EE126: Probability and Random Processes **SP'10**

Midterm — February 18

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Problem 1. (**Multiple Choice 20%**)

- *Assume* $P(A) = 0.2$, $P(B) = 0.6$, $P(A \cup B) = 0.5$. *Then* $P[A|B] = ...$
  (Circle one): 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1.

- *Assume that* $X = n$ *with probability* $\alpha 3^{-n}$ *for* $n \geq 1$. *Then* $E(X) = ...$
  (Circle one): 0.25, 0.5, 1, 1.5, 2, 2.5, 3, 4, 5, 6, depends on α

- *Assume that* $\Omega = [0, 1]$, $\mathcal{F} = \{\emptyset, [0, 0.3], (0.3, 1], [0, 1]\}$ *and* $P([0, 0.3]) = 0.4$. *Which of the following functions of* ω *are random variables? (Circle the correct answers):* $X(\omega) = 1\{\omega \leq 0.4\}$, $X(\omega) = 1$, $X(\omega) = 2 + 3 \times 1\{\omega \leq 0.3\}$, $X(\omega) = 1\{\omega \leq 0.5\}$.

- *Assume that* $X \geq 0$, $E(X) = 4$, *and* $\text{var}(X) = 0.3$. *Circle the statements that are certainly true:*
  $P(X > 10) \leq 0.4$, $P(X > 10) \leq 0.5$, $P(X > 10) \leq 0.6$, $P(|X - 4| > 1) \leq 0.2$, $P(|X - 4| > 2) \leq 0.075$, $P(|X - 1| > 3) \leq 0.2$.

- *Assume that* X *is exponentially distributed with rate 2. Then (circle the true statements):*
  $E(X) = 2$, $E(X) = 0.5$, $P[X > 2|X > 1] > P(X > 2)$,
  $P[X > 1|X > 2] < P(X > 1)$, $P[X > 3|X > 1] = P(X > 2)$.

- *Assume that* X *is uniformly distributed in* $[0, 1]$. *Then (circle the correct statements):* $E(\cos(X)) = \sin(1)$, $E(X^n) = 2n$, $E(X^n) = 1/(n + 1)$, $\text{var}(X) = 1/4$, $\text{var}(X) = 1/12$.

  $\int^1$

- *Let* (X, Y) *be uniformly picked in the triangle with vertices* $(0, 0), (0, 1), (1, 0)$. *Circle the correct statements:*
  X, Y *are independent.* X, Y *are positively correlated.* X, Y *are uncorrelated.*

- *Complete the following sentence:* A random variable is a ....

Problem 2. *(**20%**) A random number generator of type n selects a number uniformly in the set $\{1, 2, \ldots, n\}$. You are given such a generator and you are told that its type is n with probability $n2^{-n-1}$ for $n \geq 1$. Let X be the value that the random number generator selects. Given X, what is the probability that the type of the generator is n, for $n \geq 1$?*

Problem 3. *(**20%**) The point (X, Y) is picked uniformly in the triangle with vertices $(0, 0), (0, 1), (1, 1)$. Calculate $\text{cov}(X, Y)$, the covariance of X and Y. Recall that $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$.*

Problem 4. *(**20%**) Let X be the random variable selected as follows: with probability* 0.2*,* $X = 0.8$*; with probability* 0.3*,* $X = 1.4$*; with probability* 0.5*,* X *is picked uniformly in* $[0, 2]$*. (a) Give the pdf* $f(x)$ *of* X*; (b) Give the cpdf* $F(x)$ *of* X*; (c) Calculate* $E(X)$*; (d) Calculate* var(X)*.*

Problem 5. *(10%) Let $F(x)$ be the cpdf of a random variable X. Prove that $F(x)$ is right-continous.*

Problem 6. *(10%) For $n \geq 1$, let $X_n(\omega) = 1\{\omega \in A_n\}$ where the events A_n are such that $P(A_n) = 1/(4n^2)$. Show that $X := \sum_{n=1}^{\infty} X_n$ is finite with probability 1.*