Department of EECS - University of California at Berkeley

EECS 126 - Probability and Random Processes - Spring 2008

Midterm 1: 2/28/2007

Name (Last, First):

SID:

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1. (15%)

There are two coins. Coin 1 is fair. Coin 2 is such that $P(H) = 0.6$.

1) You flip the two coins together repeatedly. What is the probability that coin 1 yields H before coin 2?

2) You are given one of the two coins, with equal probabilities. You flip the coin twice and you get H both times. What is the probability that you got coin 1?

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2. (20%)

You throw a dart at a circular target with radius 1. You miss the target with probability 0.2. If you hit the target, the dart location is uniformly distributed inside the target. Let X be the distance from dart to the center of the target when you hit it and $X = 2$ when you miss the target.

1) What is the p.d.f. of X;

2) Plot the c.p.d.f. of X;

3) Calculate $Var(X)$, the variance of X.

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3. (20%) You pick a point ω uniformly in the square $[0,1]^2$ and you designate the coordinates of the point by $X(\omega)$ and $Y(\omega)$.

1) Calculate $E(|X-Y|)$.

2) Calculate $P[X \leq x \mid |X-Y| > 0.5]$ for $x \in [0,1]$.

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4. (15%)

Assume that humanity will either survive 10 billion years or ten million years, with equal probabilities. For simplicity, assume that the population is constant and about equal to 8 billion people, in both cases. Assume also that you are picked randomly human, among all humans who will ever live. You observe that humanity has been around for about 5 million years. What is the probability that humanity will survive ten billion years, given your observation?

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5. (15%)

A randomly picked 126 student has a 20% chance of being a genius and an 80% chance of being very smart but somehow short of genius. A genius gets a score on the first midterm that is uniformly distributed in $[70, 100]$. A very smart student gets a score that is uniformly distributed in $[0, 100]$. A genius has a probability 80% of going to graduate school and a very smart student has a probability 20% of going to graduate school. What is the probability that a randomly picked student who gets a score of 80 will go to graduate school?

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6. (15%)

Assume that $P(X = n) = (1-p)^{n-1}p, n \geq 1$ where $p \in (0, 1)$. Calculate $E(X^k)$ for $k \geq 1$.
