EE 126, Fall 2001
Midterm #2
Professor Anantharam

The exams starts at 3:40 p.m. sharp and ends at 5:00 p.m. sharp.
There are 5 problems. The maximum score is 50 points.
The exam is open book and notes.

Problem #1 - *25 points*

For each of the following statements, indicate whether you believe that the statement is true or believe it is false, and give a brief explanation of your reasoning. A correct answer without a valid explanation gets 1 points. A correct answer with a valid explanation gets 5 points.

(a) If X is a Gaussian random variable the X, $2X$, and $3X$ are jointly Gaussian random variables.

(b) Let X, Y, and Z be random variables, which you may assume have a joint density. Let $W = Y + Z$. Then

$$E[W \mid X] = E[Y \mid X] + E[Z \mid X]$$

(c) Let X, Y, and Z be random variables, which you may assume have a joint density. Let $W = Y + Z$. Then

$$E[X \mid W] = E[X \mid Y] + E[X \mid Z]$$

(d) If X is Gaussian and Y is uncorrelated with X, then X and Y are independent.

(e) If Gaussian random variables X and Y have the same mean and the same second moment then they have the same fourth moment.

Problem #2 - *7 points*

Let $X \sim N(2,2)$. Let the conditional density of Y given X be given by

$$f_{Y|X}(y|x) = (1/((3/2^{1/2})*(2*PI)^{1/2}))*e^{(-1/2)*(2/9)*(y-(1/2)*(x-2)-3)^2},$$

i.e. conditional on X, Y is Gaussian with mean $(1/2)(X - 2) + 3$ and variance $9/2$. Find the density of Y.

Problem #3 - *6 points*

I throw three darts at a disk of radius R_0 centered at the origin. Each dart lands on the disk and the point at which it lands is distributed according to

$$f_{R\,THETA}(r, theta) = \begin{cases} (3*r^2/(2*PI*R_0^3)) & \text{if } 0 <= r <= R_0 \text{ and } -PI <= THETA <= PI \\ 0 & \textit{elsewhere} \end{cases}$$

Problem #4 - *7 points*

X and Y are jointly Gaussian mean zero random variables. You are told that the the variance of $X + Y$ is 1, the variance of $X - Y$ is 1, and the $Cov(X + Y, X - Y) = 0$. Find the joint density of X and Y.

Problem #5 - *5 points*

X and Y are independent random variables, each of which is exponentially distributed with parameter 1. Find the linear MSE estimate of $X + Y$ given $X - Y$.