EE120, Fall 98
Midterm 1 Solutions
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Problem 1 (45 pts)

[10 pts]
a) y[n] = 2x[n] - .5 y[n-1]
y[n] + .5 y[n-1] = 2x[n]

[15 pts]
b) Easy way: use h[n] from part (c):
x[n] = sum(k=-infinity->n) h[k] = sum(k=-infinity->n) 2(-.5)^k * u[k] = sum(k=0->n) (-.5)^k
For n < 0, s[n] = 0
For n >= 0: use sum(k=0->n) a^k = (1-a ^ (k+1))/(1 - a)
s[n] = 2 * (1-(-.5) ^ (n + 1))/(1-(-.5)) = 2 * (2/3 + 1/3 (-1/2) ^ n)
For all n: s[n] = (4/3 + 2/3(-1/2) ^ n) u [n]

Hard way: solve difference equation
y[n] + .5 y[n-1] = 2x[n]
x[n] = u[n], zero-initial conditions: y [-1] = 0
Homo. soln: (y^n)[n] = .5 (y^n)[n-1] = 0
Char eqn: r + .5 = 0
(y^n)[n] = A (-.5) ^ n, n>=0
Part. soln: x[n] = 1, n >=0
(y^p)[n] = c, n >=0
Total soln: y[n] = 4/3 + A(-.5)^n, n>=0
Translate initial condition:
y[n] = -.5y[n-1] + 2 x[n]
For n = 0: y[0] = -.5y[-1] + 2 x[0]
y[0] = -.5 * 0 + 2 = 2
Find A by satisfying initial condition:
y[0] = 2 = 4/3 + A * (-.5) ^ 0 = 2
A = 2/3
y[n] = s[n] = 4/3 + 2/3 * (-.5) ^ n, n >= 0
Since s[n] = 0, n < 0,
s[n] = (4/3 + 2/3(-.5) ^ n) u[n]

[10 pts]
c) Find a closed-from expression for the impulse response h[n] for all n. (You can do this using the result of part (b).
Alternatively you can write down the result by inspection.

Easy way (inspection of block diagram):
y[n] = h[n] when x[n] = delta [u] and y [-1] = 0
Under these conditions:

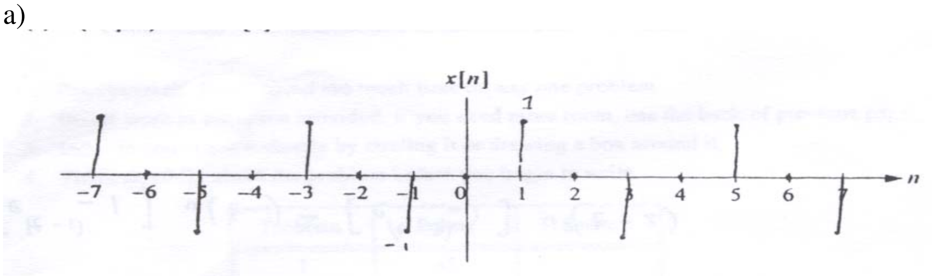
y[0] = 2 due to input
y[1] = 2(-.5) due to fed-back output
y[2] = 2(-.5)(-.5) due to fed-back output
etc.
y[n] = h[u] = 2 * (-.5) ^ n, n>= 0
since h[n] = 0, n < 0, h[n] = 2(-.5)^ n * u[n]

Hard way from s[n]:
h[n] = s[n] - s[n-1]
= [4/3 + 2/3(-.5)^n] y[n] - [4/3 + 2/3 (-.5) ^ (n-1)] y [n-1]
n = 0: h[0] = 2
n >=1: h[n] = 4/3 + 2/3(-.5) ^ n - 4/3 -2/3 (-.5) ^ (n-1)
h[n] = 2 (-.5) ^ n * u[n]

[10 pts]
let w = omega
d) y[n] + .5y[n-1] = 2x[n]
x[n] = e^(jwn), y[n] = H(e^(jw))e^(jwn)
H(e^(jw))e^(jwn) + .5 H(e^(jw))e(jwn)e(-jw) = 2e^(jwn)
H(e^(jw)) = 2 / (1 + .5e^(-jw))

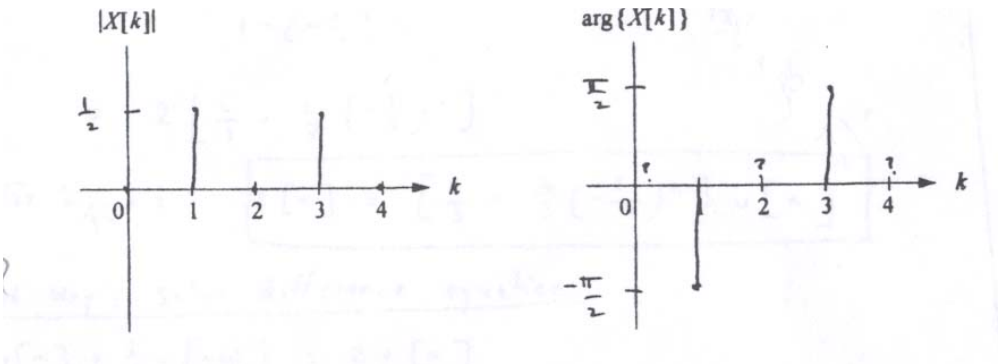
Problem 2 (35 pts)

[10 pts]
a)



[15 pts]
let w = omega_0
b) N = 4, w = pi/2
x[n] = 1/N * sum(n=<N>){x[n]e^(-jwn)}
= 1/4 * sum(n= -2->1){x[n]e^(-jwn)}
= 1/4 * [-e^(jk*pi/2) + e^(-jk*pi/2)]
= -j/2 * sin(k*pi/2)

[10 pts]
c) abs(X[k]) = .5 abs(sin(k*pi/2))
arg{X[k]} = arg (-j/2) + arg {sin (k*pi/2)}
= -pi/2 if sin(k*pi/2) > 0
pi if sin(k*pi/2) < 0
When abs(X[k]) = 0, it doesn't matter what you choose for arg.



Problem 3

h(t) = u(t-1) - u(t-2)
x(t) = (e^t) u(t)
y(t) = x(t) * h(t) = (e^ t) u(t)
= {[(e ^ t) u(t)] * u(t)} * [delta(t-1) - delta(t-2)]
[(e ^ t) u(t)] * u(t) = integral(-infinity->t) {(e ^ tau) u(tau) d tau}
= (e ^ t) - 1   if t >= 0
0   if t < 0
= [(e^t) - 1] u(t)
y(t) = [e^(t-1) -1] u(t-1) - [e ^(t-2) - 1] u(t-2)