

Midterm — September 23

*Lecturer: Jean C. Walrand**GSI: Daniel Preda***Problem 1. (Short Problems 25%)**

- Give an example of two events A and B such that $P[A|B] = P(B) > P(A)$.

- Complete the sentence: *A random variable is ...*

- Give an example of two random variables X and Y that are uncorrelated but not independent.

- Prove that $\text{var}(aX) = a^2\text{var}(X)$.

- Assume that X is exponentially distributed with rate 1, so that its probability density function is $f_X(x) = \exp\{-x\}$. Calculate $E(|X - 1|)$.

Problem 2. (20%) *The EE126 students fall into three categories. 10% of the students work extremely hard and have a 40% chance of getting an A in the class. 20% work very hard and have a 25% chance of getting an A. The remaining 70% work hard and have a 10% chance of getting an A.*

(a) *What fraction of the students get an A?*

(b) *Given that a student did not get an A, is it more likely that he worked extremely hard, very hard, or just hard? What is the probability of that most likely event given that he did not get an A?*

(c) *Assume that you value an A in EE126 at \$10,000.00 more than not getting an A because of the improved chance of getting into a great graduate school. Assume also that working extremely hard means 300 hours of work, working very hard means 200 hours, and working hard means 100 hours. Say that you value your time at \$10.00 per hour. If you want to maximize your expected payoff minus the value of your time, should you work extremely hard, very hard, or only hard?*

Problem 3. (20%) You pick a point (X, Y) uniformly inside the unit circle $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$. Calculate $\text{var}(X)$. [Hint: You may want to consider $X^2 + Y^2$.]

Problem 4. (15%) *Show that*

$$\text{var}(X + Y) = \text{var}(X) + \text{var}(Y) + 2\text{cov}(X, Y).$$

Problem 5. (20%) You choose two points X and Y independently and uniformly in $[0, 1]$. You then look at the three segments that were created by cutting $[0, 1]$ in the two places X and Y . What is the probability that the largest segment is less than 0.4?