University of California at Berkeley
College of Engineering
Deparment of Electrical Engineering and Computer Sciences
EECS 120: Signals and Systems
Spring Semester 1999
Midterm #1 Solution

Problem 1 (9 points, 3 each)

Are these functions periodic? If so, what is the period?

a. $\sin t + \sin 2t$ YES, with period $T = 2\pi$.

b. $\sin 5t + \cos(7t + \pi/4)$ YES, with period $T = 2\pi$.

c. $\sin 5t + \cos 7\pi t$ NO. The ratio of the periods is not a rational number.

Problem 2 (15 points, 3 each)

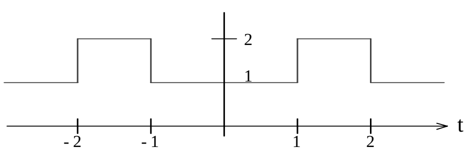
Determine whether each is a power signal, energy signal, or neither. Also calculate the power or energy for each.

a. $\sin(t)\cdot\cos(t) = 0.5 \sin 2t$ is periodic with period $T_0 = \pi$. This is a POWER signal.

$$P_\infty = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left|\sin(t)\cos(t)\right|^2 dt$$

$$= \frac{1}{T_0}\cdot\int_0^{T_0} \left(\frac{1}{2}\cdot\sin(2\cdot t)\right)^2 dt = \frac{1}{8\cdot\pi}\cdot\int_0^{\pi} 1 - \cos(4\cdot t)\, dt = \frac{1}{8\cdot\pi}\cdot\pi = \frac{1}{8}.$$

b. $\displaystyle\sum_{n=-\infty}^{\infty} \Pi\left(\frac{t-3\cdot n}{4}\right)$ is periodic with period T = 3. This is a POWER signal.



$$P_\infty = \frac{1}{3}\cdot\left(\int_{-1}^{1} 1^2\, dt + \int_{1}^{2} 2^2\, dt\right) = 2$$

c. $\displaystyle\sum_{n=-\infty}^{\infty} \delta(t-n)\cdot\sin(\pi\cdot t) = 0$. This is an ENERGY signal with $E_\infty = 0$.

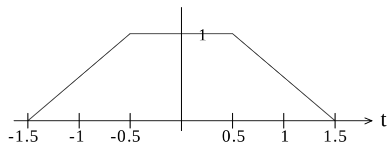
d. $\sqrt{\delta\left(t-\frac{1}{4}\right)\cdot\cos(\pi\cdot t)} = \sqrt{\delta\left(t-\frac{1}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)}$

This is an ENERGY signal.

$$E_\infty = \int_{-\infty}^{\infty} \cos\left(\frac{\pi}{4}\right)\cdot\delta\left(t-\frac{1}{4}\right) dt = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Problem 2 cont'd.

e. $\Pi(t) * \Pi(t/2)$



This is an ENERGY signal.

$$E_\infty = \int_{-1.5}^{-0.5} (1.5+t)^2\,dt + \int_{-0.5}^{0.5} 1^2\,dt + \int_{0.5}^{1.5} (1.5-t)^2\,dt = 1.667 \quad = \frac{5}{3}$$

Problem 3 (10 points)

$$y(t) = e^{-t}u(t) * \sum_{n=0}^{\infty} \delta(t-n)$$

Find the value of $y(0)$, $y(1)$, $y(2)$, and $y(\infty)$.

For integer values of $t = N > 0$, $y(t) = \sum_{n=0}^{N} e^{-n}$

$y(0) = 1$
$y(1) = 1 + e^{-1}$
$y(2) = 1 + e^{-1} + e^{-2}$

$$y(\infty) = \sum_{n=0}^{\infty} e^{-n} = \frac{1}{1-e^{-1}} = \frac{e}{e-1}$$

Problem 4 (13 points, 3/6/4)

$$x(t) = \sin^2(t) \longrightarrow \boxed{h_1(t) = e^{-t}\,u(t)} \longrightarrow y(t)$$

a. Find the Fourier series expansion of *x(t).*

$$x(t) = \sin^2(t) = \frac{1-\cos 2t}{2} = \frac{1}{2} - \frac{1}{4}\cdot\left(e^{j2t} + e^{-j2t}\right) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j\cdot k\cdot\omega_0\cdot t}$$

So, what is ω_0 ? The fundamental period of the signal is $T_0 = \pi$.

So, $\omega_0 = \frac{2\cdot\pi}{T_0} = 2$. Thus, x(t) = $\sum_{k=-\infty}^{\infty} a_k \cdot e^{j\cdot k\cdot 2\cdot t}$.

The coefficients of expansion are:

$$a_0 = \frac{1}{2}$$

$$a_1 = \frac{-1}{4}$$

$$a_{-1} = \frac{-1}{4}$$

$a_k = 0$ for all other k.

b. Find the Fourier series expansion of *y(t).*

Recall for LTI system with input of form $a\,e^{j\omega t}$, the output is the product of the input and $H(\omega)$, where

$$H(\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau}\cdot h(\tau)\,d\tau = \int_{-\infty}^{\infty} e^{-j\omega\tau}\cdot e^{-\tau}\cdot u(\tau)\,d\tau = \int_{0}^{\infty} e^{-(1+j\omega)\cdot\tau}\,d\tau = \frac{1}{1+j\omega} = \frac{1-j\omega}{1+\omega^2}$$

Thus, for the expansion of y(t) = $\sum_{k=-\infty}^{\infty} b_k \cdot e^{j\cdot k\cdot 2\cdot t}$, the coefficients of expansion are

$b_k = H(\omega)\,a_k$, where $\alpha = k\,\omega_0$:

$$b_0 = \frac{1}{2}$$

$$b_1 = \frac{-1}{4}\cdot\frac{1-2j}{1+2^2} = -\frac{1}{20}\cdot(1-2j)$$

$$b_{-1} = \frac{-1}{4}\cdot\frac{1+2j}{1+2^2} = -\frac{1}{20}\cdot(1+2j)$$

$b_k = 0$ for all other k.

Problem 4 cont'd.

c. Sketch the 2-sided amplitude and phase spectrum of *x(t)* and *y(t)*. Label salient features.