Problem 1 (Short Questions) 20 Points

For each of the following statements, if you believe it is true, give a justification. If you believe it is false, give a counterexample.
(a) A linear causal continuous-time system is always time-invariant.
(b) The system with (real-valued) input x(t) and output given by

y(t) = (1+(x(t))^2)^(cos(t))

is stable.
(c) The discrete-time signal x[n] = cos(n) is a periodic signal.
(d) For an otherwise completely unknown system, it is known that when the input is given by

x(t) = cos(t) + cos(2t),

the output is

y(t) = .5(1+cos(t)+cos(2t)+cos(3t)).

This system cannot be a linear time-invariant (LTI) system.

Problem 2 (Convolution) 20 Points

The continuous-time signals x(t) and y(t) are given in Figure 1. In the figure, draw the signal z(t) given by

z(t) = (x * y)(t).

Carefully label both axes.

```
                 |x(t)                                   |y(t)
                 |                                       |
                1|_______                               1|______
                 |       |                               |
                 |       |                               |
________-1_______|_______|________       ________________|______
        |        |       1       t                       |
        |        |                                       |
        |________|-1                                     |
                 |                                       |
                 |                                       |
```

Figure 1: Convolution: z(t) = (x * y)(t)

Problem 3 (Inverse discrete-time Fourier Transform.) 15 Points

A discrete-time signal h[n] has discrete-time Fourier transform

H(e^(jw)) = (1+e^(-jw))/(1-.5e^(-jw)).

Find the signal h[n].

Problem 4 (A linear time-invariant system.) 30 Points

A linear time-invariant system with input x(t) and output y(t) satisfies

(a^2)y(t) + 2a(dy(t)/dt) + d^2(y(t))/dt^2 = x(t).

(a) (10 Points) Find the frequency response H(jw) of the considered system.
(b) (10 Points) For a=1/2, sketch the magnitude of the frequency response H(jw). Is the system rather high-pass or rather low-pass? Justify your answer.
(c) (10 Points) For what values of a is the system stable? Justify your answer. Remark: If you cannot solve the math, don't worry. Just describe clearly and concisely how you would proceed, and you will get partial credit.

Problem 5 (Filtering.) 15 points

The signal x(t) with spectrum X(jw) as shown in Figure 2 is passed through a linear time-invariant (LTI) system with impulse response

h(t) = 2sinc(2t),

where, as defined in class,

sinc(t) = sin(pi*t)/(pi*t).

Denote the output fo the system by y(t). Calculate the error between x(t) and y(t), given by

integral from -infinity to +infinity of |x(t) - y(t)|^2 dt.

```
                      |X(jw)
                      |2
______                                 ______
|     |               |                |     |
|     |               |                |     |
|     |_____          |1          _____|     |
|           |  /\    /|\    /\   |           |
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       |           | /  \  / | \  /  \ |           |
_______|___________|/____\/__|__\/____\|___________|________
    -3pi -9pi/4      -pi            pi 3pi/2  9pi/4 3pi     w
               -3pi/2
```

Figure 2: The spectrum of the signal x(t).

Problem 5 (Filtering.) 15 points