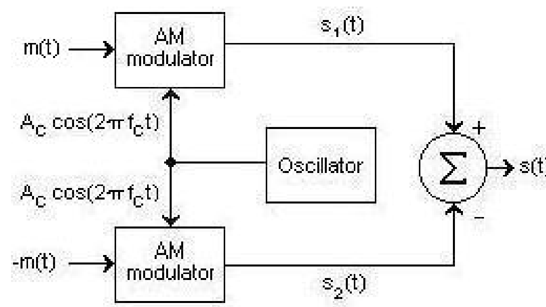
EECS 121, Spring 1996
Midterm #2

Note : Please answer all questions. Please answer with sufficient detail and clarity that there is no ambiguity about your answer.

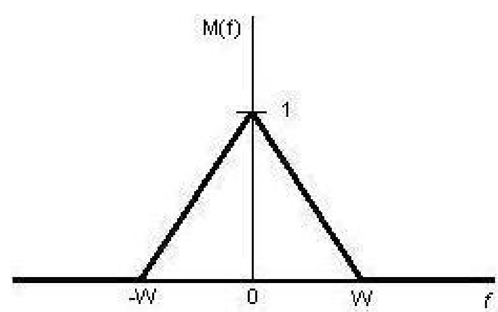
Problem #1

The diagram show is that of a *balanced modulator* which is used to generate a DSB-SC AM signal.



Here $s_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ and $s_2(t) = A_c[1 - k_a m(t)]\cos(2\pi f_c t)$ where k_a is a fixed constant called the *amplitude sensitivity* of the modulator.

The input message signal $m(t)$ is a lowpass signal having the following Fourier transform:



Here we assume that $W << f_c$.

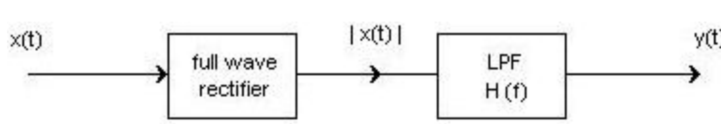
(a) Plot $S_1(f)$.
(b) Plot $S(f)$.

Problem #2

Let $x(t) = \frac{1 - \cos t}{t}$. Find the corresponding Hilbert transform $\hat{x}(t)$.

Problem #3

Consider a bandpass signal $x(t) = V(t)\cos(2\pi f_0 t + \theta(t))$ where $f_0 = 100$, $V(t) = \text{sin c}^2(t)$, and $\theta(t) = 2\pi\sin(2\pi t)$. Determine the output $y(t)$ of the following system:



Here the lowpass filter transfer function is given by

$$H(f) = \begin{cases} 1, & |f| < 1 \\ 0, & \text{otherwise} \end{cases}$$

Hint: Write $|\cos(2\pi f_0 t)|$ as a Fourier series.

Problem #4

Let X be a random variable with the density

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Here $\sigma > 0$ is a parameter. Such a random variable is said to have a Rayleigh density.

(a) Find E[X]. (*Hint:* What is the variance of a Gaussian random variable?)
(b) Find Var(X).
(c) Let $Y = X^2$. Find the density of Y.
