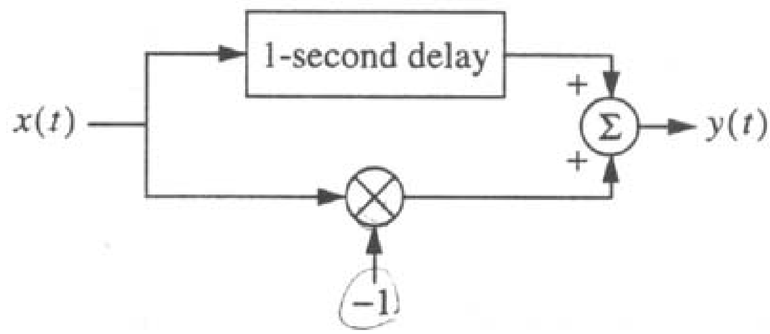
EE120 Fall 97
Midterm #1 Solutions
Professor J.M. Kahn

Problem #1 (35 pts)

A LTI system with input x(t) and output y(t) is implemented as shown.



(a) (10 pts.) Give an expression for the impulse responseh(t).

$$h(t) = -\delta(t) + \delta(t-1)$$

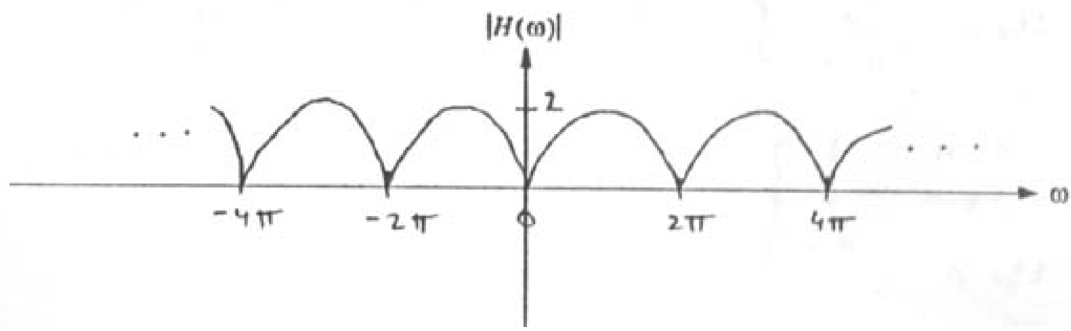
(b) (10 pts.) Give an expression for the frequency response H(w).

$$\boxed{H(\omega) = e^{-j\omega} - 1}$$

(c) (10 pts.) Find a purely real expression for |H(w)|. Sketch |H(w)|, labeling the vertical and horizontal axes of your plot.

$$|H(\omega)| = \left[(-1+e^{-j\omega})(-1+e^{j\omega})\right]^{1/2} = \left[1+1-e^{-j\omega}-e^{j\omega}\right]^{1/2}$$

$$= \left[2(1-\cos\omega)\right]^{1/2}$$



(d) (5 pts.) Suppose the input is $x(t) = \cos(w_o t)$. For what values of w_o is the output zero, i.e., $y(t) = 0$?

$$\omega_0 = 0, 2\pi, 4\pi, \ldots$$

Problem #2 (50 pts.)

A system with input x(t) and output y(t) is described by the differential equation:

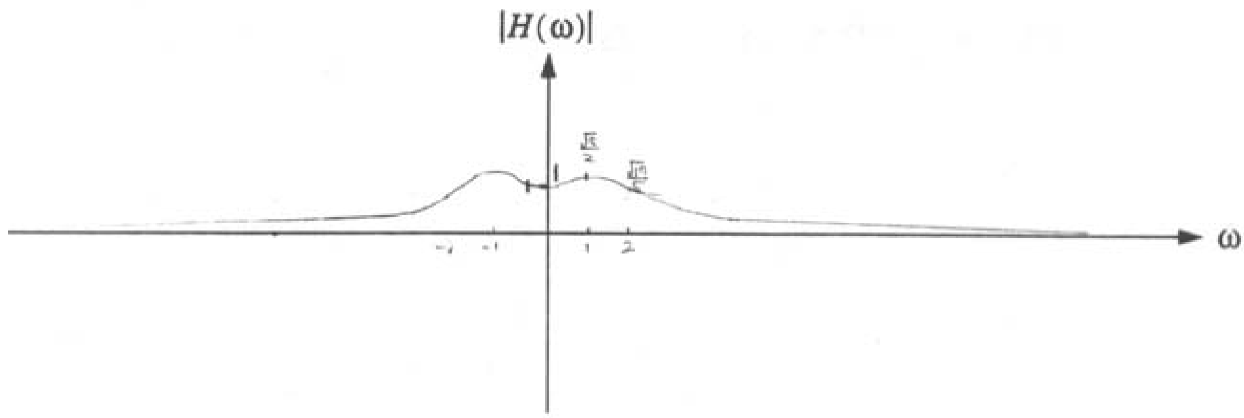
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2\frac{dx}{dt} + x.$$

(a) (10 pts.) Find an expression for the frequency response H(w).

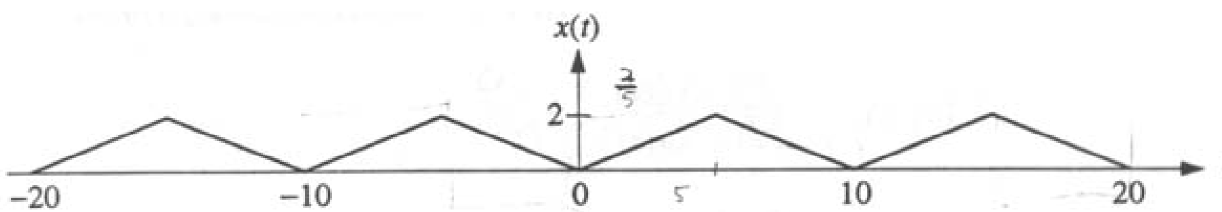
$$H(\omega) = \frac{1+2j\omega}{-\omega^2+2j\omega+1} = \boxed{\frac{1+2j\omega}{(1+j\omega)^2}}$$

(b) (10 pts.) Find a purely real expression for |H(w)|, and sketch |H(w)|, labeling the horizontal and vertical axes of your sketch. Hint: just evaluate |H(w)| for a few values of w, e.g., w = 0, 1, 2, infinity.

$$|H(\omega)| = \frac{\sqrt{1+4\omega^2}}{1+\omega^2}$$



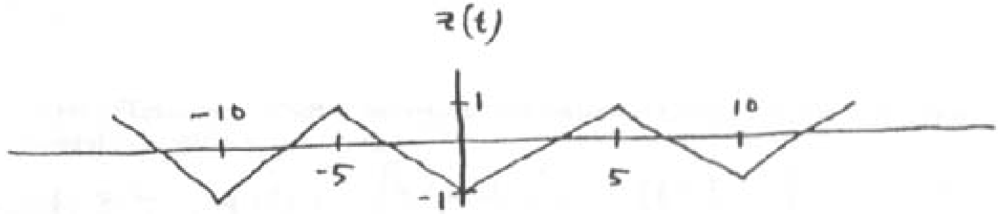
Consider the periodic signal x(t) shown below.



(c) (15 pts.) State the period T_o and the fundamental frequency w_o of the signal x(t). Give an exponential Fourier series representation of x(t).

$$T_0 = 10 \qquad \omega_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$x(t) = 1 + z(t)$$



$$z(t) = \sum_{n=-\infty}^{\infty} Z_n e^{jn\frac{\pi}{5}} \qquad Z_n = \begin{cases} 0 & n \text{ even} \\ -\frac{4}{n^2\pi^2} & n \text{ odd} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\frac{\pi}{5}} \qquad X_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0,\ n \text{ even} \\ -\frac{4}{\pi^2 n^2} & n \text{ odd.} \end{cases}$$

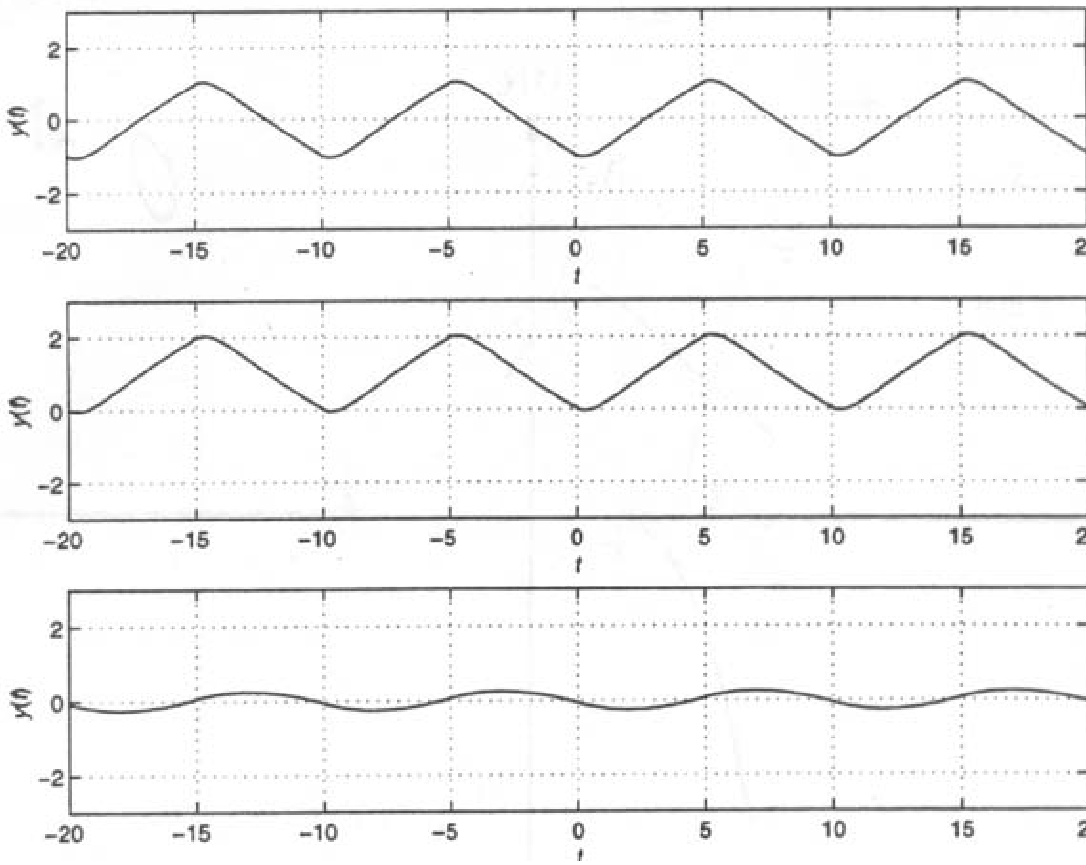
(d) (10 pts.) The signal x(t) shown above is input to the system. Give an exponential Fourier series representation of the output y(t).

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{jn\frac{\pi}{5}}$$

$$Y_n = X_n \cdot H\left(\frac{n\pi}{5}\right) = X_n \left[\frac{2j\frac{n\pi}{5} + 1}{\left(j\frac{n\pi}{5}\right)^2 + 2j\frac{n\pi}{5} + 1} \right]$$

where X_n are given in part (c).

(e) (5 pts.) Circle the drawing that you think best depicts the y(t) obtained in part (d).



Circle the second drawing because it's the only one having a non-zero d.c. level. We know that x_o != 0 and H(0) != 0.

Problem #3 (15 pts.)

Consider a signal $y(t) = r(t) \otimes \Pi(t)$, where r(t) is the unit ramp function and $\Pi(t)$ is the unit pulse function.

(a) (10 pts.) Find an expression for y(t).

$$y(t) = r(t) \otimes \left[u\left(t + \tfrac{1}{2}\right) - u\left(t - \tfrac{1}{2}\right) \right]$$

$$= p\left(t + \tfrac{1}{2}\right) - p\left(t - \tfrac{1}{2}\right)$$

$$p(t) = \tfrac{1}{2} t^2 u(t)$$

(b) (5 pts.) Sketch y(t), labeling the vertical and horizontal axes of the plot. You may find it helpful to evaluate y(t), t >= 1/2

For $t \geq \frac{1}{2}$, $y(t) = \left[\frac{1}{2}\left(t+\frac{1}{2}\right)^2 - \frac{1}{2}\left(t-\frac{1}{2}\right)^2\right]$

$$= \frac{1}{2}\left[\left(t^2 + t + \frac{1}{4}\right) - \left(t^2 - t + \frac{1}{4}\right)\right] = t$$