EECS 120 Signals & Systems
Gastpar

University of California, Berkeley: Fall 2005
November 14, 2005

Exam 2

Last name	First name	SID

- You have 1 hour and 45 minutes to complete this exam.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.
- However, two handwritten and *not photocopied* double-sided sheet of notes is allowed.
- Additionally, you receive Tables 3.1, 3.2, 4.1, 4.2, 5.1, 5.2, 9.1, 9.2 from the class textbook.
- If we can't read it, we can't grade it.
- We can only give partial credit if you write out your derivations and reasoning in detail.
- You may use the back of the pages of the exam if you need more space.

*** Good Luck! ***

Problem	Points earned	out of
Problem 1		29
Problem 2		28
Problem 3		27
Problem 4		33
Total		117

Problem 1 *(Short Questions.)* 29 Points

(a) (4 Pts) For the system in Figure 1,

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \le \omega_0 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Sketch the frequency response $G(j\omega)$ of the overall system between $x(t)$ and $y(t)$.

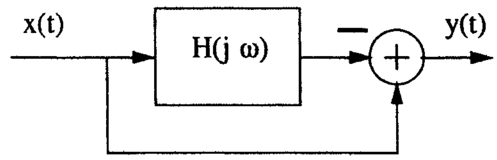

Figure 1:

(b) (15 Pts) A causal LTI system is described by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + 2y(t) = \frac{d^2}{dt^2}x(t) - x(t) \tag{2}$$

Is this system stable? Does this system have a causal and stable inverse system?

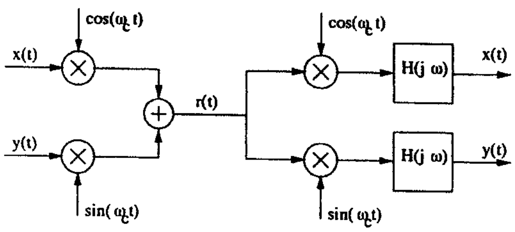

Figure 2: Quadrature modulation.

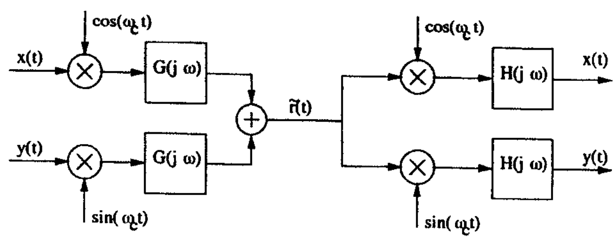

Figure 3: "Improved" quadrature modulation.

(c) (10 Pts) As you have seen in the homework, "quadrature multiplexing" is the system shown in Figure 2, where

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_M \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad G(j\omega) = \begin{cases} 1, & \text{for } |\omega| \geq \omega_c \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Both original signals are assumed to be bandlimited: $X(j\omega) = Y(j\omega) = 0$, for $|\omega| > \omega_M$; and the carrier frequency is $\omega_c > \omega_M$. The interesting feature is that the effective bandwidth of the signal $r(t)$ is only $2\omega_M$, the same as for a regular AM system with only the signal $x(t)$. Hence, $y(t)$ can ride along for free.

Now, your colleague remembers single-sideband AM and suggests to add the filters $G(j\omega)$ as shown in Figure 3. The effective bandwidth of the transmitted signal $\tilde{r}(t)$ is only ω_M, half as much as in the original quadrature multiplexing system! Show that the "improved" system will not work. *Hint:* Find a pair of example spectra $X(j\omega)$ and $Y(j\omega)$ for which $R(j\omega)$ is *not* zero, but $\tilde{R}(j\omega) = 0$ for all ω. Then, argue (in a few keywords) why this invalidates the "improved" quadrature modulation.

Problem 2 *(Discrete-time processing of continuous-time signals.)* 28 Points

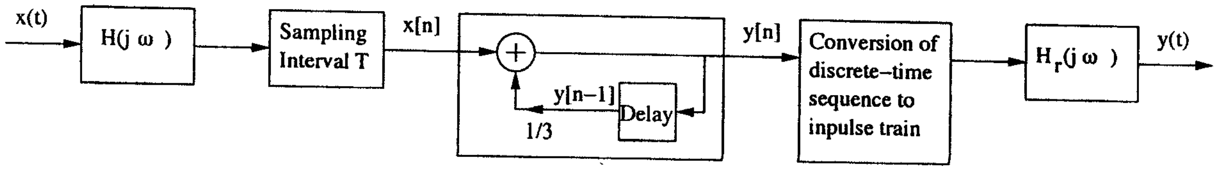

Figure 4:

For the system in Figure 4,

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \le \frac{\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad H_r(j\omega) = \begin{cases} T, & \text{for } |\omega| \le \frac{\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

(a) (20 Pts) Give the formula for the overall system response $G(j\omega)$, relating $x(t)$ and $y(t)$. Also give a sketch of the magnitude $|G(j\omega)|$, paying particular attention to the labeling of the frequency axis. *No derivation is necessary to get full credit.*

(b) (8 Pts) For $x(t) = e^{j\pi t/(2T)}$, determine the corresponding output signal $y(t)$. Your answer should not contain an integral, but apart from that, there is no need to simplify it down.

Problem 3 *(Sampling System Design.)* 27 Points

The signal $x(t)$ has the Fourier transform shown in Figure 5.

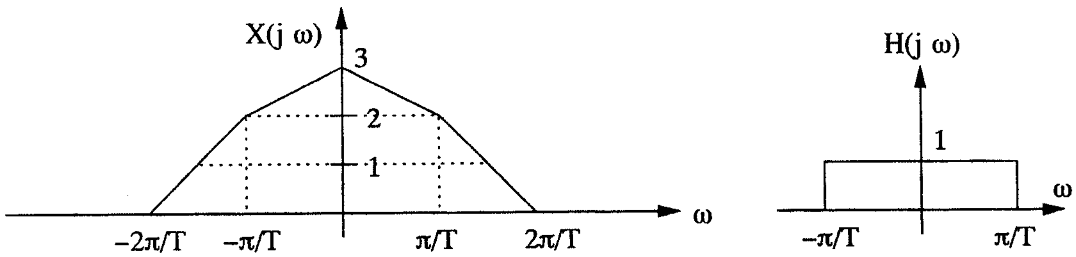

Figure 5:

(a) (5 Pts) As a function of T (as in Figure 5), determine the smallest sampling frequency $\omega_s = 2\pi/T_s$ (where T_s is the sampling interval) for which perfect reconstruction can be guaranteed for the signal $x(t)$. A graphical justification (sketch with labels on the frequency axis) is sufficient.

(b) (10 Pts) Consider the signal $y(t) \doteq h(t) * x(t)$, where $h(t)$ is the impulse response of the filter $H(j\omega)$ in Figure 5. Sketch the spectra of the two discrete-time signals

$$
\begin{aligned}
x[n] &= x(nT) &(5)\\
y[n] &= y(nT), &(6)
\end{aligned}
$$

where T is the *same* as in Figure 5. Which effect explains the difference between $x[n]$ and $y[n]$?

(c) (12 Pts) The goal is now to implement a sampler with sampling interval $T_0 = T/2$, where T is as in Figure 5. Unfortunately, such a fast sampler is not available in the current technology. Instead, you have access to the following devices:

- samplers with sampling interval T, where T is the *same* as in Figure 5 (any number)
- anti-aliasing filters with the frequency response given in Figure 5 (any number)
- continuous-time signal adders/subtractors (any number)
- *any* discrete-time processing devices (ideal filters included).

Draw the block diagram of a system that takes as an input the signal $x(t)$ (with spectrum as shown in Figure 5) as outputs the discrete-time signal $x_0[n] = x(nT_0)$. *Hint:* To maximize your chance of partial credit, give spectral plots of intermediate signals in your system.

Problem 4 *(PAM.)* 33 Points

Two pulses are suggested for a PAM system:

$$q_1(t) = \begin{cases} 1, & |t| \leq T/4 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad q_2(t) = \begin{cases} -1 & -T/4 \leq t < 0 \\ 1, & 0 \leq t \leq T/4 \\ 0, & \text{otherwise} \end{cases}$$

The PAM signal is then

$$x_m(t) \;=\; \sum_{n=-\infty}^{\infty} s[n] q_m(t - nT), \text{ for } m = 1, 2. \tag{7}$$

Throughout this problem, **we assume that the data signal is merely** $s[n] = 1$**, for all** n**.**

(a) (6 Pts) Find the powers P_1 and P_2 of the two PAM signals $x_1(t)$ and $x_2(t)$.

(b) (10 Pts) Give the formula for the Fourier series coefficients of the signal $x_1(t)$, and explicitly evaluate the coefficients a_0, a_1 and a_{-1}. Then, do the same for the signal $x_2(t)$.

(c) (7 Pts) To actually transmit our PAM signal, we first low-pass filter it:

$$\tilde{x}_m(t) \quad = \quad h(t) * x_m(t), \text{ where } H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \frac{10\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Then, we transmit the signals $y_1(t) = \tilde{x}_1(t)\cos(\frac{40\pi}{T}t)$ and $y_2(t) = \tilde{x}_2(t)\cos(\frac{40\pi}{T}t)$. Sketch the Fourier transforms of these two signals in the plots provided below, carefully labeling the frequency axis. In the magnitude plots (i.e., $|Y_1(j\omega)|$ and $|Y_2(j\omega)|$, respectively), the amplitudes need not be exact. *Remark:* The current labels on the frequency axis in the plots are for your convenience only. If you prefer, you can cross them out and start from scratch.

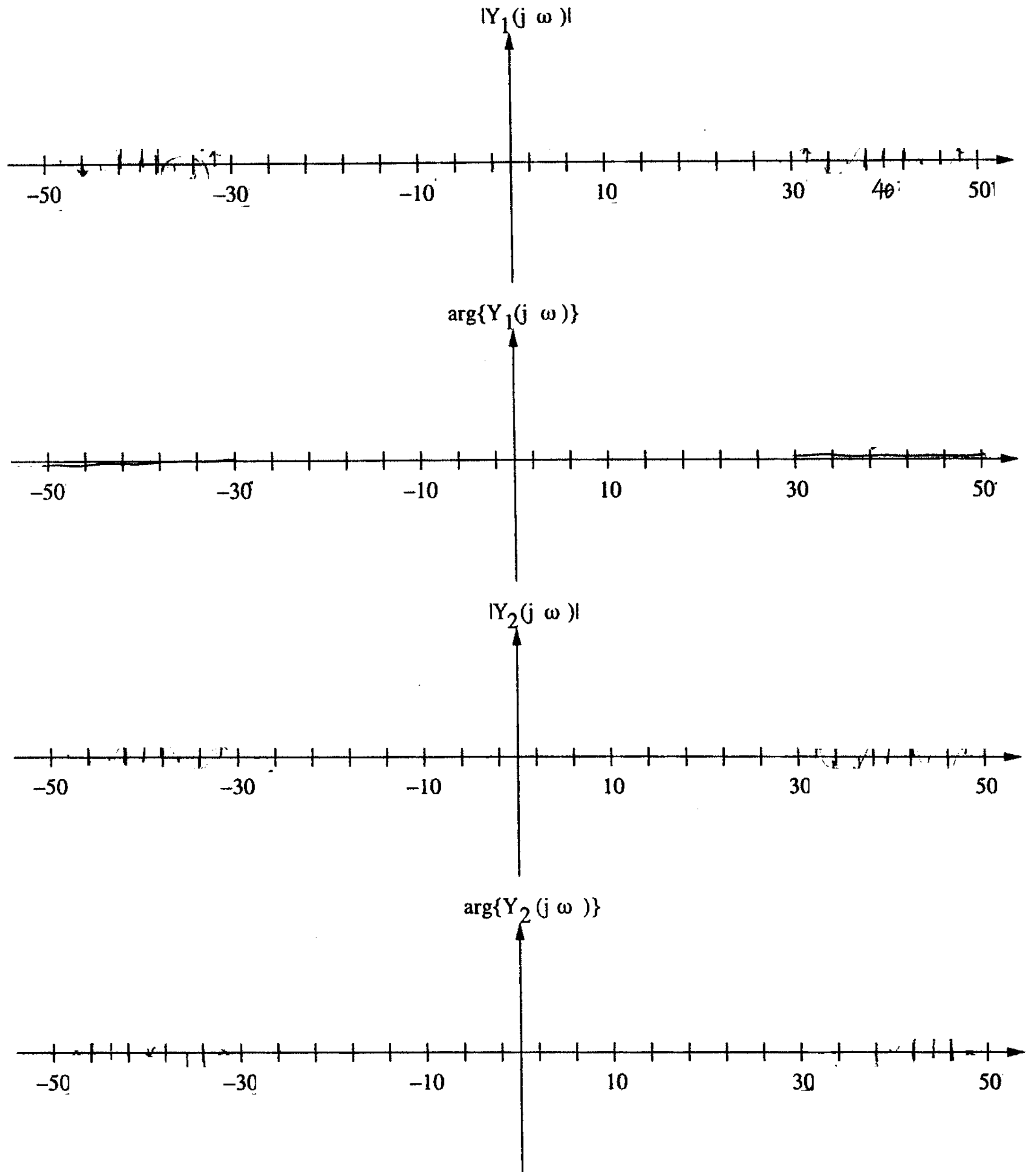

Figure 6:

(d) (10 Pts) The communication channel's effect on the signal can be described by the following band-pass filter:

$$H_{channel}(j\omega) = \begin{cases} \sin^2(\omega T/4 - \pi) & \text{for } 36\pi/T < |\omega| < 38\pi/T \\ 1, & \text{for } 38\pi/T \leq |\omega| \leq 42\pi/T \\ \sin^2(\omega T/4 - \pi) & \text{for } 42\pi/T < |\omega| < 44\pi/T \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

The channel output signal is then $z_1(t) = y_1(t) * h_{channel}(t)$ and $z_2(t) = y_2(t) * h_{channel}(t)$, respectively. Assuming that $s[n] = 1$, for all n, find the power of $z_1(t)$ and $z_2(t)$. These are the received powers. Which pulse is more efficient for transmission across this channel?