Midterm 1

Prof. Kannan Ramchandran

October, 24, 2000 7:00pm - 9:00pm

The midterm has FIVE(5) Questions. Please make sure that there are FIFTEEN (8 blank) pages following this page.

- DO NOT open the exam until instructed to do so.
- This is a closed book exam.
- You are allowed ONE side of a 8.5x11 inch sheet.
- You have 120 minutes to finish this exam.
- Box your final answers.
- Partial marks will not be awarded to answers that have no proper reasoning.
- Answers arrived at with the aid of programmable calculators, which do not show insight into the problem will not fetch any credit.
- Remember to write your name and SID on the top right corner of every sheet of paper.
- You may use the empty pages to do your work.

NAME: ______________________________

SID: ______________________________

- **Problem 1** (25 points)

I.) For each of the following systems, determine if the system is linear, causal, shift-invariant, and BIBO stable. Note that x[n] and y[n] denote the system input and output respecitively. Indicate "**Y**" for Yes, "**N**" for No, and "**X**" for "cannot be determined due to insufficient information."

I.a.) (4 points) $y[n] = cos\left(\sqrt{|n|}\right) x[n]$.

____ Linear ____ Causal ____ Shift-Invariant ____ Stable

I.b.) (4 points) The response of the system to an input of $\delta[n-1]$ is $(0.5)^n u[n]$.

____ Linear ____ Causal ____ Shift-Invariant ____ Stable

II.) (2 points) If an LSI system has a unit pulse response h[n] given and $0.1 < |h[n]| < 0.2$ for all n, is the system BIBO stable?

____ Stable ____ Unstable ____ Cannot be determined

III.) (4 points) The sequence $x[n] = \{\ldots, 0, \underset{\uparrow}{-1}, 2, 2, 0, 0, \ldots\}$, where the arrow indicates x[0] is input to an LSI system with unit pulse response $h[n] = (-1)^n [u[n+1] - u[n-2]]$. Determine the value of n for which $|y[n]|$ is maximum, and find this maximum value.

____ n_{max} ____ $y[n_{max}]$

IV.) (5 points) An LSI system with unit-pulse response h[n] has the transfer function:

$$H(z) = \frac{z^2 - 16}{(z - 0.25)(z + 4)(z - 2)}$$

It is known that $|h[n]| < 4$ for all $n \leq -4$, but the sum $\sum_{n=-\infty}^{\infty} |h[n]|$ diverges (i.e. is not finite).

The ROC of H(z) is ____________.

V.) (6 points) A system has a transfer function:

$$H(z) = \frac{(z+3)^4}{\left(z-\frac{1}{2}\right)(z+2)^2}$$

In the spaces below, specify all possible ROC's and indicate the properties that apply to the system associated with each ROC.

ROC	Stable	Causal	Non-causal

- **Problem 2** (15 points)

I.) (7 points) The two-sided z-transform of a stable system is:

$$H(z) = \frac{1}{z^7 \left(z + \frac{1}{3}\right)^2}$$

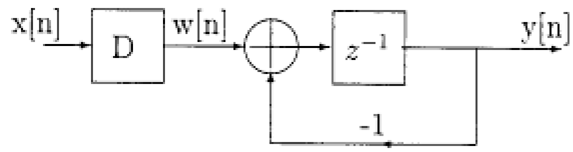
Find h[n] for all n.

II.) (8 points) A system's input-output behavior is characterized by the equation:

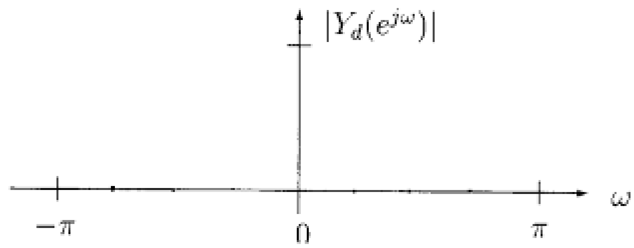
$$y[n] = -y[n-6] + 2x[n] + x[n-5]$$

Assume that the system is causal (i.e. y[-1]=0). Find the unit pulse response h[n] of the system for n=0,1,5,6,10,11, and 120.

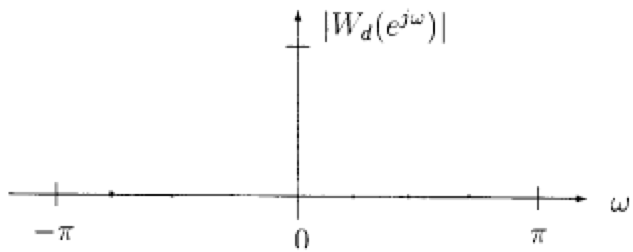
- **Problem 3** (10 points) The input to the system shown below is $x[n] = cos\left(\frac{3\pi}{4}n\right)$ for $-\infty < n < \infty$.



I.) (2 points) If the block D is removed (i.e. x[n]=w[n]), then sketch the magnitude spectrum of $Y_d(e^{j\omega})$ and give a closed form expression for y[n].



II.) (3 points) Now suppose that D is a down-sampler by a factor of 2. Sketch the magnitude spectrum $W_d(e^{j\omega})$ of $w[n]$ and give a closed form expression for w[n].

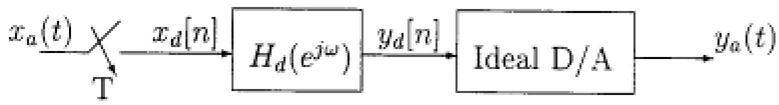


III.) (3 points) Derive a closed-form experession for $y[n]$.

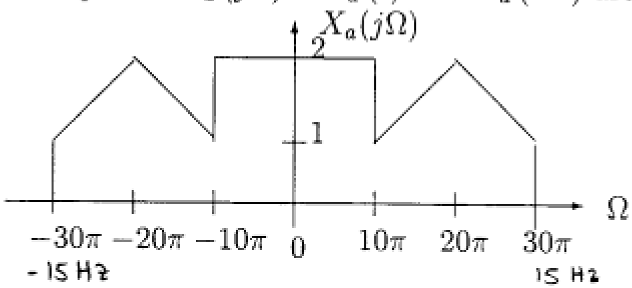
IV.) (2 points) Is the system time-invariante? <u>Prove</u> your answer to get credit.

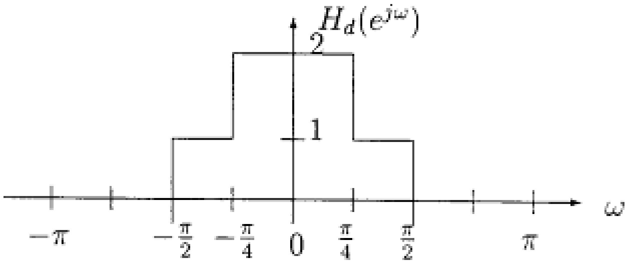
- **Problem 4** (25 points)

  Consider the following digital processor for analog signals.



The spectra $X_a(j\Omega)$ of $x_a(t)$ and $H_d(e^{j\omega})$ are both real, and as shown below:

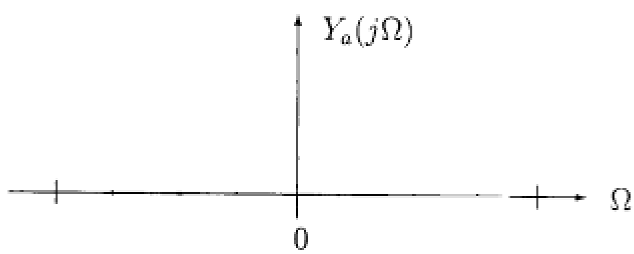




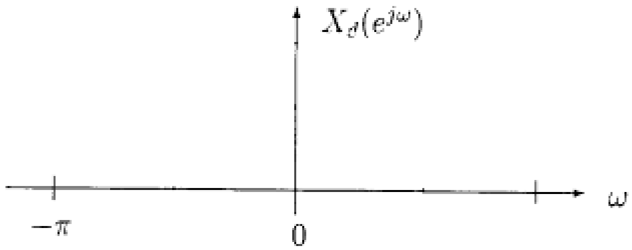
In all the plots, be sure to label the axes, and mark the values at the transition points.

I.) (4 points) Find the largest T that will prevent aliasing at the sampler.

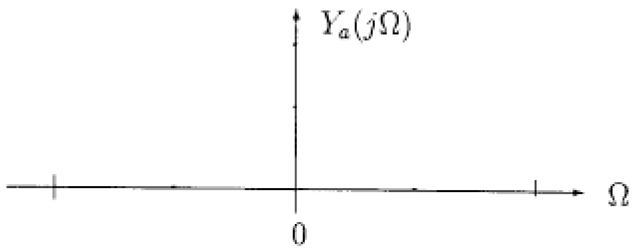
II.) (7 points) Plot $Y_a(j\Omega)$, assuming $T = \frac{1}{40}$ seconds.



III.) (7 points) Plot $X_d(e^{j\omega})$, assuming $T = \frac{1}{20}$ seconds.

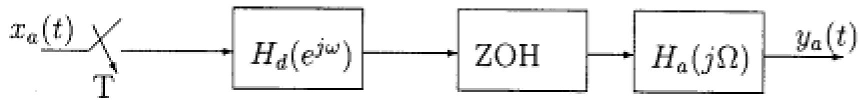


IV.) (7 points) Plot $Y_a(j\Omega)$, assuming $T = \frac{1}{20}$ seconds.



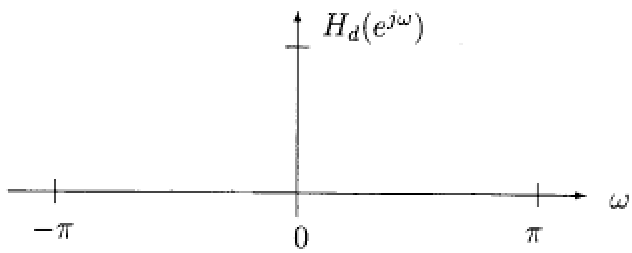
- **Problem 5** (25 points)

  You are assigned the task of designing a signal processing system according to the diagram below to implement a bandpass filter passing frequencies between 2kHz and 6kHz and suppressing all other frequencies. The input signal is bandlimited to 10kHz.

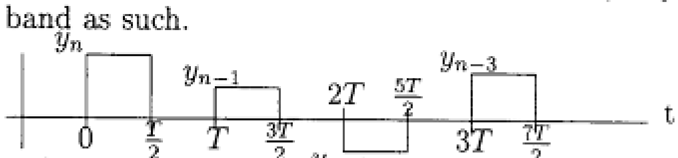


1. (5 points) What is the minimum sampling frequency required?

II.) (8 points) For this sampling frequency, assuming the D/A is ideal, sketch the magnitude of the ideal desired response of the digital filter over the frequencies $\omega \in [-\pi, \pi]$.



III.) (12 points) You are offered a great deal on a ZOH circuit that only holds for half the sample interval and then returns to zero (see sketch below). Determine the ideal frequency response of the analog reconstruction filter $H_a(j\Omega)$ such that the the D/A reconstruction in this system is ideal. Plot the ideal magnitude response over the frequencies which must be controlled and specify any "don't care" regions of the frequency band as such.



ZOH Operation