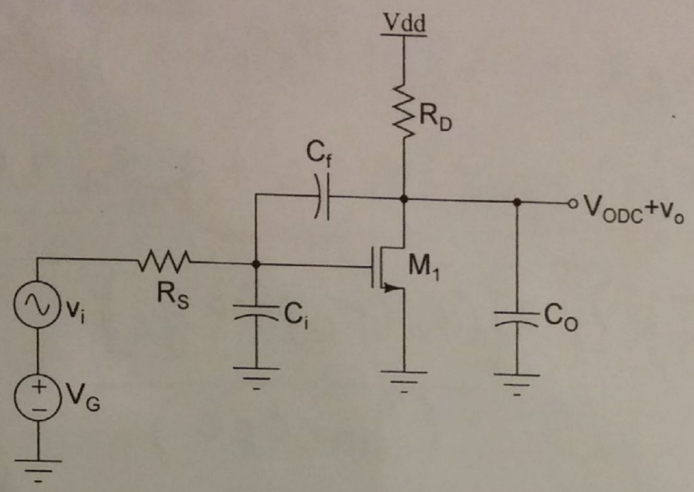
1. (60 points) Frequency Response. For the circuit shown below, assume M_1 operates in saturation and has the following defining parameters: W, L, μ_n, C_{OX} , V_{TH} and λ.



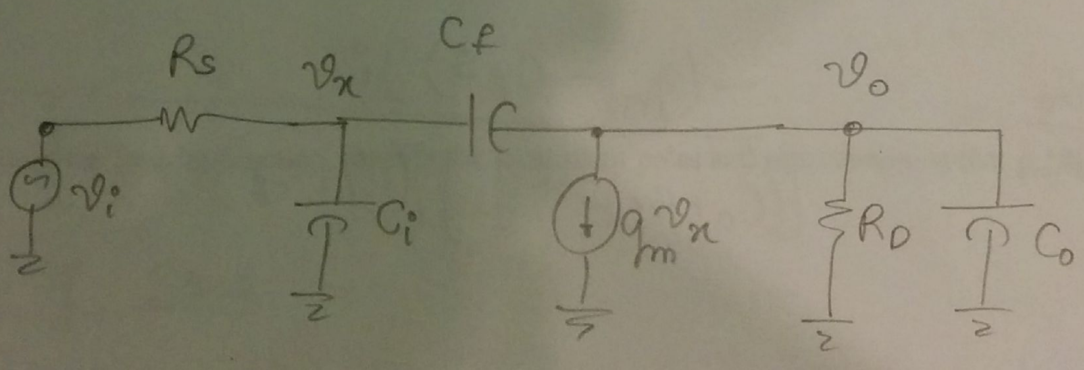
(a) (8 points) Express the small-signal parameters g_m and r_o for M_1 for the DC bias condition $V_G > V_{TH}$ in terms of the defining parameters and V_G.

$$I_D = \mu C_{ox} \frac{W}{L} (V_g - V_{Th})^2 (1+\lambda V_{DS}) \rightarrow 1$$

$$g_m = \frac{2 I_D}{V_g - V_{Th}} \rightarrow 2, \quad r_o = \frac{1}{\lambda I_D} \rightarrow 3$$

Substitude for I_D in 2 and 3

(b) (8 points) Draw the small signal model for the circuit shown above.



<u>Transfer function</u> $R_L = 1/g_L$ $g_s = \frac{1}{R_s}$ $s = j\omega$

KCL at node V_o

$$(v_o - v_x) C_f s + v_o g_L + v_o C_o s + g_m v_x = 0$$

$$v_o (C_o s + g_L + C_f s) - v_x C_f s + g_m v_x = 0$$

$$v_x = \frac{-v_o\left[(C_o + C_f)s + g_L\right]}{(g_m - C_f s)}$$

<u>KCL at node V_x</u>

$$(v_x - v_i) g_s + v_x C_i s + (v_x - v_o) C_f s = 0.$$

$$v_x \left[g_s + s(C_f + C_i)\right] - C_f s v_o = v_i g_s$$

$$-\left[\frac{\left(g_s + s(C_f + C_i)\right)\left(g_L + s(C_f + C_o)\right)}{g_m - C_f s} + C_f s\right] v_o = v_i g_s$$

$$\frac{v_o}{v_i} = \frac{-(g_m - C_f s) g_i}{\left[(C_o + C_f)s + g_L\right]\left[(C_i + C_f)s + g_i\right] + C_f s (g_m - C_f s)}$$

(c) (16 points) Determine the small signal transfer function $H(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$. Use only the capacitors explicitly shown. Assume $r_o \gg R_L$ for simplicity.

$$\frac{v_o(j\omega)}{v_i(j\omega)} = \frac{-(g_m - C_f j\omega)\, g_S}{[(C_o + C_f)j\omega + g_L][(C_i + C_f)j\omega + g_S] - j\omega C_f(g_m - C_f s)}$$

$$g_S = 1/R_S$$

$$g_L = 1/R_L$$

(see previous page)

(d) (8 points) From the transfer function, identify the location of poles and zeros assuming that $g_m R_L \gg 1$.

Location of zero

$$g_m - j\omega C_f = 0$$

$$\boxed{\omega_z = \frac{g_m}{C_f}}$$

Locating the poles

Consider only denominator.

$$\Rightarrow \left[(C_o+C_f)s+g_L\right]\left[(C_i+C_f)s+g_s\right]+C_f s(g_m-C_f s)$$

$$\Rightarrow (C_o+C_f)(C_i+\cancel{C_f})s^2+\left[g_s(C_o+C_f)+g_L(C_i+C_f)+g_m C_f\right]s+g_s g_L \; \cancel{-C_f^2 s^2}$$

Converting back to R_L and R_s

$$\Rightarrow R_s R_L(C_oC_i+C_oC_f+C_iC_f)s^2+s\left[R_L(C_o+C_f)+R_s(C_i+C_f)+C_f g_m R_L R_s\right]+1$$

$$\Rightarrow R_sR_L(C_oC_i+C_oC_f+C_iC_f)s^2+s\left[R_L(C_o+C_f)+R_s(C_i+C_f(1+g_mR_L))\right]+1$$

Denominator of the form

$$s^2/P_1P_2+s\left[\frac{1}{P_1}+\frac{1}{P_2}\right]+1$$

Since $g_mR_L \gg 1$ $\quad \frac{1}{P_1} \approx R_s(C_i+C_f(1+g_mR_L))$

$$P_1=\frac{1}{R_s(C_i+C_f(1+g_mR_L))}$$

← Same result as when C_f is split using miller theorm.

Finding P_2: see last page.

(e) (8 points) We know that at low frequencies, the output is 180° out of phase with respect to input. As the input frequency increases, there is additional phase added due to the pole(s) and zero(s). Now, what is the phase of $H(j\omega) = v_o/v_i$ at very high frequencies ? *(hint: The zero also does something interesting)*

We have $\dfrac{(1 - s/z_1)}{(1 + s/p_1)(1+s/p_2)}$ $(s = j\omega)$

$\angle v_o/v_i = -\tan^{-1}\left(\frac{\omega}{z_1}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right) - \tan^{-1}\left(\frac{\omega}{p_2}\right)$

net phase added

$\angle v_o/v_i \to -270°$ as $\omega \to \infty$

(f) (6 points) You will learn in later courses (if you still want to stay in EE ☺) that this kind of a zero is actually a bad thing. One way to get rid of it is with the circuit shown below. From the transfer function you obtained above, find only the new location of zero including R_F *(hint: Just modify* $\frac{1}{j\omega C_F}$ *to* $\frac{1}{j\omega C_F} + R_F$*)*

$\frac{1}{j\omega C_F} \longrightarrow \frac{1}{j\omega C_F} + R_F \Rightarrow j\omega C_F \to \frac{j\omega C_F}{1 + R j\omega C_F}$

Zero

$g_m - \frac{j\omega C_F}{1 + R_F j\omega C_F} = 0 \Rightarrow g_m + (g_m R_F - 1) j\omega C_F = 0$

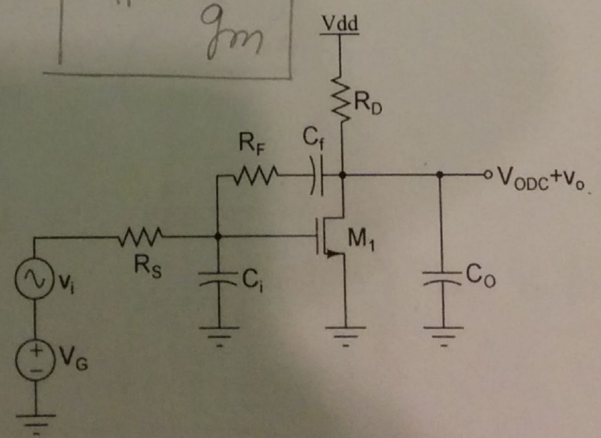
$\boxed{\omega_z = \frac{g_m}{(1 - g_m R_F) C_F}}$

(g) (6 points) Find the value of R_F that will cancel the bad zero completely.

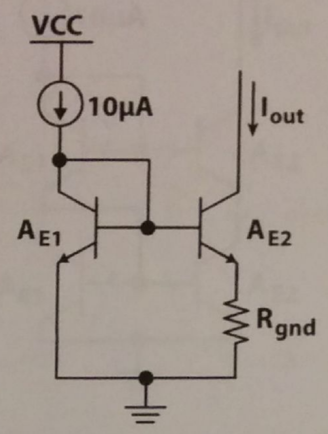
To get rid of zero or push it to infinity

$1 - g_m R_F = 0$

$\boxed{R_F = \frac{1}{g_m}}$



2. (30 points). DC bias calculations. For the circuit shown below, assume R_{gnd}=0, $V_A=\infty$ and β is large enough to neglect the base current.



(a) (6 points) Find the relation between A_{E1} and A_{E2}, so that I_{out}=1mA.

β is large

$$\frac{A_{E2}}{A_{E1}} = \frac{1mA}{10\mu A} = 100 \Rightarrow \boxed{A_{E2} = 100 A_{E1}}$$

(b) (10 points) With your sizing in Part (a), you discover that the current I_{out} is only 90% of 1mA. After a lot of debugging, you discover that the ground connection of the mirror transistor is improper and has a resistance R_{gnd} as shown. Find the value of R_{gnd}.

$I_{out} = 900\mu A$

β is large

$$V_{BE1} = V_T \ln\left(\frac{10\mu A}{I_S}\right)$$

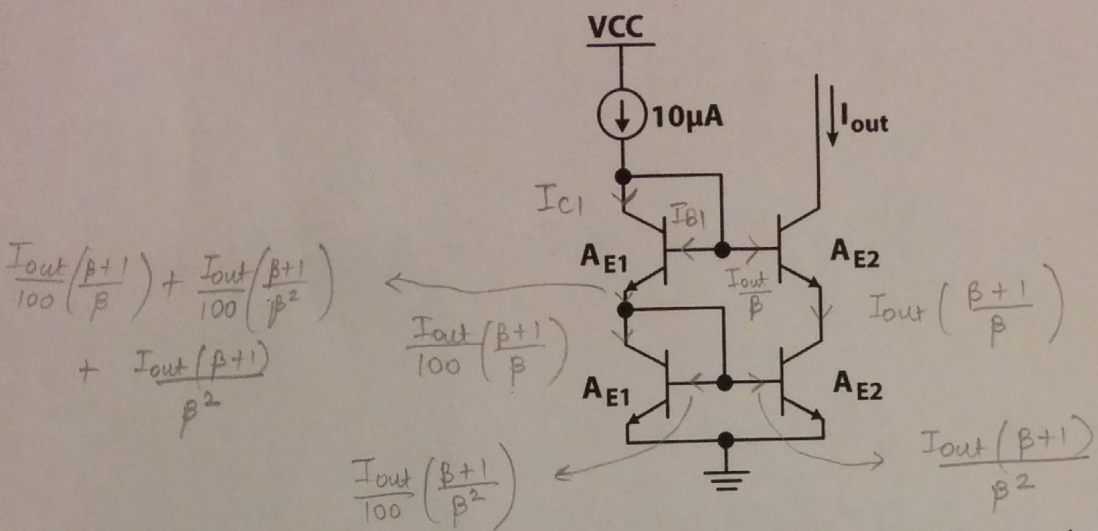
$$V_{BE2} = V_T \ln\left(\frac{900\mu A}{100 I_S}\right)$$

$$900\mu A \times R_{gnd} = V_{BE1} - V_{BE2}$$

$$= V_T \ln\left(\frac{10\mu A}{I_S}\right) - V_T \ln\left(\frac{900\mu A}{100 I_S}\right)$$

$$= V_T \ln\left(\frac{1000}{900}\right)$$

$$\Rightarrow \boxed{R_{gnd} = 3.04\Omega}$$



(c) (14 points) In order to boost the output impedance, we now utilize a cascode current source. With your sizing in (a) for A_{E2}, $V_A=\infty$ and $\beta=100$, find the value of I_{out}. Note that β is finite. (Hint: Express the emitter, base and collector currents of each transistor in terms of I_{out}).

$A_{E2} = 100 A_{E1}$, $\beta = 100$

Mark the currents

$$I_{C1} = \left(\frac{\beta}{\beta+1}\right)\left[\frac{I_{out}}{100}\left(\frac{\beta+1}{\beta}\right) + \frac{101\, I_{out}}{100}\left(\frac{\beta+1}{\beta^2}\right)\right]$$

$$= \frac{I_{out}}{100} + \frac{101\, I_{out}}{100\beta}$$

$$I_{B1} = \frac{I_{out}}{100\beta} + \frac{101\, I_{out}}{100\beta^2}$$

$$I_{C1} + I_{B1} + \frac{I_{out}}{\beta} = 10\,\mu A$$

$$\Rightarrow \frac{I_{out}}{100}\left(\frac{\beta+1}{\beta}\right) + \frac{101\, I_{out}}{100}\frac{(\beta+1)}{\beta^2} + \frac{I_{out}}{\beta} = 10\,\mu A$$

With $\beta = 100$,

$$I_{out}\left[\frac{101}{100^2} + \frac{101^2}{100^3} + \frac{1}{100}\right] = 10\,\mu A$$

$$\Rightarrow \boxed{I_{out} = 330\,\mu A}$$

Question 1d

$$\frac{1}{P_1 P_2} = R_L R_S (C_o C_i + C_o C_f + C_i C_f)$$

$$\frac{1}{P_1} = R_S (C_i + C_f (1 + g_m R_L))$$

$$\frac{\cancel{R_S}(C_i + C_f (1 + g_m R_L))}{P_2} = R_L \cancel{R_S} (C_o C_i + C_o C_f + C_i C_f)$$

$$P_2 = \frac{C_i + C_f (1 + g_m R_L)}{R_L (C_o C_i + C_o C_f + C_i C_f)}$$

$$P_2 = \frac{1}{R_L \left(\frac{C_o C_i + C_o C_f + C_i C_f}{C_i + C_f (1 + g_m R_L)} \right)}$$

If we assume $g_m R_L \gg 1$

$$P_2 = \frac{g_m \cancel{R_L} C_f}{\cancel{R_L} (C_o (C_i + C_f) + C_i C_f)}$$

$$P_2 = \frac{g_m C_f}{(C_o (C_i + C_f) + C_i C_f)}$$