EE123

Midterm 1

Prof. Avideh Zakhor

October 22, 2003 9:30am - 11:00am

The midterm has THREE(3) Questions. Please make sure that there are FOUR(4) pages following this page.

- This is a closed book exam.
- You are allowed ONE double sided 8.5x11 inch sheet.
- This exam contributes to 40% of the total score.
- Partial marks will not be awarded to answers that have no proper reasoning.
- Answers arrived at with the aid of programmable calculators, which do not show insight into the problem will not fetch any credit.

- **Problem 1** (20 points)

(a) (6 points) Find, in terms of $x[n]$ and $y[n]$, the sequence whose Fourier transform is $X(e^{j\omega})Y^*(e^{j\omega})$.

(b) (7 points) Show that

$$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega.$$

(c) (7 points) Determine the numerical value of the sum

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/4)}{2\pi n}\frac{\sin(\pi n/6)}{5\pi n}$$

- **Problem 2** (35 points)

The signal $y[n]$ is the output of an LTI system with impulse response $h[n]$ for a given $x[n]$. Given $y[n]$ is stable and has a z-transform as in figure 1 and $x[n]$ is stable with a z-transform as in figure 2:

(a) (5 points) What is the region of convergence of $Y(z)$?

(b) (5 points) Is $y[n]$ left sided, right sided, or two sided?

(c) (5 points) What is the ROC of $X(z)$?

(d) (5 points) Is $x[n]$ a causal sequence?

(e) (5 points) What is $x[0]$?

(f) (5 points) Draw the pole-zero plot of $H(z)$, and specify its ROC.

(g) (5 points) Is $h[n]$ anticausal?

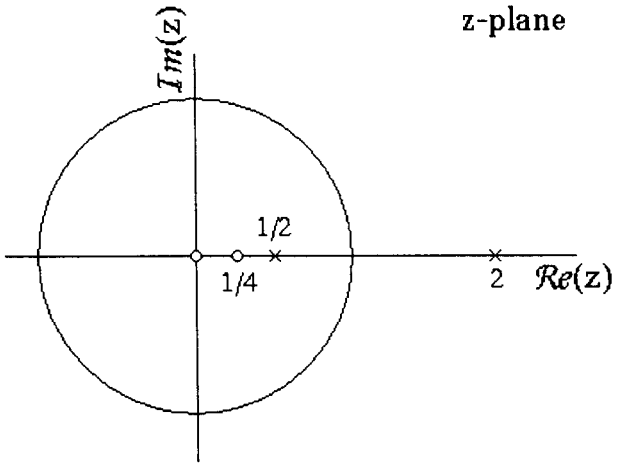

Figure 1: Z-transform of $y[n]$

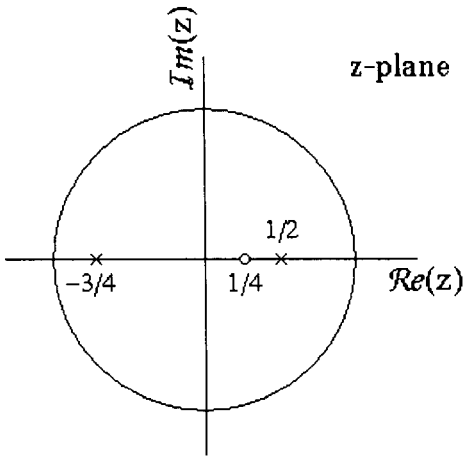

Figure 2: Z-transform of $x[n]$

- **Problem 3** (45 points)

Given the system in figure 3. Furthermore, it is known that:

$$w[n] = \begin{cases} x_1[n/2] & \text{n is even} \\ x_2[(n-1)/2] & \text{n is odd} \end{cases}$$

Assume $x_1[n]$ and $x_2[n]$ come from sampling the signals $x_{c1}(t)$ and $x_{c2}(t)$ at a rate T.

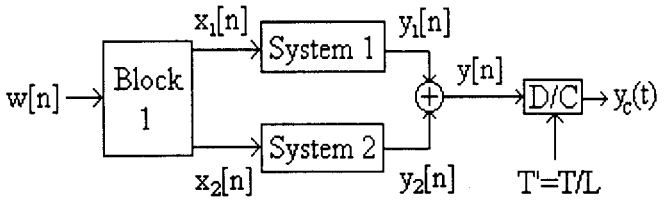

Figure 3: Overall system of question 3

Assume also that the continuous time signals both have a highest frequency of Ω_N and that $T = \pi/\Omega_N$.

(a) (10 points) Draw a block diagram of a system that takes as input $w[n]$ and outputs $x_1[n]$ and $x_2[n]$. Your system may consist of adders, multipliers, time shifts, upsamplers, downsamplers, and LTI filters. This is the equivalent of block 1 in figure 3. State whether or not your system is linear, time invariant, causal, and/or stable.

(b) (15 points) The ith system ($i = 1$ or 2) is defined by the block diagram in figure 4. The lowpass filter $H_k(e^{j\omega})$, which is the same for both systems, has gain L and cutoff frequency π/L. The highpass filters $H_i(e^{j\omega})$ have unity gain and cutoff frequency ω_i. The ω_i's have the following relationship (Assume $\omega_1 > \pi/2$):

$$\omega_2 = \omega_1 + \pi/L$$

$$\omega_2 + \pi/L \leq \pi$$

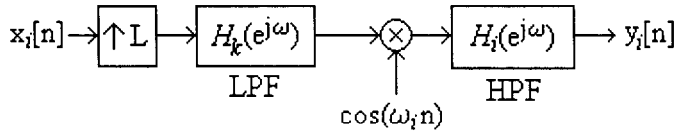

Figure 4: Block Diagram of System i

Assume that $\Omega_N = 2\pi \text{ x } 5 \text{ x } 10^3$. Find ω_1 and the smallest L so that $y_c(t)$ is zero, except in the band of frequencies $2\pi \text{ x } 10^5 \leq \Omega \leq (2\pi \text{ x } 10^5) + 2\Omega_N$.

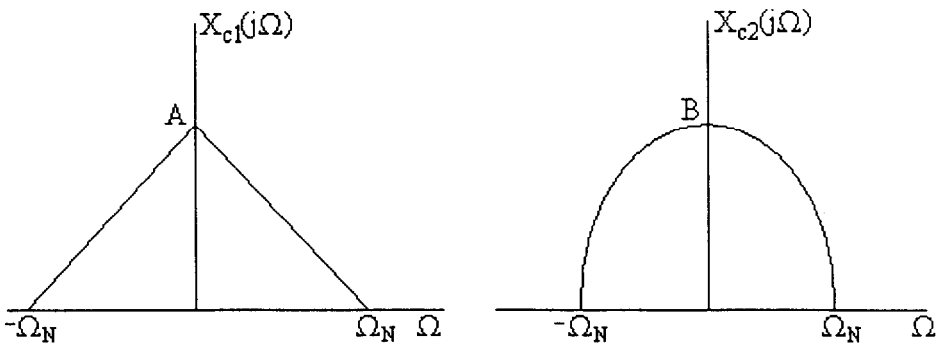

Figure 5: Fourier transforms of $x_{c1}(t)$ and $x_{c2}(t)$.

(c) (20 points) Assume that the continuous-time Fourier transforms of the two original input signals are as sketched in figure 5. Sketch the Fourier transforms at each point in the system (i.e. Plot the Fourier transforms of the signals $x_1[n]$, $x_2[n]$, the spectra of all intermediate signals in systems 1 and 2, and the spectrum of the outputs $y[n]$ and $y_c(t)$).