentration is unchanged, and the majority-carrier mobility is doubled, $\underline{\text{the resistivity is halved}}$ (i.e. ρ is reduced by a factor of 2.)

Problem 2 **[15 points]: PN Junctions**

Consider a Si PN junction diode, maintained at $T = 300\text{K}$, with a structure and potential distribution as shown.





Note that the width of the depletion region is $0.4\,\mu m = 0.4 \times 10^{-4}\,cm$.

a) Calculate the built-in potential, V_0. **[4 pts]**

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026 \ln\left(\frac{10^{16} \cdot 10^{16}}{10^{20}}\right) = 0.026 \ln\left(10^{12}\right)$$

$$= 12 \cdot 0.026 \ln(10) = 12 \cdot 0.06 \approx \boxed{0.72\,V}$$

b) What is the applied voltage, V_D? Is this diode [✓ forward] or reverse biased? (Circle one.) **[7 pts]**

$$W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 - V_D)} = \sqrt{\frac{4\varepsilon_{si}}{qN}(V_0 - V_D)} \quad \text{where } N = 10^{16}\,cm^{-3}$$

$$\Rightarrow \quad V_0 - V_D = \frac{q W_{dep}^2 N}{4\varepsilon_{si}} = \frac{(1.6 \times 10^{-19})(0.4 \times 10^{-4})^2(10^{16})}{4 \times 10^{-12}} = 0.64\,V$$

$$V_D = V_0 - 0.64\,V = 0.72\,V - 0.64\,V = \boxed{0.08\,V}$$

Since $V_D > 0$, the diode is forward biased

c) Calculate the areal junction (depletion) capacitance. **[4 pts]**

$$C_{dep} = \frac{\varepsilon_{si}}{W_{dep}} = \frac{10^{-12}\,F/cm}{0.4 \times 10^{-4}\,cm} = \boxed{2.5 \times 10^{-8}\,F/cm^2}$$

Problem 3 [15 points]: Bipolar Junction Transistor Design

a) Why is the base region doped less heavily than the emitter region? **[3 pts]**

The ratio of carrier diffusion into the base (which determines I_C) to carrier diffusion into the emitter (which determines I_B) is proportional to N_E/N_B. Thus, to achieve large current gain $\beta \equiv I_C/I_B$, N_E should be much larger than N_B.

b) Why is the base region doped more heavily than the collector region? **[3 pts]**

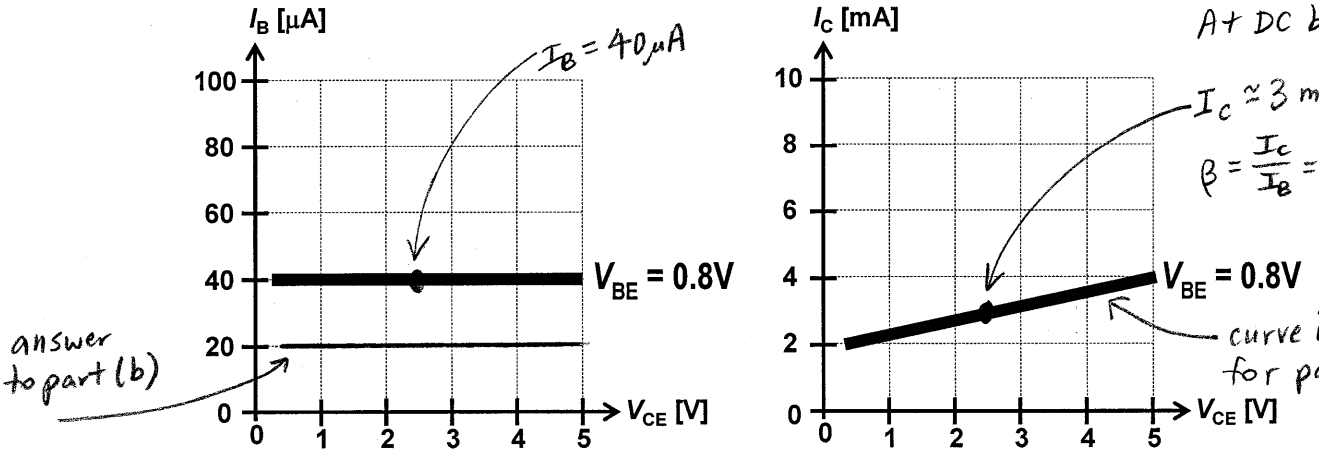
To minimize base-width modulation (i.e. to maximize the Early voltage V_A and hence the BJT output resistance r_o), the width of the collector-junction depletion region in the base should be minimized, by making N_B much greater than N_C so that most of the depletion region resides within the collector.

c) Indicate in the table below (by checking the appropriate box) how the BJT parameters would change, if the **base width were to be increased** (*e.g.* by 2×). Provide **qualitative reasoning** for your answers. **[9 pts]**

BJT Parameter	Parameter will			Brief Justification **(No equations or formulas!)**
	increase	decrease	not change significantly	
Reverse saturation current, I_S		✓		The minority-carrier concentration gradient would decrease, so that the minority-carrier diffusion current through the base (into the collector) would decrease.
Common-emitter DC current gain, β		✓		So long as the quasi-neutral base width is much shorter than the minority-carrier diffusion length, I_B is primarily supplying carrier diffusion into the emitter, which does not depend on W_B. I_C, due to carrier diffusion in the base, would decrease.
Early voltage, V_A	✓			The percentage change in quasi-neutral base width (W_B), for a given change in base-collector reverse bias, would be smaller, so base-width modulation would be decreased.

Problem 4 [15 points]: **Bipolar Junction Transistor *I-V* and Small-Signal Model**

Consider a BJT with the *I-V* characteristics as shown below.



At DC bias condition:

$I_C \simeq 3\text{ mA}$

$$\beta = \frac{I_C}{I_B} = \frac{3\text{ mA}}{40\mu A} = 75$$

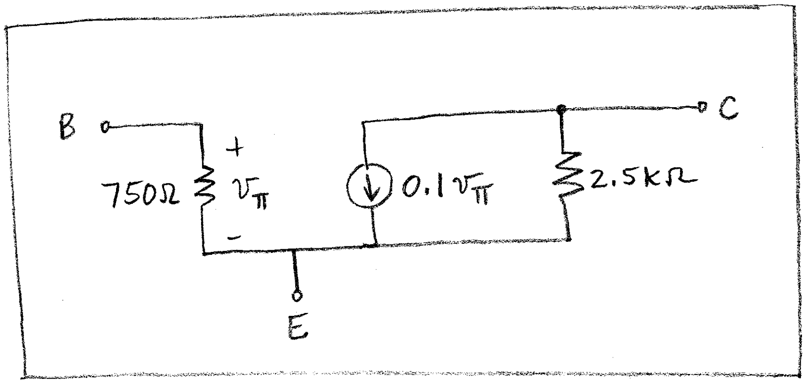
curve is unchanged for part (b)

a) Draw the small-signal model for the BJT for the DC bias condition $V_{BE} = 0.8\text{V}$, $V_{CE} = 2.5\text{V}$. **[12 pts]** (Indicate numerical values and units for r_π, g_m, and r_o, and label the transistor terminals.)

$$g_m = \frac{I_C}{V_T} = \frac{3\times10^{-3}\text{ A}}{0.026\text{ V}} \simeq 0.1\text{ S}$$

$$r_\pi = \frac{\beta}{g_m} \cong \frac{75}{0.1} = 750\,\Omega$$

$r_o \equiv$ the inverse slope of the I_c vs. V_{CE} characteristic $\simeq \frac{5\text{V}}{2\text{mA}} = 2.5\text{ k}\Omega$



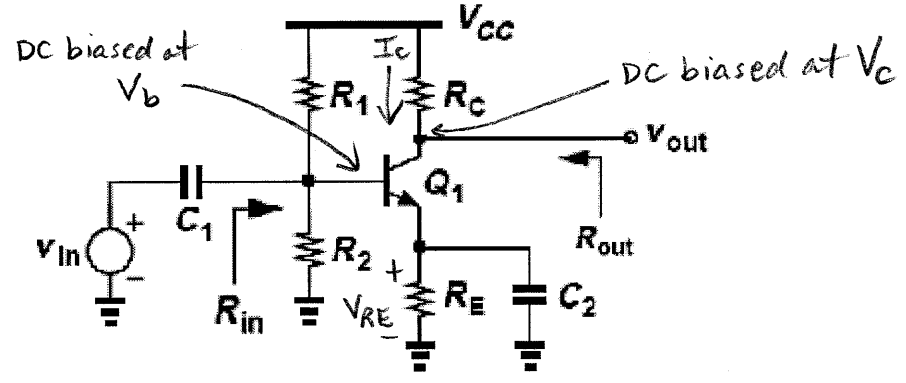
b) Show qualitatively (**by sketching curves on each of the plots above**) how the *I-V* characteristics would change **if the emitter dopant concentration were to be increased by a factor of 2. [3 pts]**

If N_E increases by 2X, $I_B \propto \frac{1}{N_E}$ would decrease by 2X.

I_c is not strongly dependent on N_E, so the I_c vs. V_{CE} characteristic would not be affected significantly.

Problem 5 **[20 points]: BJT Amplifier**

Consider the BJT amplifier stage shown below, operating at $T = 300K$ with a bias current $I_C = 0.1mA$. Assume $I_S = 1\times10^{-16}A$, $\beta = 100$, $V_A = \infty$. $R_C = 10\ k\Omega$, $R_E = 5\ k\Omega$, and $V_{CC} = 2.5V$. Note that $e^{0.72/0.026} \cong 10^{12}$.



a) What is the purpose of R_1 and R_2? **[2 pts]**

to establish the DC bias voltage for the base of the BJT.

b) What is the purpose of R_E? **[2 pts]**

to reduce the error in I_C (hence g_m, r_π) resulting from errors in the values of R_1 and R_2.

c) Is the BJT operating in the active mode? Justify your answer. **[5 pts]** ✓ $\boxed{\text{YES}}$

Since $I_C = 10^{12} I_S$, $V_{BE} = 0.72\,V > 0$ ✓

Since $I_E \simeq I_C = 0.1mA$, the voltage dropped across R_E is $(0.1mA)(5k\Omega) = 0.5V$

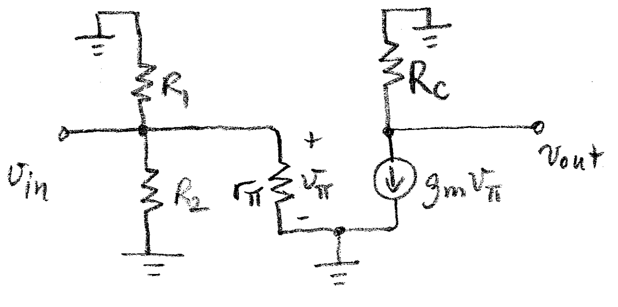
$\Rightarrow V_b = V_{RE} + V_{BE} = 0.5V + 0.72V = 1.22V$

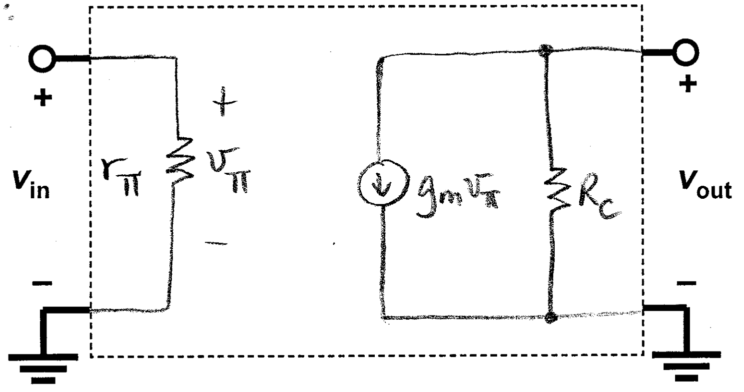
$V_c = V_{CC} - I_C R_C = 2.5 - (0.1mA)(10k\Omega) = 2.5V - 1V = 1.5V$

Since $V_c > V_b$, collector junction is reverse-biased. ✓

d) Draw (in the box provided) the **most simplified circuit** that can be used for AC analysis **to determine the small-signal voltage gain, A_v.** You can assume that C_1 and C_2 are large, so that their impedances are negligible at the small-signal frequency of interest. Label the various circuit elements. **[6 pts]**

Shorting out the capacitors and V_{CC}:





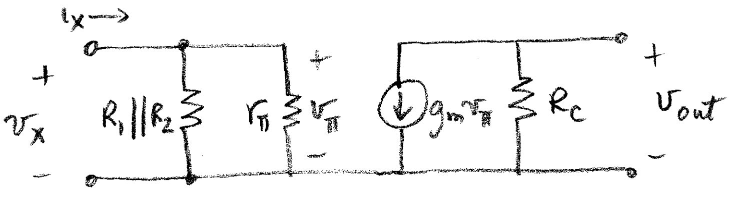
Problem 5 (continued)

e) Write expressions for the small-signal voltage gain (A_v), input resistance (R_{in}), output resistance (R_{out}). **[5 pts]**

$A_v = -g_m R_C$

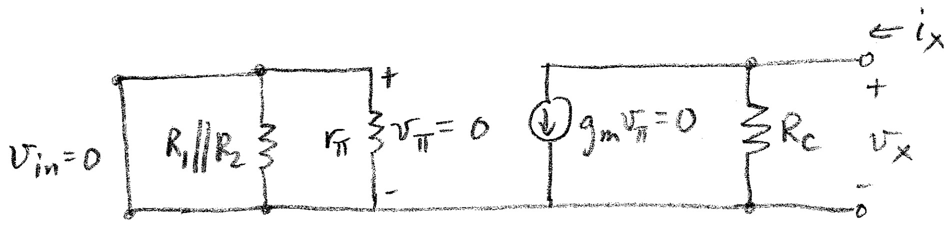
(from circuit in part (d))

Circuit for analysis of $R_{in} = \frac{v_x}{i_x}$:



$R_{in} = R_1 \| R_2 \| r_\pi$

Circuit for analysis of $R_{out} = \frac{v_x}{i_x}$:



$R_{out} = R_C$