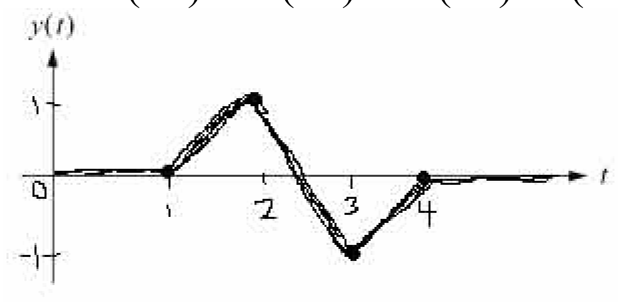
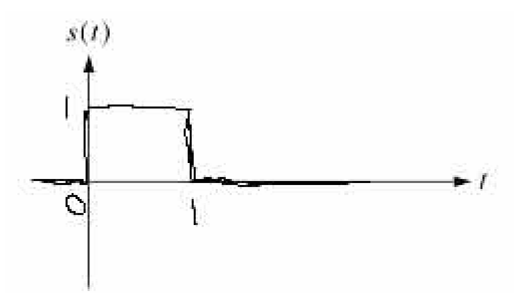
1.

$y(t) = [u(t-1) - u(t-2)]*[u(t) - u(t-1) + u(t-2)]$
$\quad = r(t-1) - 2r(t-2) + r(t-3) - r(t-2) + 2r(t-3) - r(t-4)$
$\quad = r(t-1) - 3r(t-2) + 3r(t-3) - r(t-4)$



2.

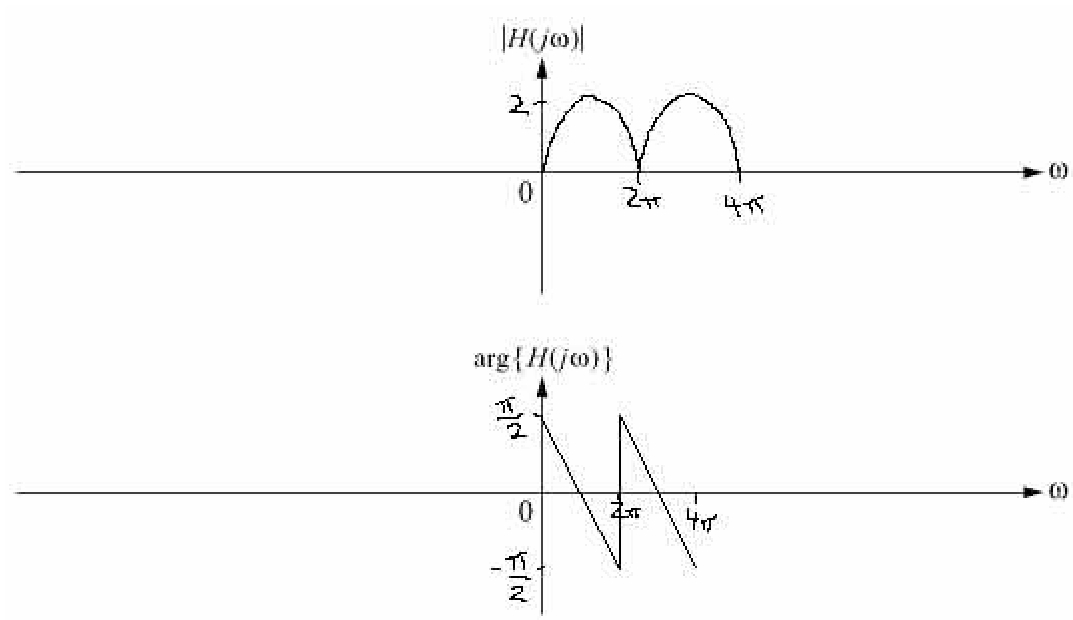
a) $y\_s\ (t) = r(t) - r(t-1)$
$s(t) = d(y\_s\ (t))/dt = u(t) - u(t-1)$



b) $h(t) = d(s(t))/dt = \delta(t) - \delta(t-1)$
$H(j\omega) = \int_{-\infty,\infty} [\delta(t) - \delta(t-1)]*\exp(-j\omega t)*dt$
$\quad = 1 - \exp(-j\omega)$

c) $H(j\omega) = \exp(-j\omega/2)*2j*(\exp(j\omega/2) - \exp(-j\omega/2))/(2j)$
$\quad = 2j\exp(-j\omega/2)\sin(\omega/2)$
$|H(j\omega)| = |2j|\ |\exp(-j\omega/2)|\ |\sin(\omega/2)| = 2|\sin(\omega/2)|$
$\arg[H(j\omega)] = \arg(2j) + \arg(\exp(-j\omega/2)) + \arg(\sin(\omega/2))$
$\quad = \pi/2 - \omega/2 + \{0 \text{ if } \sin(\omega/2) > 0, -\pi \text{ if } \sin(\omega/2) < 0$
$\quad = -\omega/2 + \{\pi/2 \text{ if } \sin(\omega/2) > 0, -\pi/2 \text{ if } \sin(\omega/2) < 0$

$H(j\omega)$ seems to be periodic with period 4π, so plot for $0 \leq \omega < 4\pi$. We find that period is actually 2π.



3.

a) $y[n] + \frac{1}{2} y[n-1] = x[n] + x[n-1]$

b) By inspection, $H(\exp(j\Omega)) = (1 + \exp(-j\Omega))/(1 + \frac{1}{2}\exp(-j\Omega))$

c) $x[n] = \cos(\Omega_0 n)$

$\Rightarrow$ $y[n] = |H(\exp(j\Omega_0))| \cos(\Omega_0 n + \arg[H(\exp(j\Omega_0))])$

Want Ω_0 such that $|H(\exp(j\Omega_0))| = 0$. Look at numerator.

$$|1 + \exp(-j\Omega_0)| = |2*\exp(-j\Omega_0/2)*(\exp(j\Omega_0/2) + \exp(-j\Omega_0/2))/2|$$
$$= |2|\ |\exp(-j\Omega_0/2)|\ |\cos(\Omega_0/2)|$$
$$= 2|\cos(\Omega_0/2)|$$

For $0 \le \Omega_0 < 2\pi$, $\cos(\Omega_0/2) = 0 \Rightarrow \Omega_0 = \pi$

d) Assume $x[n] = u[n] = 1, n \ge 0$

$y[n] = s[n]$

$y[-1] = 0$

Homogenous

$y^{(h)}[n] + (1/2)y^{(h)}[n-1] = 0$ $\qquad$ $y^{(h)}[n] = c_1(r^n), n >= 0$

$r + \frac{1}{2} = 0$ $\qquad$ $r = -\frac{1}{2}$ $\qquad$ $y^{(h)}[n] = c_1((-\frac{1}{2})^n), n >= 0$

Particular

$x[n] = 1, n >= 0$ $\qquad$ $y^{(p)}[n] = b, n >= 0$

$y^{(p)}[n] + (½)y^{(p)}[n – 1] = x[n] + x[n – 1]$

$b + ½ b = 1 + 1$ $\qquad$ $b = 4/3$ $\qquad$ $y^{(p)}[n] = 4/3, n >= 0$

Translate IC

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y[-1] = 0 Find y[0]
y[n] = - ½ y[n – 1] + x[n] + x[n – 1]
At n = 0
y[0] = - ½ y[-1] + x[0] + x[-1]
y[0] = - ½ (0) + 1 + 0 = 1

Match IC

$y[n] = c_1(- ½)^n + 4/3, n >= 0$
$y[0] = c_1(- ½)^0 + 4/3 = c_1 + 4/3 = 1$
$c_1 = -1/3$
$y[n] = -1/3(- ½)^n + 4/3, n >= 0$
$s[n] = [-1/3(- ½)^n + 4/3]$ $\quad$ y[n]

4.

a) By inspection: $h(t) = \delta(t) – \alpha\, \delta(t-\tau) + \alpha^2\, \delta(t-2\tau) - \ldots$
$= \sum_{n=0,\infty} (-\alpha)^n\, \delta(t-n\tau)$

b) Stable if $\int_{-\infty,\infty} |h(t)|dt < \infty$

$\int_{-\infty,\infty} |h(t)|dt = \int_{-\infty,\infty} |\sum_{n=0,\infty} (-\alpha)^n\, \delta(t-n\tau)|dt$
$= \int_{-\infty,\infty} \sum_{n=0,\infty} |\alpha|^n\, \delta(t-n\tau)dt$
$= \sum_{n=0,\infty} |\alpha|^n \int_{-\infty,\infty} \delta(t-n\tau)dt$
$= \sum_{n=0,\infty} |\alpha|^n$

If $|\alpha| < 1$, sum converges to $1/(1 - |\alpha|)$ and system is stable.