UNIVERSITY OF CALIFORNIA, BERKELEY
Department of Electrical Engineering and Computer Sciences

EE128 — Fall 2009
Feedback Control — Prof. Jose M. Carmena

FINAL EXAM
December 15th 2009

NAME: ___________________________________ , ___________________________
(print) Last First

STUDENT ID#:________________

SIGNATURE: ___________________

General Instructions:

1. **Do not begin this test nor turn this page until you are explicitly asked to.**
2. **This is a CLOSED BOOK, CLOSED NOTES, NO CALCULATOR exam.**
3. **This exam should take you less than 180 minutes.**
4. **Please make sure to TURN OFF ALL NOISE MAKING DEVICES LIKE CELL PHONES and PAGERS.**
5. **SHOW YOUR WORK on this exam. Make your methods clear to the grader so you can receive partial credit.**
6. **MAKE SURE THE EXAM HAS 17 NUMBERED PAGES. There are 14 questions and 41 points in total.**

1) [3 points] Given a state-space representation with the following A and D matrices, what are size of the vectors x, u, and y? What are the dimensions of the controllability matrix $\mathcal{C}$ and observability matrix $\mathcal{O}$? For state feedback with a full-state estimator, what are the necessary dimensions of feedback gain matrix K and the observer gain matrix L?

$$A = \begin{bmatrix} 0 & 3 & 2 \\ 0 & 3 & 1 \\ 2 & -2 & -3 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

2) [2 points] Given Ackermann's formula $\boldsymbol{K} = [\mathbf{0} \quad \ldots \quad \mathbf{0} \quad \mathbf{1}]\boldsymbol{\mathcal{C}}^{-1}\boldsymbol{a}(\boldsymbol{A})$, where $\mathcal{C}$ is the controllability matrix and $a(s)$ is the characteristic equation, what two system conditions will cause this method to fail (make $\mathcal{C}$ noninvertible)?

3) [2 points] For the following system, you have the choice between two different sensors: $y_1 = x_1$ and $y_2 = x_2$. Which would you choose and why?

$$\dot{x} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

4) [2 points] For a combined plant and full-state estimator system with state $\begin{bmatrix} x \\ e \end{bmatrix}$, where x is the state of the plant and $e = \hat{x} - x$ is the estimator error, what are the combined system's eigenvalues?

5) [2 points] We wish to draw the Nyquist plot for the following open-loop transfer function.

$$G(s) = \frac{s^2+2s+2}{s^3(s+1)(s-1)}$$

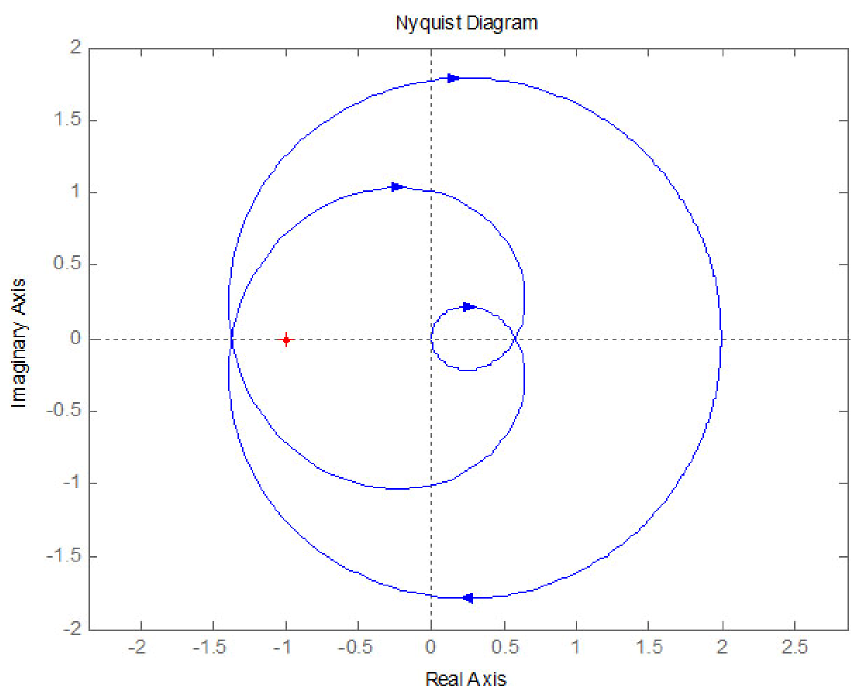
If we decide to *include* the poles at the origin in our original contour:

a) What is the value of P in the criterion $Z = N + P$?
b) In the Nyquist plot, what will be the phase change (# of loops) at infinity?

6) [2 points] Why do we examine $s = j\omega$ for frequency response? Why do we examine $s = j\omega$ for Nyquist?

7) [3 points] The three primary design parameters for a lead compensation using the frequency response method are crossover frequency, phase margin, and low-frequency gain. *Briefly* explain what characteristics of the system's dynamic response are determined by each of these three parameters.

8) [3 points] The following plot is the Nyquist plot for the open-loop transfer function $G(s) = \frac{2K(s^2-2s+2)}{s^3+3s^2+4s+2}$ with $K = 1$. The set of the real axis crossings is $\{-1.38,\ 0,\ 0.58,\ 2\}$. Fill in the table below. For what values of K is this system stable? (leave the bounds as fractions)



Interval	$(-\infty, -1.38)$	$(-1.38, 0)$	$(0, 0.58)$	$(0.58, 2)$	$(2, \infty)$
N					

9) [2 points] For a controllable system (A, B, C, D), we apply full-state feedback $u = -Kx$. Is the new combined system still controllable? Now we apply full-state feedback with reference $u = -Kx + r$. What is the new controllability matrix?

10) [3 points] Fill in the table below with either 'Yes', 'No,' or 'NEI' for not enough information. $\sigma(A)$ refers to the *spectrum* of A, which is the set of eigenvalues of A. $p(G)$ refers to the set of poles of $G(s)$.

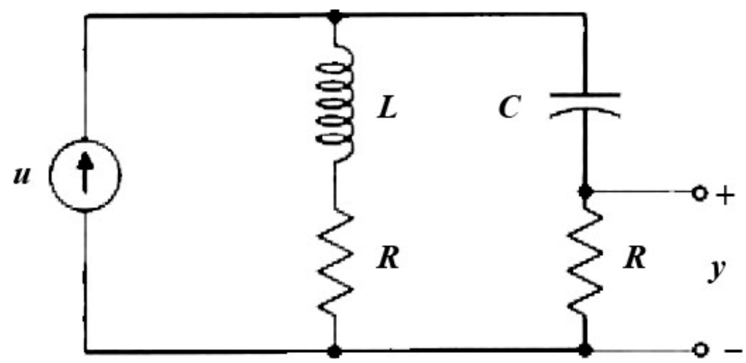
System	**Internally stable?**	**Input-output (BIBO) stable?**
$\sigma(A) = \{-1 \pm j, -0.5\}$		
$\sigma(A) = \{-1 \pm j, 0.5\}$		
$p(G) = \{-3, -1, -0.5\}$		
$p(G) = \{-3, -1, 0.5\}$		

11) [2 points] Back when we used PID control on transfer functions, the integral control was of the form $k_i \int e\,dt$ and primarily used to eliminate steady-state error. The same concepts can be applied to state-space. Below we have our numerical model of the cart-pendulum system with full-state feedback gains $K = [K_1 \quad K_2 \quad K_3 \quad K_4]$. The encoders were always zeroed at the start of simulation, and having $\theta = 0$ be not perfectly vertical caused the system to continually oscillate around the equilibrium point as it tried to set the pendulum at a non-vertical angle. This measurement bias can be compensated for by introducing an integral term to the controller. Augment the state with the new state variable $z = \int x\,dt$ and add the integral term $k_i \int x\ dt$ to the control law. Write out the new state space matrices A_i, B_i, C_i, D_i and the new gain matrix K_i.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6.8 & -1.5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 15.5 & 25.7 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.5 \\ 0 \\ -3.5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

12) [5 points] Given a system defined by the following electric circuit:



a) Write the state equations for the system. Let $x_1 = i_L$ and $x_2 = v_C$.

b) What condition(s) on R, L, and C will guarantee that the system is controllable? Observable?

c) Discuss the relationship between the poles of the transfer function $G(s)$ and the eigenvalues of A. Given that physically we are constrained to $R > 0$, $C > 0$, and $L > 0$, is this system stable?

[Extra space for Problem 12]

13) [4 points] Given a *KG(s)* system with open-loop poles at $s = -1 \pm j,\ -4$, estimate the range of values in which a real zero will make the system:

- Always stable
- Unstable as K increases
- Always unstable

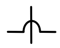
Assuming the zero is placed at $s = z$ and falls within the range where the system becomes unstable as K increases, at what value of $K > 0$ does the system become unstable?

14) [6 points] You are given the following system:

$$\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -3 & 0 \\ 2 & 2 & 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x + [0] u$$

a) Is this system controllable? Observable?

b) Draw a block diagram representation of this system from its current equations of motion. If your wires end up crossing, make sure to show that they cross using a symbol similar to: 

c) The notions of observability and controllability make little sense in this representation. Using the state transformation $z = Tx$, transform this system into modal form. Use the eigenvectors $v_1 = [1 \quad 1 \quad -1]^T$, $v_2 = [1 \quad -1 \quad 0]^T$, $v_3 = [0 \quad 0 \quad 1]^T$. Looking at the new state matrices B^* and C^*, which modes (z_1, z_2, z_3), if any, are uncontrollable? Unobservable?

d) Draw a block diagram representation for the system in modal form. What do you notice about the relationship between uncontrollable modes and the input u? Between unobservable modes and the output y?

e) Can you find a state-space representation that shares the same transfer function as the modal form, but is both controllable and observable?

[Extra space for Problem 14]

1. Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$u(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^4
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin(at)$
18	$\frac{s}{(s^2+a^2)}$	$\cos(at)$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$
21	$\frac{a^2+b^2}{s((s+a)^2+b^2)}$	$1-e^{-at}(\cos(bt)+\frac{a}{b}\sin(bt))$

2. Differentiation Property:

$$L\{f^m(t)\} = s^m F(s) - s^{m-1} f(0^-) - s^{m-2} f'(0^-) - \ldots - f^{(m-1)}(0^-)$$

3. Final value theorem:

If all poles of $sY(s)$ are in the left half s-plane, then:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

4. Partial Fraction Expansion

$$F(s) = K\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)} = \frac{C_1}{(s-p_1)} + \frac{C_2}{(s-p_2)} + \cdots + \frac{C_n}{(s-p_n)}$$

Coefficients: $$C_i = (s-p_i)F(s)\Big|_{s=p_i}$$

5. Second-order transfer function parameters and system type

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \approx \frac{1.8}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \le \zeta \le 1$$

$$M_p \cong 5\% \to \zeta = 0.7$$

$$t_s = \frac{4.6}{\zeta\omega_n}$$

Errors as a function of system type

	Input		
Type	Step	Ramp	Parabola
Type 0	$1/(K_p+1)$	∞	∞
Type 1	0	$1/K_v$	∞
Type 2	0	0	$1/K_a$

$$K_p = \lim_{s\to 0} L(s), \quad n = 0,$$

$$K_v = \lim_{s\to 0} sL(s), \quad n = 1,$$

$$K_a = \lim_{s\to 0} s^2 L(s), \quad n = 2,$$

TABLE 7.3 Important Equations in Chapter 7

Name	Equation
Control canonical form	$\mathbf{A}_c = \begin{bmatrix} -a_1 & -a_2 & \cdots & \cdots & -a_n \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix},$
	$\mathbf{B}_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{C}_c = [\, b_1 \quad b_2 \quad \cdots \quad \cdots \quad b_n \,], \quad D_c = 0.$
State description	$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u$
Output equation	$y = \mathbf{H}\mathbf{x} + Ju$
Transformation of state	$\mathbf{A} = \mathbf{T}^{-1}\mathbf{F}\mathbf{T}$
	$\mathbf{B} = \mathbf{T}^{-1}\mathbf{G}$
	$y = \mathbf{H}\mathbf{T}\mathbf{z} + Ju = \mathbf{C}\mathbf{z} + Du,$ where $\mathbf{C} = \mathbf{H}\mathbf{T}$, $D = J$
Controllability matrix	$\mathcal{C} = [\, \mathbf{G} \quad \mathbf{F}\mathbf{G} \quad \cdots \quad \mathbf{F}^{n-1}\mathbf{G} \,]$
Transfer function from state equations	$G(s) = \frac{Y(s)}{U(s)} = \mathbf{H}(s\mathbf{I} - \mathbf{F})^{-1}\mathbf{G} + J$
Transfer-function poles	$\det(p_i\mathbf{I} - \mathbf{F}) = 0$
Transfer-function zeros	$\alpha_z(s) = \det \begin{bmatrix} z_i\mathbf{I} - \mathbf{F} & -\mathbf{G} \\ \mathbf{H} & J \end{bmatrix} = 0$
Control characteristic equation	$\det[s\mathbf{I} - (\mathbf{F} - \mathbf{G}\mathbf{K})] = 0$
Ackermann's control formula for pole placement	$\mathbf{K} = [\, 0 \quad \cdots \quad 0 \quad 1 \,]\mathcal{C}^{-1}\alpha_c(\mathbf{F})$
Reference input gains	$\begin{bmatrix} \mathbf{F} & \mathbf{G} \\ \mathbf{H} & J \end{bmatrix} \begin{bmatrix} \mathbf{N}_{\mathbf{x}} \\ N_{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$
Control equation with reference input	$u = N_u r - \mathbf{K}(\mathbf{x} - \mathbf{N}_x r)$
	$= -\mathbf{K}\mathbf{x} + (N_u + \mathbf{K}\mathbf{N}_x)r$
	$= -\mathbf{K}\mathbf{x} + \bar{N}r$
Symmetric root locus	$1 + \rho G_0(-s)G_0(s) = 0$

TABLE 7.3 Important Equations in Chapter 7 (cor

Name	Equation
Estimator-error characteristic equation	$\alpha_e(s) = \det[s\mathbf{I} - (\mathbf{F} - \mathbf{LH})] = 0$
Observer canonical form	$\dot{\mathbf{x}}_o = \mathbf{F}_o\mathbf{x}_o + \mathbf{G}_o u,$ $y = \mathbf{H}_o\mathbf{x}_o,$ where $\mathbf{F}_o = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & 0 & \dots & \vdots \\ \vdots & \vdots & \ddots & & & 1 \\ -a_n & 0 & & 0 & & 0 \end{bmatrix}$ $\mathbf{G}_o = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \mathbf{H}_o = [1 \quad 0 \quad 0 \quad \cdots \quad 0]$
Observability matrix	$\mathcal{O} = \begin{bmatrix} \mathbf{H} \\ \mathbf{HF} \\ \vdots \\ \mathbf{HF}^{n-1} \end{bmatrix}$
Ackermann's estimator formula	$\mathbf{L} = \alpha_e(\mathbf{F})\mathcal{O}^{-1}\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$
Compensator transfer function	$D_c(s) = \frac{U(s)}{Y(s)} = -\mathbf{K}(s\mathbf{I} - \mathbf{F} + \mathbf{GK} + \mathbf{LH})^{-1}\mathbf{L}$
Reduced-order compensator transfer function	$D_{cr}(s) = \frac{U(s)}{Y(s)} = \mathbf{C}_r(s\mathbf{I} - \mathbf{A}_r)^{-1}\mathbf{B}_r + D_r$
Controller equations	$\dot{\hat{\mathbf{x}}} = (\mathbf{F} - \mathbf{GK} - \mathbf{LH})\hat{\mathbf{x}} + \mathbf{L}y + \mathbf{M}r$ $u = -\mathbf{K}\hat{\mathbf{x}} + \bar{N}r$
Augmented state equations with integral control	$\begin{bmatrix} \dot{x}_I \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{H} \\ \mathbf{0} & \mathbf{F} \end{bmatrix}\begin{bmatrix} x_I \\ \mathbf{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{G} \end{bmatrix}u - \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix}r + \begin{bmatrix} 0 \\ \mathbf{G}_1 \end{bmatrix}w$

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.