08|01

Sample Final (Spring 2000 Final) (Solutions)

EE105

a) $|I_{D3}| = 50\mu A = \underbrace{\mu_p C_{ox}}_{25\mu A/V^2} \underbrace{(W/2L)_3}_{8} (V_{SG3} + \underbrace{V_{Tp}}_{-1V = V_{Tp0}})^2$

Solving for V_{SG3}, $V_{SG3} = 1.5V$ $\Rightarrow$ $\boxed{V_{BATT3} = 2.5 - 1.5 = 1V}$

$I_{D4} = 100\mu A = \mu_n C_{ox} (W/2L)_4 (V_{GS4} - V_{Tn})^2$

Solving for V_{GS4}, $V_{GS4} = 1.5V$ $\Rightarrow$ $\boxed{V_{BATT4} = -2.5V + 1.5V = -1V}$

b) $\overset{0V}{\cancel{V_{OUT}}} = \overset{0V}{\cancel{V_{IN}}} + V_{SG1} - V_{SG2} \Rightarrow V_{SG2} = V_{SG1}$

$V_{SG1} = -\underbrace{V_{Tp}}_{V_{T0p}} + \left[\dfrac{2|I_{D1}|}{\mu_p C_{ox}(W/L)_1}\right]^{1/2} = 1V + \left[\dfrac{2(50)}{25(16)}\right]^{1/2} = 1.5V = V_{GS2}$

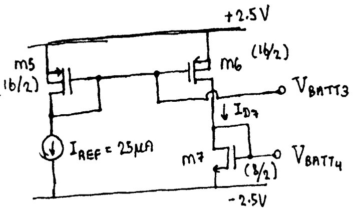
$V_{GS2} = V_{Tn} + \sqrt{\dfrac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$ where $V_{Tn} = V_{T0n} + \gamma_n\left(\sqrt{V_{SB} - 2\Phi_p} - \sqrt{-2\Phi_p}\right)$ (back-gate effect)

$= 1V + (0.3)\left[\sqrt{\underbrace{0-(-2.5)}_{V_{SB}}-(-1)} - \sqrt{-(-1)}\right]$

$= 1.261V$

$(W/L)_2 = \dfrac{2I_{D2}}{\mu_n C_{ox}(V_{GS2} - V_{Tn})^2} = \dfrac{2(100\mu A)}{(50\mu A/V^2)(1.5-1.261)^2 V^2} = 70.2 \Rightarrow \boxed{W_2 = 140.3\ \mu m}$

c)



We want $V_{BATT3} = 1V$ and $V_{BATT4} = -1V$ as found in part ⓐ. One way to solve for the size of m5 is to use what we know in part a. In part a, $I_{REF} = 50\mu A$ and in order to have the same $\underbrace{V_{BATT3}}$, $(W/L)_5 = \frac{1}{2}(W/L)_3 = 16/2$.
↳ which leads to the same V_{SG}

We then make m6 the same size as m5 for convenience. We then know $|I_{D6}| = 25\mu A = I_{D7}$. [100µA vs. 25µA]

$V_{BATT4} = -1V \Rightarrow$ Same V_{GS} as in part ⓐ but I_D now scaled by $\frac{1}{4}$!

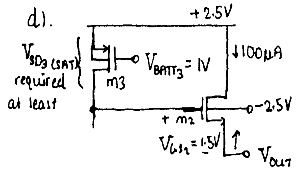
$\therefore (W/L)_7 = \frac{1}{4}(W/L)_4 = 8/2$.

*) Combinations of the following will work too.

1) $(W/L)_5 = (16/2)$, $(W/L)_6 = (64/2)$, $(W/L)_7 = (32/2)$

2) $(W/L)_5 = (16/2)$, $(W/L)_6 = (8/2)$, $(W/L)_7 = (4/2)$

d).



$V_{SD3(SAT)} = 1.5V - 1V = 0.5V$

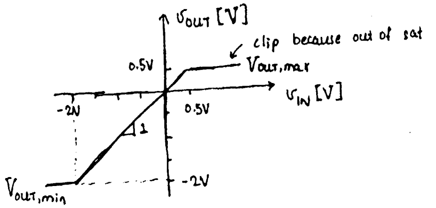
$V_{OUT,max} = \underbrace{(2.5V - 0.5V)}_{V_{G2,max}} - \underbrace{1.5V}_{V_{GS2}\text{ required to keep } I_{D2} \text{ at } 100\mu A}$

$\boxed{\therefore V_{OUT,max} = 0.5V}$

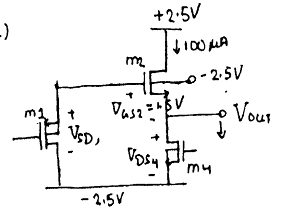
f) $V_{OUT} = v_{IN} + \cancel{V_{SG1}} - \cancel{V_{GS2}} \approx v_{IN}$

Looks like CD-CD amp. V. Buffer.

Remember the max & min output voltage.



e)



The 1st thing we need to check for $V_{OUT,min}$ is whether we keep m4 in saturation.

$V_{DS4(SAT)} = V_{GS4} - V_{T0n} = 1.5 - 1 = 0.5V$

$\therefore V_{OUT,min} = V_{DS4(SAT)} - 2.5V = -2V$. ✓

But this may be too low for the transistors on the left side to be in saturation. Let's check m1.

$V_{G2,min} = V_{S1,min} = -2.5V + V_{SD1(SAT)}$

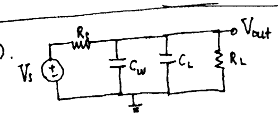
$= -2.5 + 1.5 - 1 = -2V$

Again we need $V_{GS2} = 1.5V$ to keep I_{D2} @ 100

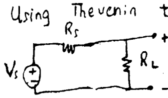
$\therefore V_{OUT,min}$ in this case is $V_{G2,min} - 1.5V$
$= -3.5V$. ×

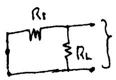
$\boxed{\therefore V_{OUT,min} = -2V}$

② a).



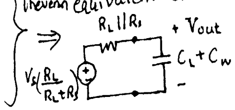
$\Rightarrow$ Using Thevenin technique,





$R_{th} = R_s \| R_L$

Thevenin equivalent circuit



Standard one-pole eq:

$\boxed{\dfrac{V_{out}}{V_s} = \left(\dfrac{R_L}{R_L+R_s}\right)\left(\dfrac{1}{1 + j\omega/\omega_{-3dB}}\right)}$

$\omega_{-3dB} = \dfrac{1}{(R_L \| R_s)(C_W + C_L)}$

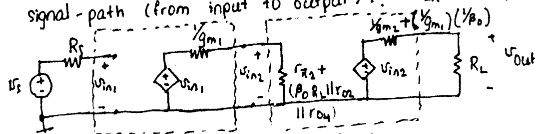
b). $\left|\dfrac{V_{out}}{V_s}\right|_{\omega \to 0} = A_{v_{DC}} = \dfrac{R_L}{R_s + R_L} = \dfrac{10}{10+50} = \dfrac{1}{6}$

$\omega_{-3dB} = \dfrac{1}{(10k\Omega \| 50k\Omega)(100+500)fF} = 200\ M\ rad/sec = 2\pi\,(31.8 MHz)$

$\angle V_{out}/V_s = -\tan^{-1}\left(\dfrac{\omega}{\omega_{-3dB}}\right) = -\tan^{-1}\left[\dfrac{2\pi(25MHz)}{2\pi(31.8MHz)}\right] = -38^\circ$

$\boxed{\therefore v_{out}(t) = \underbrace{\dfrac{25mV}{6}}_{4.2mV} \cos\left[2\pi(25MHz)\,t - 38^\circ\right]}$

2c) Both stages look CD-ish and CC-ish respectively. Trace the transistors in the signal-path (from input to output)!! In this problem, they are m1 and Q2.



2d) $\frac{v_{out}}{v_s} = \left[\frac{r_{\pi_2} + \beta_0 R_L}{\underbrace{1/g_{m_1} + r_{\pi_2} + \beta_0 R_L}_{\approx 1}} \right] \left[\frac{R_L}{(1/g_{m_2} + \frac{1}{\beta_0 g_{m_1}}) + R_L} \right] \approx 1 \left[\frac{10k\Omega}{(250\Omega + 50\Omega) + 10k\Omega} \right] \approx 0.966 \approx 1$

$R_L \ll r_{o_2}, r_{o_4}$

$g_{m_1} = \sqrt{2\mu_p C_{ox} (W/L)_1 |I_{D_1}|}$

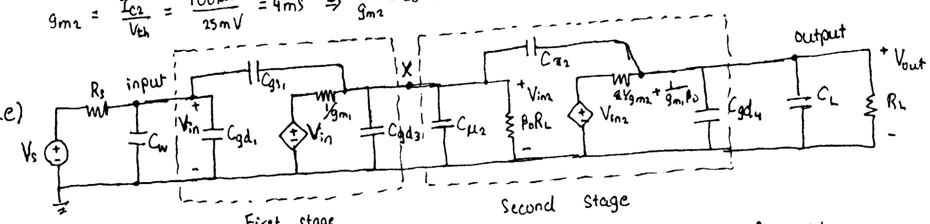
$= \sqrt{2(25)(16)(50)}$

$= 200\mu S$

$1/g_{m_1} = 5000\Omega \ll \beta_0 R_L = (100)(10k\Omega) = 1M\Omega$

$g_{m_2} = \frac{I_{C2}}{V_{th}} = \frac{100\mu A}{25mV} = 4mS \Rightarrow \frac{1}{g_{m_2}} = 250\Omega$

2e)



2f) $\tau_{IN} = R_s \{ C_w + C_{gd_1} + (1 - A_{v_{Cgs_1}}) C_{gs_1} \}$

$C_{gd_1} = (0.4)(32) = 12.8fF$ (C_{ov})

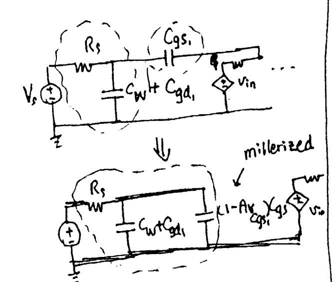
$C_{gs_1} = \frac{2}{3} C_{ox} W_1 L_1 + C_{ov} W_1 = 98.1fF$

$A_{v_{Cgs_1}} = A_{v_1} = \frac{R_{incc}}{R_{incc} + \underbrace{1/g_{m_1}}_{5k\Omega}}$ where $R_{incc} = r_{\pi_2} + \beta_0 (R_L || r_{o_2} || r_{o_4})$

$= 25k\Omega + 100(10k\Omega || 500k\Omega || 200k\Omega)$

$\approx 960k\Omega$

$= \frac{960k\Omega}{965k\Omega} \approx 0.994$



$\therefore \tau_{IN} = (50k\Omega) \{ 100fF + 12.8fF + (1 - 0.994)(98.1fF) \}$

$= (50k\Omega)(113.4fF)$

$\boxed{\therefore \tau_{IN} \approx 5.67ns}$

2g) $\tau_X = (\frac{1}{g_{m_1}}) \{ C_{gd_3} + C_{\mu_2} + (\overbrace{1 - A_{v_2}}^{0.0029}) C_{\pi_2} \} = (5k\Omega)(12.8fF + 25fF + 5.95fF) \Rightarrow \boxed{\tau_X = 0.219ns}$

$A_{v_2} = \frac{R_L}{(\frac{1}{g_{m_2}} + \frac{1}{\beta_0 g_{m_1}}) + R_L} = \frac{10k\Omega}{300\Omega + 10k\Omega} = 0.971$

$C_{\pi_2} = C_{jE} + g_{m_2}\tau_F = 25fF + (4\times10^{-3}S)(45\times10^{-12}s) = 205fF$

$C_{\mu_2} = 25fF$

$C_{gd3} = C_{gd_1} = 12.8fF$

h) We still need to solve time-constant at output node.

$\tau_{OUT} = [\underbrace{R_L}_{10k\Omega} || \underbrace{(\frac{1}{g_{m_2}} + \frac{1}{\beta_0 g_{m_1}})}_{300\Omega}] (\underset{500fF}{C_L} + \underset{12.8fF}{C_{gd_4}}) \approx 0.149ns$

$\therefore \omega_{-3dB} = \frac{1}{\tau_{IN} + \tau_X + \tau_{OUT}} = \frac{1}{5.67 + 0.219 + 0.149} \times 10^9 \text{ rad/sec} \approx 1.66 \times 10^8 \text{ rad/sec}$

$\boxed{\therefore f_{-3dB} = 26.4 MHz}$

3 a) $I_A = 40\mu A \frac{(W/L)_2}{(W/L)_1} = 40\mu A (\frac{8}{4}) = \boxed{80\mu A}$

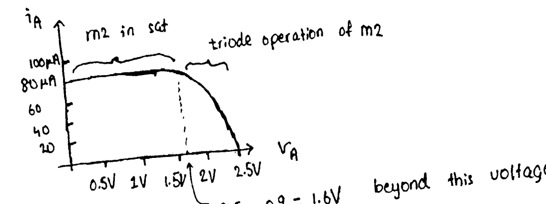
b). $r_{oc} = r_{o_2} = \frac{1}{\lambda_{p_2} |I_{D_2}|}$ where $\lambda_{p_2} = \frac{0.1}{L_2 [\mu m]}$ $\Rightarrow \therefore L_2 = (5\times10^5\Omega)(8\times10^{-5}A)(0.1)$

$\boxed{\therefore L_2 = 4\mu m}$

c) $I_A \approx 80\mu A$ when m2 is saturated.

Once m2 goes out of saturation, I_A drops dramatically.

$V_{SD_2 SAT} = V_{SG_2} + V_{Tp} = \sqrt{\frac{2|I_{D_2}|}{\mu_p C_{ox} (W/L)_2}} = \sqrt{\frac{2(80)}{25(8)}} \Rightarrow V_{SD_2 SAT} = 0.9V.$



$2.5 - 0.9 = 1.6V$ beyond this voltage, m2 goes out of saturation.

d) $V_A = 1.25V$ ensures that MOSFET is saturated.

$C_a = C_{db_2} + C_{gd_2}$

$C_{db_2} = \frac{C_{j0} W_2 L_{diff}}{\sqrt{1 - (V_{DB}/\phi_{Bp})}} = \frac{(0.2fF/\mu m^2)(32)(6)\mu m^2}{\sqrt{1 - (-1.25V)/V}} = 25.6fF$

$C_{gd_2} = C_{ov} \cdot W_2 = (0.4fF/\mu m)(32\mu m) = 12.8fF$

$+ \Rightarrow \boxed{C_a = 38.4fF}$

3\. e) $Z_{oc} = r_{oc} \,\|\, \frac{1}{j\omega C_a} = \frac{1}{\frac{1}{r_{oc}} + j\omega C_a} = \frac{r_{oc}}{1 + j\omega C_a r_{oc}}$

$|Z_{oc}| = \frac{\overset{\leftarrow 500k\Omega}{r_{oc}}}{\sqrt{1+(\omega C_a r_{oc})^2}} = 300k\Omega \Rightarrow 1 + (\underset{2\pi f}{\omega} \underset{38.4fF}{C_a} \underset{500k\Omega}{r_{oc}})^2 = \left(\frac{500}{300}\right)^2 \Rightarrow \boxed{f = 11\text{ MHz}}$

f) $V_{A,max} = V^+ - \underbrace{V_{SG4}}_{1.9} - \underbrace{V_{SD2\,SAT}}_{0.9}$ where $V_{SG4} = -V_{Tp} + \sqrt{\frac{2|I_{D4}|}{\mu_p C_{ox}(W/L)_4}} = 1 + \sqrt{\frac{2(80)}{25(8)}} = 1.9V$

$= 5 - 1.9 - 0.9$

$\boxed{V_{A,max} = 2.2V}$

g) $r_{oc} = r_{o2}(1 + g_{m2} R_{s2}) = 250k\Omega\,[1 + (179\times10^{-6})(3.74\times10^3)] = 417k\Omega \Rightarrow \boxed{r_{oc} = 417k\Omega}$

$R_{s2} = \frac{1}{g_{m4}} \,\|\, \frac{1}{g_{m3}} = 5.6k\Omega \,\|\, 11.2k\Omega = 3.74k\Omega$

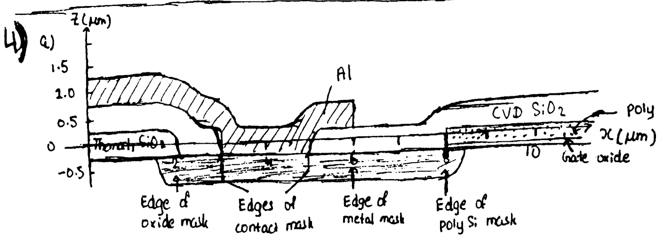
$g_{m4} = \sqrt{2\mu_p C_{ox}(W/L)_4 |I_{D4}|} = \sqrt{2(25)(8)(80)} = 179\mu S$

$g_{m3} = \sqrt{2\mu_p C_{ox}(W/L)_3 |I_{D3}|} = \sqrt{2(25)(4)(40)} = 89\mu S$

$r_{o2} = \frac{1}{\lambda |I_{D2}|} = \frac{1}{(0.05/V)(80\mu A)} = 250k\Omega$

$g_{m2} = g_{m4}$

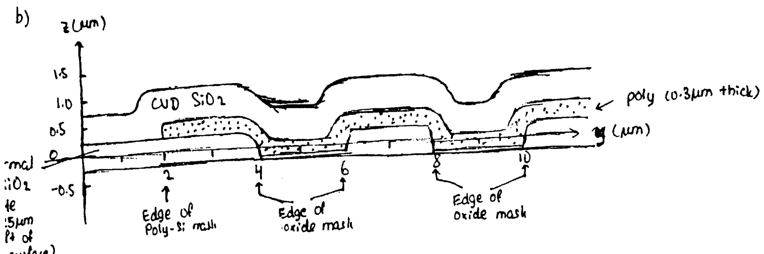
4\) a)



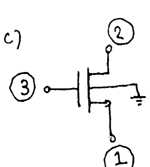
$n = \underset{\nearrow\text{ electrons}}{}\ \bar{N}_d - N_a = \left[\frac{10^{14}}{5\times10^{-5}} - 10^{17}\right] cm^{-3}$ ($\swarrow \frac{Q_d}{x_j}$)

$\boxed{n \approx 1.9\times10^{18}\,cm^{-3}}$

b)



c)



$V_1 = 0V$
$V_2 = 0.01V$
$V_3 = 1.25V$

Operation $\Rightarrow$ Triode

Channel width $\Rightarrow$ 2µm

Channel Length $\Rightarrow$ 27µm + 4(3) + 24µm (2) + 27µm = 114µm
↖ length of center line of channel

Note that the device is N-type and the source is at the lower potential than the drain.

4 d). Triode region with $\underbrace{V_{DS}}_{10mV} \ll \underbrace{V_{GS} - V_{Tn}}_{250mV}$

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-13}}{1.5\times10^{-6}} (F/cm^2) \left(\frac{10^{-8} cm^2}{\mu m^2}\right)$

$C_{ox} = 2.3\ fF/\mu m^2$

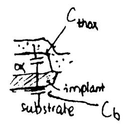
$I_D = \mu_n C_{ox}(W/L)(V_{GS} - V_{Tn} - \overset{small}{\cancel{\frac{V_{DS}}{2}}})V_{DS}$

$\approx (325\frac{cm^2}{Vs})(2.3\times10^{-15}F/\mu m^2)(10^8 \mu m^2/cm^2)(\frac{2}{114})(1.25-1)(0.01)$

$\boxed{I_D \approx 32.8\,nA}$

e). $C_3 = C_{gb} + \overbrace{\tfrac{1}{2}C_{ox} WL + C_{ov}\cdot W}^{C_{gs}\text{ in triode}}$ where $C_{ov} = C_{ox}\cdot(L_D)$ ↙ 0.5µm ↙ see ion-implantation junction depth (0.5µm in lateral direction too)

$= (2.3fF/\mu m^2)(0.5\mu m) = 1.15fF/\mu m$



$C_{gb} = C_{thox}$ in series with C_b where $C_{thox} = \frac{\epsilon_{ox}}{t_{thox}} = \frac{3.45\times10^{-13}}{5\times10^{-5}} (F/cm^2) = 69\ aF/\mu m^2$

$= \frac{(0.069)(5.175)}{0.069 + 5.175} (A_{gb})$ $\qquad C_b = \frac{\epsilon_s}{X_d} = \frac{1.035\times10^{-12}}{2\times10^{-6}} F/cm^2 = 5.175 fF/\mu m^2$ (X_d ↗ 200Å)

$C_{gb} = 0.068\ fF/\mu m^2\ (336\mu m^2) = \underline{22.8fF}$

$A_{gb} = \underbrace{(30\times18 + 4\times4)}_{\text{total poly area}} - \underbrace{((28\times2)\times2 + (24\times2)\times2 + (2\times2)\times3)}_{\text{Channel area}} = 556\mu m^2 - 220\mu m^2 = 336\mu m^2$

$C_{gs} = \tfrac{1}{2}(2.3)(220) + (1.15\times2) = \underline{255.3fF}$

$\therefore C_3 = C_{gb} + C_{gs} = 22.8fF + 255.3fF = \boxed{278fF}$

Good Luck on the Final !