

Problem 1 (Short Questions) 20 Points

For each of the following statements, if you believe it is true, give a justification. If you believe it is false, give a counterexample.

- (a) A linear causal continuous-time system is always time-invariant.
- (b) The system with (real-valued) input $x(t)$ and output given by

$$y(t) = (1+(x(t))^2)^{\cos(t)}$$

is stable.

- (c) The discrete-time signal $x[n] = \cos(n)$ is a periodic signal.
- (d) For an otherwise completely unknown system, it is known that when the input is given by

$$x(t) = \cos(t) + \cos(2t),$$

the output is

$$y(t) = .5(1+\cos(t)+\cos(2t)+\cos(3t)).$$

This system cannot be a linear time-invariant (LTI) system.

Problem 2 (Convolution) 20 Points

The continuous-time signals $x(t)$ and $y(t)$ are given in Figure 1. In the figure, draw the signal $z(t)$ given by

$$z(t) = (x * y)(t).$$

Carefully label both axes.

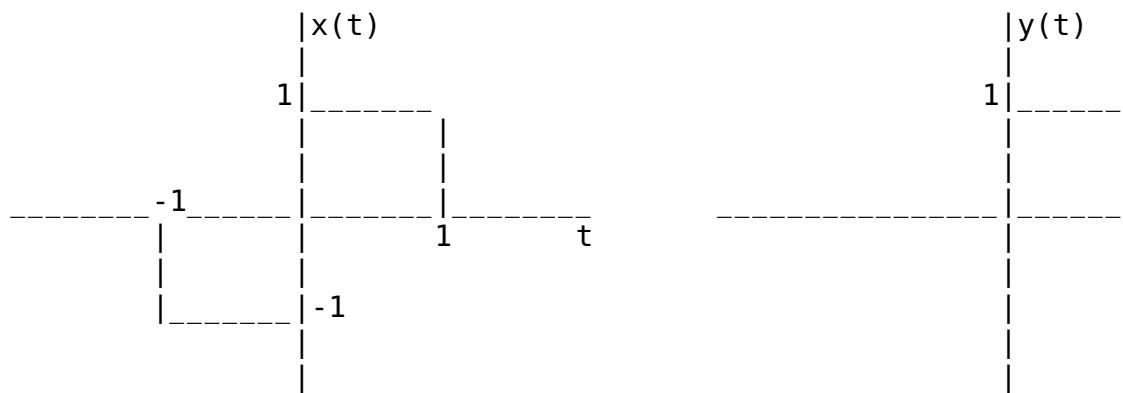


Figure 1: Convolution: $z(t) = (x * y)(t)$

Problem 3 (Inverse discrete-time Fourier Transform.) 15 Points

A discrete-time signal $h[n]$ has discrete-time Fourier transform

$$H(e^{j\omega}) = (1 + e^{-j\omega}) / (1 - 0.5e^{-j\omega}).$$

Find the signal $h[n]$.

Problem 4 (A linear time-invariant system.) 30 Points

A linear time-invariant system with input $x(t)$ and output $y(t)$ satisfies

$$(a^2)y(t) + 2a(dy(t)/dt) + d^2(y(t))/dt^2 = x(t).$$

- (a) (10 Points) Find the frequency response $H(j\omega)$ of the considered system.
- (b) (10 Points) For $a=1/2$, sketch the magnitude of the frequency response $H(j\omega)$. Is the system rather high-pass or rather low-pass? Justify your answer.
- (c) (10 Points) For what values of a is the system stable? Justify your answer. Remark: If you cannot solve the math, don't worry. Just describe clearly and concisely how you would proceed, and you will get partial credit.

Problem 5 (Filtering.) 15 points

The signal $x(t)$ with spectrum $X(j\omega)$ as shown in Figure 2 is passed through a linear time-invariant (LTI) system with impulse response

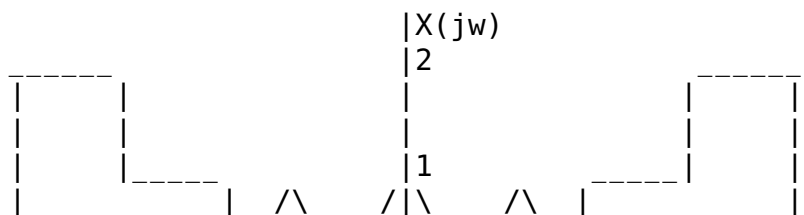
$$h(t) = 2\text{sinc}(2t),$$

where, as defined in class,

$$\text{sinc}(t) = \sin(\pi t) / (\pi t).$$

Denote the output of the system by $y(t)$. Calculate the error between $x(t)$ and $y(t)$, given by

$$\int_{-\infty}^{+\infty} |x(t) - y(t)|^2 dt.$$



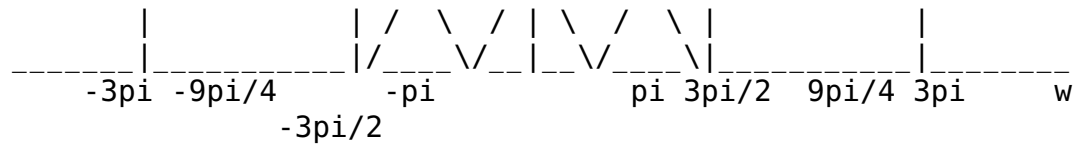


Figure 2: The spectrum of the signal $x(t)$.