EECS120: Signals and Systems Midterm 1

Write name and ID number on each page of your solutions

Problem 1.1 *Fourier Transforms and Simple Filtering*

Justify your answers for full credit.

a. *10pts What is the CTFT of the unit pulse* $y(t) = \begin{cases} 1 & t \in [-\frac{1}{2}, +\frac{1}{2}] \\ 0 & otherwise \end{cases}$ *?*

b. *10pts If* $b(t)$ *is a periodic signal with period* T*, and it is represented by the Fourier Series* $b(t) = \sum_{k=-\infty}^{+\infty} B_k e^{j\frac{2\pi}{T}kt}$*, then what is the Fourier Series representation of* $c(t) = b(t-1)$*?*

c. *15pts* *Consider discrete-time signals with period* 2. *Model these as 2-d vectors. Consider an LTI system that has impulse response* $h(0) = 1, h(1) = 2$. *Write this system as a matrix and give its eigenvectors and corresponding eigenvalues.*

d. *10pts* *A discrete time LTI system has DTFT* $1 - e^{-j\omega}$. *What is its impulse response?*

Problem 1.2 *True/False. Do at least two of the following for full credit.*

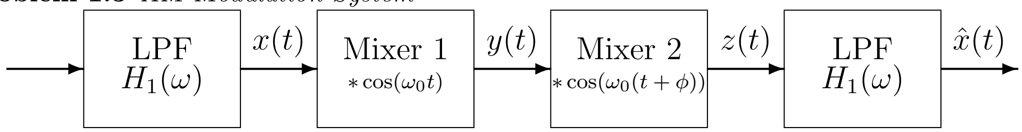
If the bold statement is true, give a proof for it. If the statement is false, show a counterexample or proof that it is false.

a. *20 pts* *Let L be a system that acts on continuous time signals as follows:* $[Lx](t) = x(t)\cos(t)$. **Then, L is L.T.I.**

b. 20 pts *Let L be an LTI system that acts on signals that are defined on Z_N, the discrete time interval $\{0, 1, 2, \ldots, N-1\}$ viewed as positions along the circumference of a circle. Delays and shifts on such signals are to be interpreted in a "wrap around" manner with $[D_\tau x](t) = x(t - \tau \bmod N)$. Let $x_\omega(t) = e^{j\omega t}$.* **Then for every real ω there exists a constant λ_ω so that $Lx_\omega = \lambda_\omega x_\omega$.**

c. *20 pts* *Let L be a linear system that acts on continous time signals. Let* $x_\omega(t) = e^{j\omega t}$. *There exists a complex valued function* $\lambda(\omega)$ *so that for a particular subset of real* $\omega \in \Omega$, *the system* L *has the property that* $[Lx_\omega](t) = \lambda(\omega)x_\omega(t)$. **Then, for the class of signals that can be written** $y(t) = \sum_{i=1}^{N} \alpha_i x_{\omega_i}(t)$ **(where the** $\omega_i \in \Omega$**), the system** L **is LTI.**

Problem 1.3 *AM Modulation System*



In the above continuous time system, consider the LPF to be ideal and to perfectly pass through all frequencies less than 2.

$$H_1(\omega) = \begin{cases} 1 & \text{if } |\omega| < 2 \\ 0 & \text{otherwise} \end{cases}$$

and $\omega_0 = 10$ *so that*

$$y(t) = x(t)\cos(\omega_0 t)$$

and

$$z(t) = y(t)\cos(\omega_0(t+\phi))$$

a. 15pts Suppose $\phi = 0$ *and* $x(t) = \sin(t)$*. What are* $y(t), z(t), \hat{x}(t)$*?*

b. 15pts Suppose $x(t) = \sin(t)$ *but* $\phi \neq 0$*. What is* $\hat{x}(t)$ *as a function of* ϕ*? Please plot the power of* $\hat{x}(t)$ *as a function of* ϕ*.*

c. *10pts Suppose now that $y(t)$ was corrupted by some potentially interfering signal and so the input to mixer 2 was now $y'(t) = y(t) + \sin(\omega_n t)$ rather than just $y(t)$. For what values of ω_n would you see an undesirable component in $\hat{x}(t)$?*