EE 120: Signals & Systems
Department of Electrical Engineering and Computer Sciences — Midterm Exam #2
University of California, Berkeley — April 11, 2016, 2:10-4:00pm

Instructions:

- There are **four** questions on this midterm. Answer each question in the space provided, and **clearly label the parts of your answer**. You can use the additional blank pages at the end for scratch paper if necessary. **Do NOT write answers on the back of any sheet or in the additional blank pages. Any such writing will NOT be graded.**
- Each problem is worth 25 points, and you may solve the problems in any order.
- Show all work. If you are asked to prove something specific, you must give a derivation and not quote a fact from your notes sheet. Otherwise, you may freely use facts and properties derived in class; just be clear about what you are doing!
- None of the questions requires a very long answer, so avoid writing too much! Unclear or long-winded solutions may be penalized.
- You may use one double-sided sheet of notes. **No calculators are allowed** (or needed).

Your Name:

Your Student ID:

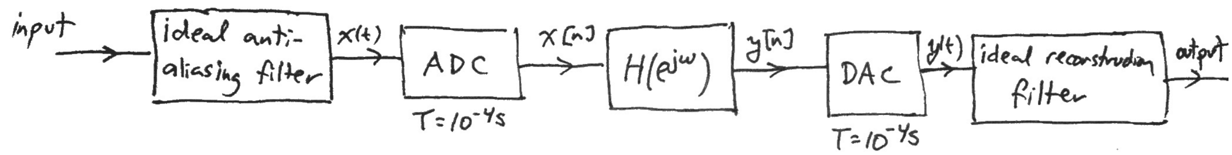
Name of Student on Your Left:

Name of Student on Your Right:

For official use – do not write below this line!

Q1	Q2	Q3	Q4	Total

Problem 1. (DT Processing of a CT Signal) A bandlimited continuous time signal is known to contain a 60-Hz component, which we want to remove by processing with the system shown below.



a) Assuming $T = 10^{-4}$ s, what is the highest frequency that the continuous time input signal can contain if aliasing is to be avoided?

b) Draw the frequency responses of the ideal anti-aliasing and ideal reconstruction filters assuming $T = 10^{-4}$ s. You may assume the ADC produces samples $x[n] = x(nT)$ for integer values of n, and the DAC generates the weighted impulse train $y(t) = \sum_{n=-\infty}^{\infty} y[n]\delta(t - nT)$.

c) The discrete-time system to be used has frequency response

$$H(e^{j\omega}) = \frac{(1 - e^{-j(\omega-\omega_0)})(1 - e^{-j(\omega+\omega_0)})}{(1 - 0.9e^{-j(\omega-\omega_0)})(1 - 0.9e^{-j(\omega+\omega_0)})}.$$

Compute the filter gain $|H(e^{j\omega})|^2$, and make a rough sketch of it.
(Hint: $\frac{(1.8-1.8\cos(\omega-\omega_0))}{(1.81-1.8\cos(\omega-\omega_0))} \approx 1$ unless ω is very close to ω_0...)

d) What value should be chosen for ω_0 in order to eliminate the 60-Hz component?

e) In words, why might it be preferable to perform the signal processing in discrete time, instead of continuous time? Give a real-life example where this might apply.

(Additional space for Problem 1)

Problem 2. (Laplace Transform) Consider the causal LTI system describe by the differential equation

$$y''(t) + 6y'(t) + 9y(t) = x''(t) + 3x'(t) + 2x(t).$$

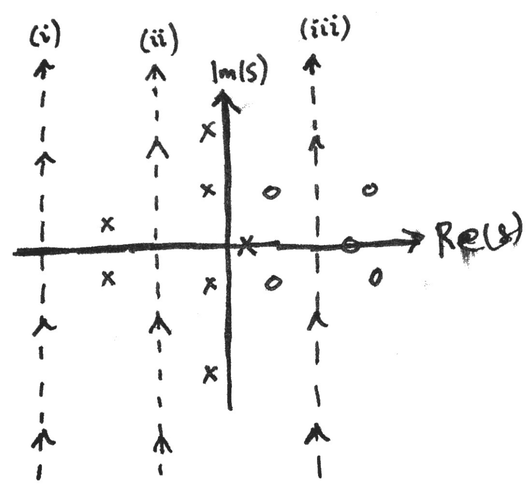
a) Find the transfer function $H(s)$ corresponding to this system, and determine whether the system is stable.

b) Find the impulse response $h(t)$ corresponding to this system.

c) The inverse of this system (taking $y(t)$ back to $x(t)$) can also be described by a causal LTI system in differential equation form. Find the differential equation describing the inverse and its transfer function $G(s)$.

d) What is the impulse response $g(t)$ corresponding to the inverse system you found in part (c)?

e) Suppose a rational function $X(s)$ has a pole-zero diagram as shown below. If you integrate the function $X(s)e^{st}$ over the variable s along the illustrated paths (in the vertical direction indicated by the arrows), what type of time domain signal will you obtain? For each path labeled (i)-(iii), respond with 'left-sided', 'right-sided' or 'two-sided'.



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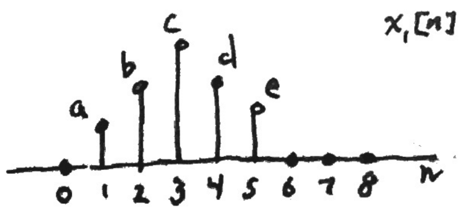
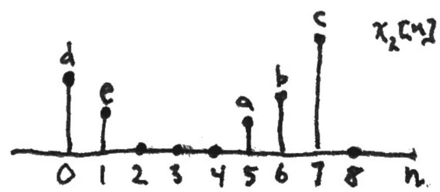
Problem 3. (DFT)

a) Consider a 20-point finite duration signal $x[n]$ such that $x[n] = 0$ outside $0 \le n \le 19$, and let $X(e^{j\omega})$ denote the DTFT of $x[n]$. You have a computer program that can compute a DFT, but not the DTFT. However, you want to evaluate $X(e^{j\omega})$ at $\omega = 4\pi/5$. Give a method for doing so.

b) Two eight-point sequences are shown below. Signal $x_1[n]$ has 8-point DFT $X_1[k]$, and signal $x_2[n]$ has 8-point DFT $X_2[k]$. Determine the relationship between $X_1[k]$ and $X_2[k]$.





c) Suppose $x_c(t)$ is a periodic continuous-time signal with period 10^{-3}s and for which the Fourier series is

$$x_c(t) = \sum_{k=-9}^{9} a_k e^{j2\pi kt/10^{-3}}.$$

The Fourier series coefficients are zero for $|k| > 9$. The signal $x_c(t)$ is sampled with a sampling period $T = \frac{1}{6}10^{-3}$s to form $x[n]$. That is,

$$x[n] = x_c\left(\frac{n10^{-3}}{6}\right).$$

i) Is $x[n]$ periodic and, if so, what is the period?

ii) Is the sampling rate large enough to avoid aliasing?

iii) Compute the 6-point DFT of $\{x[0], \ldots, x[5]\}$ in terms of the a_k's.

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Problem 4. (Overcoming Quantization Noise) Let $x_c(t)$ be a power signal with (continuous time) autocorrelation function

$$C_{x_c x_c}(\tau) := \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_c(t) x_c(t+\tau) dt = \frac{\sin(2\pi B\tau)}{\pi\tau}.$$

Suppose $x_c(t)$ is sampled according to $x[n] = x_c(nT_s)$, where the sampling period T_s satisfies $T_s < \frac{1}{2B}$. We will assume that $x[n]$ is a power signal with

$$C_{xx}[k] = C_{x_c x_c}(kT_s). \tag{1}$$

a) Give an intuitive (non-mathematical) explanation for why (1) should hold, and compute $S_{xx}(e^{j\omega})$.

b) In practice, analog-to-digital converters cannot perfectly sample $x[n] = x_c(nT_s)$, since doing so would require an infinite number of bits. Instead, they quantize the number $x_c(nT_s)$ to an interval of length Δ, which corresponds to the *resolution* of the analog-to-digital converter. In this manner, we model the quantized samples, denoted by $y[n]$, as

$$y[n] = x[n] + e[n],$$

where $e[n]$ is the *quantization error*, and is assumed to be a random number between $-\Delta/2$ and $\Delta/2$, uncorrelated from one sample to the next. Stated another way, $e[n]$ is uncorrelated with $x[n]$ and has autocorrelation function

$$C_{ee}[k] = \frac{\Delta^2}{12}\delta[k].$$

Compute $S_{xy}(e^{j\omega})$ and $S_{yy}(e^{j\omega})$.

c) Specify the optimal filter $h[n]$ (i.e., the Wiener filter) for estimating the true sample values $x[n]$ from the quantized samples $y[n]$.

d) For $\hat{x}[n] = y[n] * h[n]$, the mean square error (MSE) satisfies

$$\lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} (x[n] - \hat{x}[n])^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(S_{xx}(e^{j\omega}) - \frac{|S_{xy}(e^{j\omega})|^2}{S_{yy}(e^{j\omega})} \right) d\omega. \tag{2}$$

Use (2) to find the MSE in estimating $x[n]$ from $y[n]$ in terms of T_s, B and Δ.

e) Prove that (2) holds. (Note: this relationship holds in general for the Wiener filter, and is not specific to this problem.)

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