Microelectronic Devices and Circuits- EECS105
Final Exam

Wednesday, December 8, 1999

Costas J. Spanos
University of California at Berkeley
College of Engineering
Department of Electrical Engineering and Computer Sciences

Your Name: Official Solutions
(last) (first)

Your Signature: ______________________

1. *Print and sign your name on this page before you start.*
2. *You are allowed three, 8.5"x11" handwritten sheets. No books or notes!*
3. *Do everything on this exam, and make your methods as clear as possible.*

Problem 1 ______ */ 24*
Problem 2 ______ */ 28*
Problem 3 ______ */ 24*
Problem 4 ______ */ 24*
TOTAL ______ */ 100*

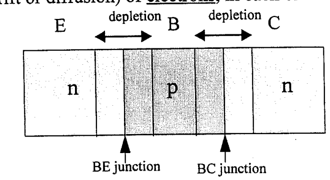
AVG = 69
$\hat{\sigma} = 17.3$

MOS Device Data (you may not have to use all of these...)
$\mu_n C_{ox} = 50\mu A/V^2$, $\mu_p C_{ox} = 25\mu A/V^2$, $V_{Tn} = -V_{Tp} = 1V$, Lmin = $2\mu m$. $V_{BS} = 0$.
$\lambda_n = \lambda_p = 0.1V^{-1}$ when L = $2\mu m$, and it is otherwise proportional to 1/L
$C_{ox} = 2.3fF/\mu m^2$, $C_{jn} = 0.1fF/\mu m^2$, $C_{jp} = 0.3fF/\mu m^2$, $C_{jswn} = 0.5fF/\mu m$,
$C_{jswp} = 0.35fF/\mu m$, $C_{ovn} = 0.5fF/\mu m$, $C_{ovp} = 0.5fF/\mu m$

npn Data $I_S = 10^{-17}A$, $\beta = 100$, $V_A = 25V$, $\tau_F = 50ps$, $C_{je} = 15fF$, $C_\mu = 10fF$
pnp Data $I_S = 10^{-17}A$, $\beta = 50$, $V_A = 25V$, $\tau_F = 50ps$, $C_{je} = 15fF$, $C_\mu = 10fF$

Problem 1 of 4: Answer each question briefly and clearly. (4 points each, total 24)

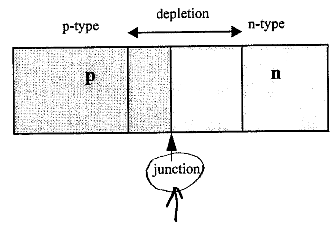
Mark in the table below the npn Bipolar Transistor in forward action mode the direction of flow, and the type of flow (drift or diffusion) of electrons, in each of the bulk and depletion regions.



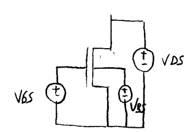
drift	■	√		√	√
diffusion		√	√	√	
→	■	√	√	√	√
←					

(place a mark at the appropriate box to indicate your answer. You can choose more than one box if appropriate.)

Where is the maximum |E| field in a forward-biased junction? Please place a mark on the graph below.



What happens to the drain current of an n-channel MOS transistor in saturation, when L and W increase proportionally? (i.e. L and W increase, but W/L stays constant. Assume that V_{GS}, V_{BS} and V_{DS} stay constant. Do take λ into account!)

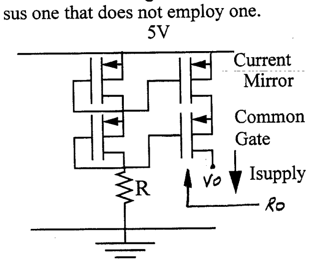


$I_D = \underbrace{\frac{1}{2}\mu_n C_{ox}(V_{GS}-V_T)^2 \cdot \frac{W}{L}}_{constant}(1+\lambda V_{DS})$

$\lambda \downarrow$ as $L \uparrow$

$\Rightarrow \underline{I_D \downarrow}$

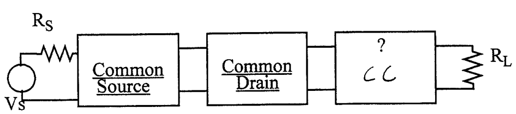
Name one advantage and one disadvantage of a MOS current source employing a CG buffer, versus one that does not employ one.



Advantage: High Ro, I supply depends on Vo less.

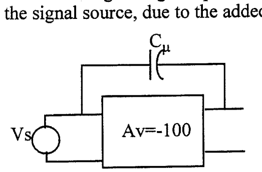
Disadvantage: Reduced Vo range

The following multistage amplifier is meant to deliver a voltage signal to a relatively small ohmic load of 1KΩ. Mark your choice of the last stage, and write a brief justification.



A CC has very low output resistance (less than a CD) and a high input resistance. It makes a good final stage voltage buffer

The following voltage amplifier has a voltage gain Av = -100. What is the capacitance "seen" by the signal source, due to the added capacitor C_μ=1pF as shown?



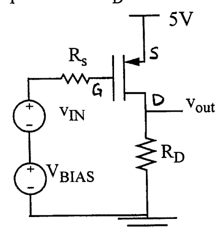
$C_{in} = C_\mu(1-A)$

$C_{in} = 1pF \cdot (1-(-100))$

$C_{in} = 101pF$

Problem 2 of 4 (28 points)

The following p-channel common source amplifier uses a rather primitive current supply: it is a simple resistor R_D=10kΩ tied between drain and ground. L = 2μm, $\lambda_p = 0.1V^{-1}$.



For each of the following questions, make sure that you show the expressions before you plug in the specific values. A correct expression is worth 70% of the credit, even if the numerical calculation is incorrect!

a) Find W/L so that when V_{BIAS}=3.5 and V_{in} = 0V, then Vout = 2.5V. Do take λ_p into account! Note that L=2μm. (4 points)

$I_D = \frac{2.5^V}{10k\Omega} = 250\mu A$

$I_D = \frac{1}{2} \cdot \frac{W}{L} \mu_p C_{ox} \cdot (V_{GS} - V_{Tp})^2 (1 + V_{DS} \cdot \lambda_p)$

$250\mu A = \frac{1}{2} \cdot 25\frac{\mu A}{V^2} \cdot (-1.5 - (-1))^2 (1 + 0.1 \cdot 2.5) \cdot \frac{W}{L}$

$\frac{2 \cdot 250\mu A}{25\mu A \cdot (0.5)^2 (1.25)} = \frac{W}{L} = 64$

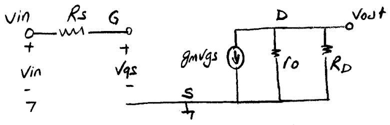
W/L = 128 / 2

b) What is the minimum and the maximum output voltage for this amplifier? Make sure you mention the limiting reason for each case (i.e. transistor X falls out of saturation, or current source Y hits its minimum voltage drop, etc.). (4 points)

MAX Vout = 4.5^V MAX Limited by: X falls out of saturation

MIN Vout = 0 MIN Limited by: GND

c) Draw the small signal model <u>for this amplifier</u>, and calculate the values of g_m and r_o <u>for this transistor</u> (4 points).



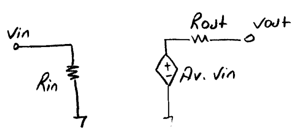
$g_m = \frac{2 \cdot I_D}{(V_{GS}+V_{Tp})} = \frac{1mA}{V}$

$r_o = \frac{1}{\lambda \cdot I_D} = \frac{1}{0.1 \times 250\mu A} = 40k\Omega$

g_m = 1mA/V

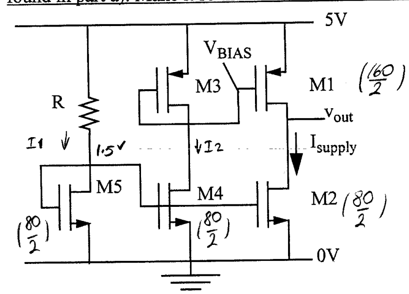
r_o = 40 kΩ

d) Draw the 2-port model for this circuit <u>as a voltage amplifier</u> and calculate the values of A_v, R_{in} and R_{out}. (4 points)



Parameter	Expression	Value
A_v	$-g_m \cdot (r_o // R_D)$	-8
R_{in}	∞	∞
R_{out}	$(r_o // R_D)$	8kΩ

e) In an attempt to increase the Av of this amp, we redesign it with a "real" biasing circuit as shown below. Calculate the value of the reference resistor and the W/L of the biasing transistors so that I_{supply} = 250μA and V_{BIAS} = 3.5V. <u>For this part do not take λ into account, and assume that the W/L of the amplifying transistor M1 is 160/2. (this is *not* the same value as the one you found in part a). Make it so that all three branches have the same current of 250μA.</u> (4 points)



$I_1 = I_2 = I_{supply} = 250\mu A$

⇒ Let all transistors have the same V_{GS}:

$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_{GS}-V_{Tn})^2 \stackrel{?}{=} \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (\;)^2$

$\Rightarrow \frac{\mu_n C_{ox}}{\mu_p C_{ox}} = \frac{(W/L)_P}{(W/L)_n} = 2$

⇒ make all NMOS transistors 80/2

Choose:

- $V_{GS5} = V_{GS1} = 1.5^V$ ⇒ $I_1 = \frac{5-1.5^V}{R} = 250\mu A$ → $\frac{3.5}{250\mu A} = R$

$14k\Omega = R$

Other choices for V_{GS_2} are possible, however, if you choose $V_{GS_2} > 1.5$, you limit voltage swing. If you choose $V_{GS_2} < 1.5$, M2 becomes too large, so cap. becomes a problem.

Transistor	W/L
M1	160/2
M2	80/2
M3	160/2
M4	80/2
M5	80/2
Resistor	**Value in Ω**
R	14000

f) What is the R_{in}, A_v and R_{out} of the new design? (4 points)

$$g_{m1} = \frac{2 \times 250\mu A}{0.5} = 1\frac{mA}{V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_n I_{supply}} = 40k\Omega \qquad \lambda_n = \lambda_p$$

The same as part c.

Parameter	Expression	Value
A_v	$-g_{m1}(r_{o1} \| r_{o2})$	-20
R_{in}	∞	∞
R_{out}	$(r_{o1} \| r_{o2})$	$20k\Omega$

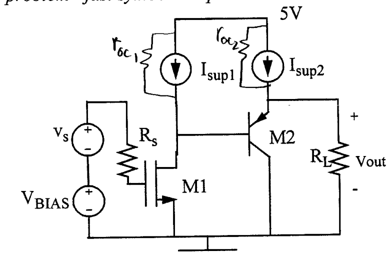
g) What is the minimum and the maximum output voltage of the new design? Make sure you mention the limiting reason for each case (i.e. transistor X falls out of saturation, or current source Y hits its minimum voltage drop, etc.). (4 points)

MAX V_{out} = 4.5 MAX Limited by: M_1 falls out of saturation

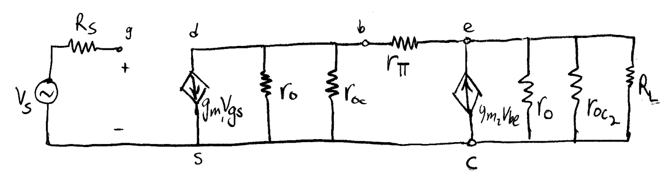
MIN V_{out} = 0.5 MIN Limited by: M_2 falls out of saturation

Problem 3/4 (24 points)

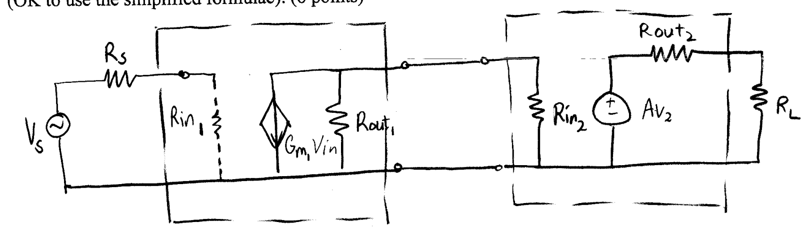
The following is a two stage voltage amplifier employing a n-channel CS stage, and a pnp CC stage. *Note that there are no numerical substitutions or calculations in parts a, b and c of this problem - just symbolic expressions!*



a) Draw the small signal model for this amplifier (Make sure to properly mark the g, s and d nodes for M1, and b, c, and e nodes for M2. Include r_{oc1}, r_{oc2} due to I_{sup1} and I_{sup2}, respectively). (6 points)

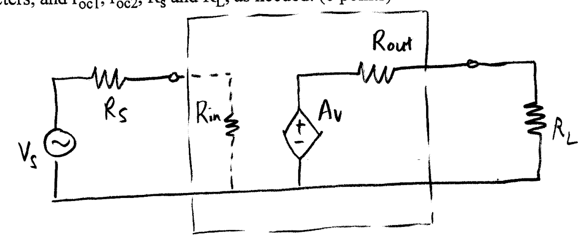


b) Draw the two stage amp 2-port model (i.e. draw one 2-port of each stage and connect them properly together. CS is a transconductance amp, CC is a voltage amp). Write the *expressions* for the quantities shown below, in terms of the device parameters, r_{oc1}, r_{oc2} and R_s and R_L as needed. (OK to use the simplified formulae). (6 points)



Parameter	Expression
Gm_1	g_{m_1}
R_{in1}	∞
R_{out1}	$r_{o_1} \| r_{oc_1}$
Av_2	1
R_{in2}	$r_\pi + \beta_o (r_{o2} \| r_{oc_2} \| R_L)$
R_{out2}	$\frac{1}{g_{m2}} + \frac{R_{out_1}}{\beta_o}$

c) Draw the overall voltage amp 2-port for the entire amp (i.e. draw one 2-port that represents the entire 2-stage amp), and derive *expressions* for the A_v, R_{in}, R_{out}, as well as v_{out} / v_s in terms of the device parameters, and r_{oc1}, r_{oc2}, R_s and R_L, as needed. (6 points)



$$A_v = -G_{m_1} (R_{out_1} \| R_{in_2})$$

$$\frac{V_{out}}{V_s} = A_v \cdot \frac{R_L}{R_L + R_{out_2}}$$

Parameter	Expression
R_{in}	∞
R_{out}	$\frac{1}{g_{m2}} + \frac{R_{out_1}}{\beta_o}$
A_v	$-g_{m_1}(r_{o_1} \| r_{oc_1} \| [r_\pi + \beta_o (r_{o2} \| r_{oc2} \| R_L)])$
v_{out}/v_s	$-g_{m_1}(r_{o_1} \| r_{oc_1} \| [r_\pi + \beta_o (r_{o2} \| r_{oc2} \| R_L)]) \cdot \frac{R_L}{R_L + \frac{1}{g_{m2}} + \frac{R_{out_1}}{\beta_o}}$

d) Assume that V_{BIAS} = 1.5V, and that the minimum voltage across the current sources is 0.5V. Find the maximum and minimum voltages at the drain of M1 and at the emitter of M2. Make sure you mention the limiting reason for each case (i.e. transistor X falls out of saturation, or current source Y hits its minimum voltage drop, etc.) (6 points)

- $V_{dmin} = V_{GS} - V_T = 1.5V - 1V = \boxed{0.5V}$ $\therefore$ $V_{emin} = 0.5V + 0.7V = \boxed{1.2V}$
- $V_{d\,max} \neq 4.5V$ because $V_e = 4.5V + 0.7V = 5.2V > V_{DD}$

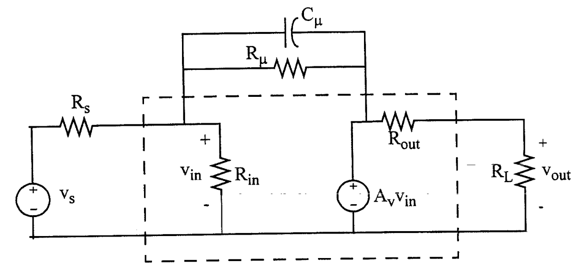
  $V_{e\,max} = \boxed{4.5V}$ $\therefore$ $V_{dmax} = 4.5V - 0.7V = \boxed{3.8V}$

Node	Min Voltage	Reason for Min Voltage	Max Voltage	Reason for Max Voltage
drain of M1	0.5V	V_{Dsat} of M1	3.8V	Min Voltage across I_{SUP2} & V_{EB} of M2
emitter of M2	1.2V	V_{Dsat} of M1 & V_{EB} of M2	4.5V	Min voltage across I_{SUP2}

Problem 4/4 (24 points)

The following is the 2-port representation of a voltage amplifier, where the "Miller" elements C_μ and R_μ have been added as shown.

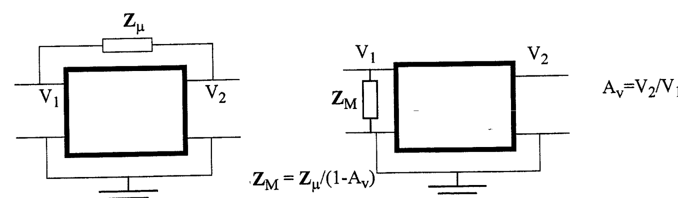
A_v = -100, R_s = 5kΩ, R_{in} = 5kΩ, R_{out} = 5kΩ, R_L = 5kΩ, C_μ = 0.4pF, R_μ = 100kΩ.



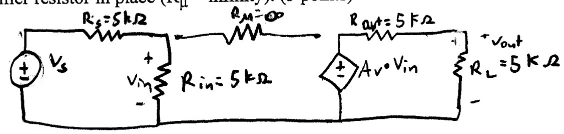
Miller Approximation Refresher:

Note that the Miller approximation applies to *any kind of complex impedance* connected between the nodes that exhibit voltage gain A_v. In general, $\mathbf{Z_M} = \mathbf{Z_\mu}/(1-A_v)$. (Here a **bold** symbol indicates a complex number). As you know, for capacitors, $\mathbf{Z_\mu} = 1/j\omega C_\mu$, so it turns out that $C_M = C_\mu(1-A_v)$. Below you will be asked to apply the Miller approximation for capacitors, as well as for resistors....

Miller approximation refresher: these two circuits are almost equivalent.



a) For the voltage amplifier shown in the previous page, find the overall DC gain (v_{out}/v_s) with no Miller resistor in place (R_μ = infinity). (5 points)



$$V_{out} = A_v \cdot V_{in} \cdot \frac{R_L}{R_L + R_{out}} \qquad V_{in} = V_s \cdot \frac{R_{in}}{R_s + R_{in}}$$

$$V_{out} = A_v \cdot V_s \cdot \frac{R_{in}}{R_{in} + R_s} \cdot \frac{R_L}{R_L + R_{out}}$$

$$\boxed{\frac{V_{out}}{V_s} = \frac{A_v \cdot R_{in} \cdot R_L}{(R_{in}+R_s)(R_L+R_{out})} = \frac{-100 \cdot 5k\Omega \cdot 5k\Omega}{(10k\Omega)(10k\Omega)} = -25}$$

b) Find the overall DC gain expression and evaluate it when $R_\mu = 100k\Omega$. (5 points)

$$R_\mu = 100k\Omega \qquad R_M = \frac{100k\Omega}{(1+100)} \approx 1k\Omega$$

$$R_{in} \| R_m = \frac{5k\Omega \cdot 1k\Omega}{6k\Omega} = 0.83\ k\Omega$$

$$\frac{V_{out}}{V_s} = \frac{A_v (R_{in}\|R_m)\ R_L}{(R_{in}\|R_m + R_s)(R_L + R_{out})} = \frac{-100 \cdot (0.83k\Omega)(5k\Omega)}{5.83k\Omega \cdot 10k\Omega} = -7.143$$

	Expression	Value for $R_\mu = 100k\Omega$
v_{out}/v_s	$\frac{A_v \cdot (R_{in}\|R_m) \cdot R_L}{(R_{in}\|R_m + R_s)(R_L + R_{out})}$ $R_m = R_\mu/(1-A_v)$	-7.143

c) Find the ω_{3db} of $|v_{out}/v_s|$ when R_μ is infinity and when R_μ is 100kΩ. *Hint: apply the Miller approximation to the resistor and the capacitor separately. It is not necessary to solve this considering the complex impedance of the resistor and capacitor taken together.* (5 points)

$$C_\mu = 0.4\ pF \qquad C_m = 101 \cdot C_\mu \approx 40\ pF$$

for $R_\mu = \infty$

$$R = R_s \| R_{in} = 2.5k\Omega$$

$$R \cdot C_m = 10^{-7}\ sec \Rightarrow \omega_{3db} = \frac{1 \cdot Rad}{10^{-7}\ sec} = 10\ Mrad/sec$$

for $R_\mu = 100k\Omega$ $\quad R_m = 1k\Omega$

$$R = R_s \| R_{in} \| R_m = \frac{2.5k\Omega \cdot 1k\Omega}{3.5k\Omega} = 0.714$$

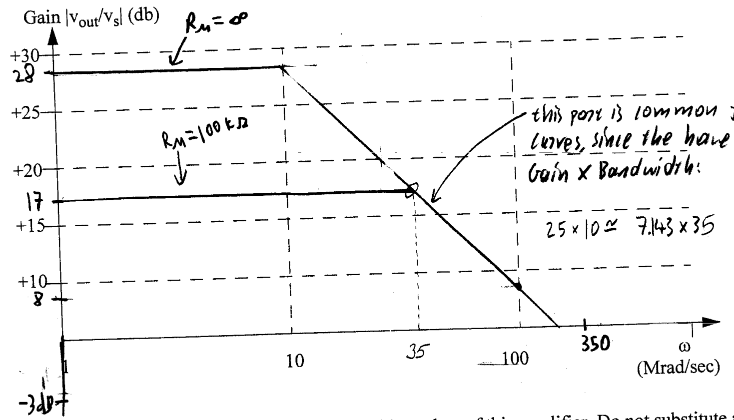
$$R \cdot C_m = 2.86 \times 10^{-8}\ sec \Rightarrow \omega_{3dB} = \frac{1 \cdot Rad}{2.86 \times 10^{-8}\ sec} = 35\ \frac{Mrad}{sec}$$

	Expression	Value in Mrad/sec
ω_{3db} for R_μ infinity	$\frac{1}{(R_s \| R_{in}) \cdot C_m}$ $C_m = C_\mu(1-A_v)$ $R_m = R_\mu(1-A_v)$	10
ω_{3db} for R_μ = 100kΩ	$\frac{1}{(R_s \| R_{in} \| R_m)\ C_m}$	35

d) Draw Bode plots on the same graph for $|v_{out}/v_s|$ when R_μ infinity and R_μ =100kΩ. (5 points)

$\boxed{20\cdot\log\left|\frac{V_{out}}{V_s}\right|_{DC,\infty} = 28\ dB}$ for $R_M = \infty$

$\boxed{20\cdot\log\left|\frac{V_{out}}{V_s}\right|_{DC,100k} \approx 17\ dB}$ for $R_M = 100\ k\Omega$



this point is common for both curves, since the have the same Gain x Bandwidth:

$25\times 10 \simeq 7.143\times 35$

e) Derive an expression for the Gain X Bandwidth product of this amplifier. Do not substitute any values. Simplify this expression assuming that $|A_v|>>1$, $R_\mu/|A_v|<<R_s$ and $R_\mu/|A_v|<<R_{in}$. (4 points)

$$Gain = \frac{R_{in}||(R_\mu/(1-A_v))}{R_s + R_{in}||(R_\mu/(1-A_v))} \cdot \frac{A_v\cdot R_L}{R_{out}+R_L}$$

$$BW = \frac{1}{C_M\cdot\left(\frac{R_\mu}{-A_v}||R_s||R_{in}\right)}$$

$$BW = \frac{1}{C_\mu(1-A_v)\left(\left(\frac{R_M}{A_v}\right)||R_s||R_{in}\right)}$$

now apply assumptions: $R_{in}||\frac{R_M}{(1-A_v)} \approx -\frac{R_M}{A_v}$; $1-A_v \approx -A_v$

$$Gain = \frac{-R_M/\cancel{A_v}}{R_s}\cdot\frac{\cancel{A_v}\cdot R_L}{R_L+R_{out}}$$

$$\boxed{BW = \frac{\cancel{A_v}}{\cancel{A_v}\cdot C_M\cdot R_M} = \frac{1}{C_\mu R_\mu}}$$

$$\boxed{Gain = \frac{-R_\mu\cdot R_L}{R_s(R_L+R_{out})}}$$

$$Gain\cdot BW = \frac{-\cancel{R_M}\cdot R_L}{R_s(R_L+R_{out})}\cdot\frac{1}{C_M\cancel{R_M}}$$

~ That's All Folks! ~

$$= \boxed{-\frac{R_L}{R_s(R_L+R_{out})C_\mu}}$$