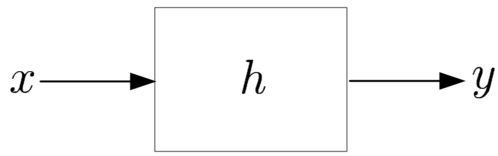
EE 120: Signals and Systems MIDTERM 3
Department of EECS 3 December 2010
UC Berkeley

LAST Name ______________________ FIRST Name ______________________

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" $\times$ 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 7.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the seven numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT3.1 (70 Points) Consider a *real-valued* signal x. We apply this signal as the input to a discrete-time LTI filter as shown below:



The filter's impulse response is $h(n) = x(-n)$, for all n.

The autocorrelation function of a signal is a measure of the similarity of the signal with itself—a self-similarity measure, so to speak. It's used in many applications, including radar, sonar, and digital communications. In particular, the autocorrelation r_{xx} of the signal x is defined as follows:

$$r_{xx}(n) = \sum_{m=-\infty}^{+\infty} x(m)\, x(m-n).$$

For the remainder of this problem, let $\widehat{X}(z)$, $\widehat{H}(z)$, $\widehat{Y}(z)$, and $\widehat{R}_{xx}(z)$ denote the Z transforms of x, h, y, and r_{xx}, respectively. Assume all of these Z transforms are rational in z.

(a) (10 Points) Show that the output signal y is the *autocorrelation* of the input signal: $y(n) = r_{xx}(n)$, for all n.

(b) (10 Points) Show that $\widehat{R}_{xx}(z) = \widehat{X}(z)\widehat{X}(1/z)$.

(c) (10 Points) Show that if ζ is a complex-valued (but not real-valued) pole of $\widehat{R}_{xx}(z)$, then so are its conjugate ζ^*, its reciprocal $1/\zeta$, and its conjugate reciprocal $1/\zeta^*$. In other words, every complex-valued pole of $\widehat{R}_{xx}(z)$ is a member of a family of *four* poles. The same is true of the zeros, using identical reasoning, but you're not asked to show it here.

(d) (10 Points) Suppose $\zeta \neq 0$ is a simple *real-valued* pole of $\widehat{R}_{xx}(z)$. The existence of what other pole (or poles) can you infer from the existence of ζ?

(e) (10 Points) Suppose the signal x is causal, but *not* of finite energy; that is,

$$\sum_{m=-\infty}^{+\infty} |x(m)|^2 = \infty.$$

Determine at least one time instance (or sample) $n = N$ at which the output signal y is infinite (i.e., $y(N) = \infty$).

(f) (10 Points) Let the input signal be defined as follows:

$$\forall n \in \mathbb{Z}, \quad x(n) = \left(\frac{1}{2}\right)^n \sin\left(\frac{\pi}{4}n\right) u(n).$$

Determine $\widehat{X}(z)$ and its corresponding region of convergence. Also provide a well-labeled pole-zero diagram of $\widehat{R}_{xx}(z)$ and specify its region of convergence.

(g) (10 Points) Suppose the signal x is causal, but *not* absolutely summable; that is, $\sum_{m=-\infty}^{+\infty} |x(m)| = \infty$. Explain why $\widehat{R}_{xx}(z)$, the Z transform of the autocorrelation function r_{xx}, is undefined. Drawing a valid pole-zero diagram (or at least the poles), consistent with the description of x, may help you).

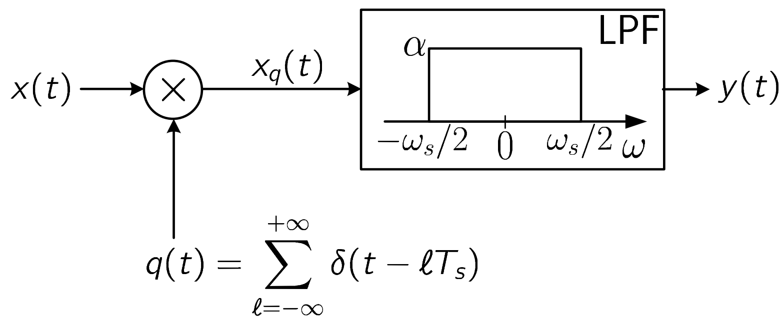
MT3.2 (20 Points) Consider the continuous-time sinusoid

$$\forall t \in \mathbb{R}, \quad x(t) = \sin(\omega_0 t),$$

where ω_0 is a positive frequency. We send this signal through the processing blocks shown in the figure below. The sampling period is

$$T_s = \frac{4\pi}{3\omega_0} \text{ seconds.}$$

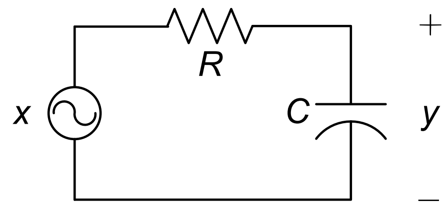
The cutoff frequency of the ideal low-pass filter is $\omega_s/2$—half the sampling frequency.



Determine a reasonably simple expression for the output signal y. Let x represent the motion of a rotating carriage wheel, and y the perception of an observer watching the wheel. Is there a qualitative difference between x and y? If so, describe it in words.

Hint: To ease your analysis, and earn at least partial credit, be sure to plot the spectra $X(\omega)$, $X_q(\omega)$, and $Y(\omega)$.

MT3.3 (15 Points) Consider the passive RC-circuit shown below:



where x and y denote the input and output, respectively. The circuit is described by the linear, constant-coefficient differential equation

$$\dot{y}(t) + \frac{1}{RC}\, y(t) = \frac{1}{RC}\, x(t)$$

and transfer function

$$\widehat{H}(s) = \frac{1/RC}{s + 1/RC}, \quad -\frac{1}{RC} < \mathrm{Re}(s).$$

(a) (10 Points) Determine a reasonably simple expression for the unit-step response of the system. Your final expression for $y(t)$ must not contain *any* new parameter (i.e., don't leave unspecified constants, other than R and C, in your expression). Be sure to identify the transient and steady-state components of the response (in the time domain).

(b) (5 Points) Determine the steady-state response of the circuit if the input signal is $x(t) = e^{i\omega_0 t} u(t)$, for all t, where ω_0 is a non-zero frequency.

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Problem Name	Points 10	Your Score
1	70	
2	20	
3	15	
Total	**115**	