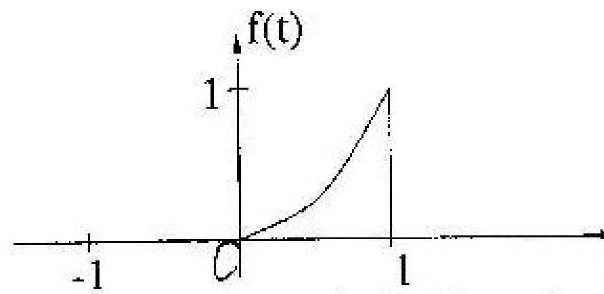
EE 120, Spring 1998
Midterm #2
Professor Lau

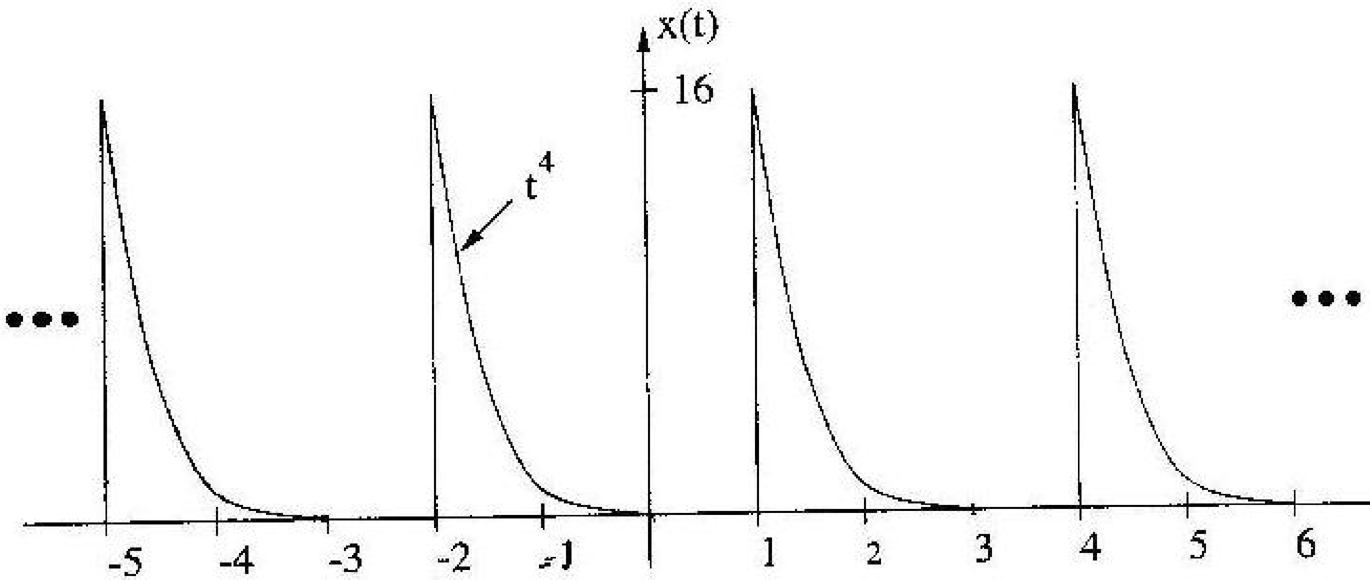
Problem #1

Consider the function

$$f(t) = \begin{cases} t^4 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$



Suppose you are told what F(w) = F {f(t)} is. Now consider the periodic function x(t):



The Fourier series expansion of x(t) is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

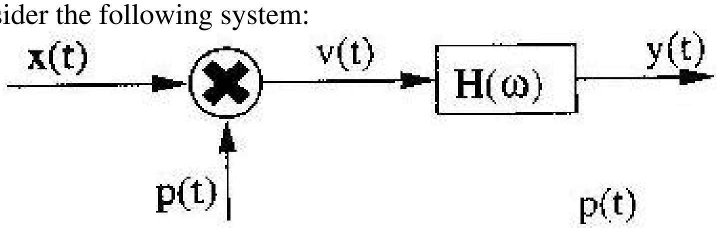
(a) What is Wo?

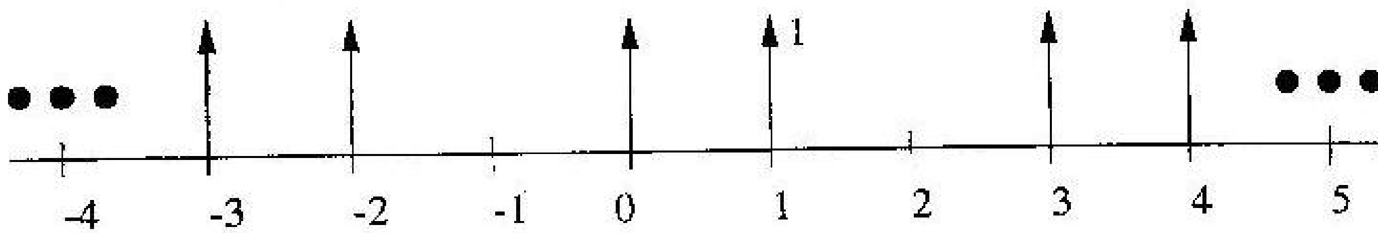
(b) Find Ak in terms of F'(w).

(c) Find $\sum_{k=-\infty}^{\infty} |a_k|^2$. (Find the actual numerical value.)

Problem #2

Consider the following system:





where x(t) is bandlimited; i.e., X(w) = 0 for |w| > B.

(a) Find P(w) = F{p(t)}. Sketch |P(w)|.

(b) What is the maximum value of B for which we can recover the original signal x(t) from the sampled signal v(t)?

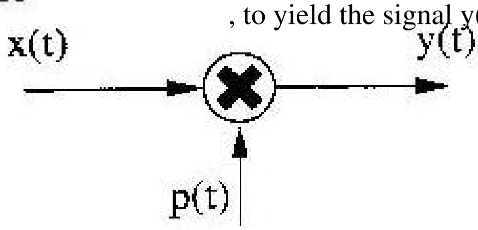
(c) If the constraint in part (b) is satisfied, sketch |H(w)| to recover x(t).

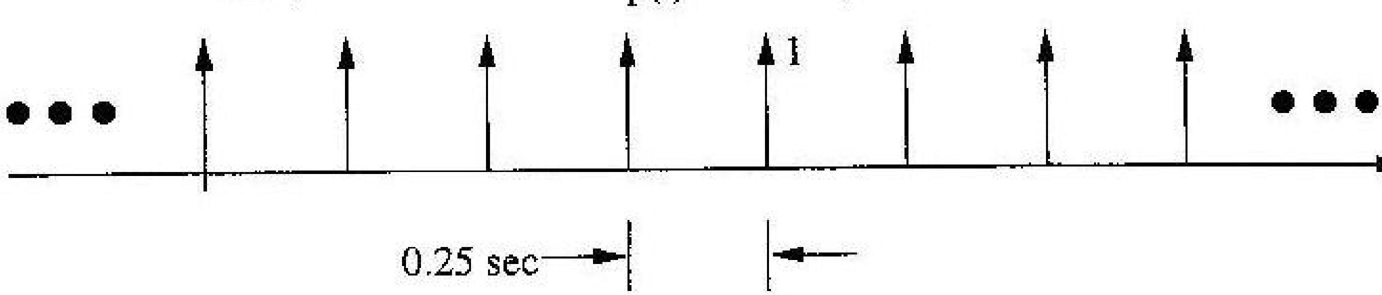
Problem #3

Consider a sine wave of frequency 5 Hz, x(t) = cos(10*pi*t), that is sampled by a comb of frequency 4 Hz, p(t) =

$$\sum_{k=\infty}^{\infty} \delta(t - \tfrac{1}{4}k)$$

, to yield the signal y(t):





(a) Sketch the spectrum (Fourier Transform) of the sampled signal y(t). (b) Is y(t) periodic? If so, what is its period? (C

Problem #4

Find the values of Tau and A which satisfy the following:

$$\Pi(t - \tau) * \sin t = A\cos(t + \frac{\pi}{4})$$

where

$$\Pi(t) = \begin{cases} 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Numerical values are not required; you may leave the result as an expression involving constants.
