**UNIVERSITY OF CALIFORNIA, BERKELEY**
**Department of Electrical Engineering and Computer Sciences**

**EE C128 / ME C134 Feedback Control Systems, Spring 2016**

# FINAL EXAM

**May 12th, 2016**

**NAME: ______________________________, ______________________________**
**(print) Last First**

**STUDENT ID:_______________**

**Note(s):**

1. **Please WRITE your NAME and SID.**
2. **MAKE SURE THE EXAM HAS 20 NUMBERED PAGES.**
3. **This is a CLOSED BOOK, CLOSED NOTES, exam. Please make sure to TURN OFF ALL CELL PHONES.**
4. **SHOW YOUR WORK on this exam. Make your methods clear to the grader so you can receive partial credit.**
5. **Please BOX ALL your answers.**
6. **Remember to specify units on answers whenever appropriate.**
7. **A two-page Chi-Chi is provided for your reference at the end of this booklet.**

**SCORE:**

**1: _______ / 20 points**

**2: _______ / 15 points**

**3: _______ / 15 points**

**4: _______ / 20 points**

**5: _______ / 15 points**

**6: _______ / 15 points**

**7: _______ / 10 Bonus points**

**TOTAL: _______ / 100 points**

You may use this page for scratch work only. Without exception, subject matter on this page will ***not*** be graded.

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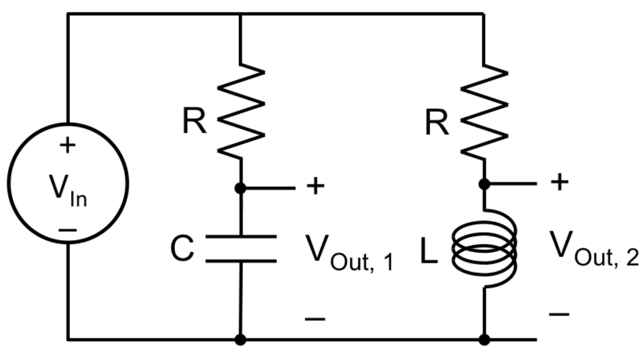
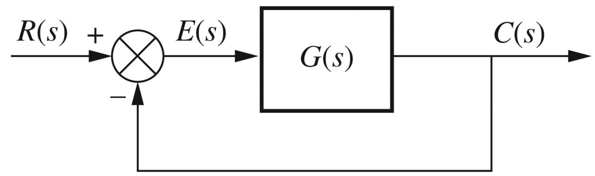
1. **[20 points]** Given the electrical network shown below:

   a) Represent the network in **state space** form. $V_{In}(t)$ is the input, and $V_{Out,1}(t)$ is the output. **Hint:** the state vector should be made of energy storage elements.
   b) Write the Controllability Matrix.
   c) Is the system Controllable? For what ratio of values (if any) of R, L, and C can the system become Uncontrollable?
   d) Write the Observability Matrix.
   e) Is the system Observable?
   f) If we now measure two outputs, $y(t) = [V_{Out,1}(t) \;\; V_{Out,2}(t)]^T$, how does this affect Observability?



**2. [15 points]** For the unity feedback system given below, with $G(s) = \frac{K}{s(s+5)}$

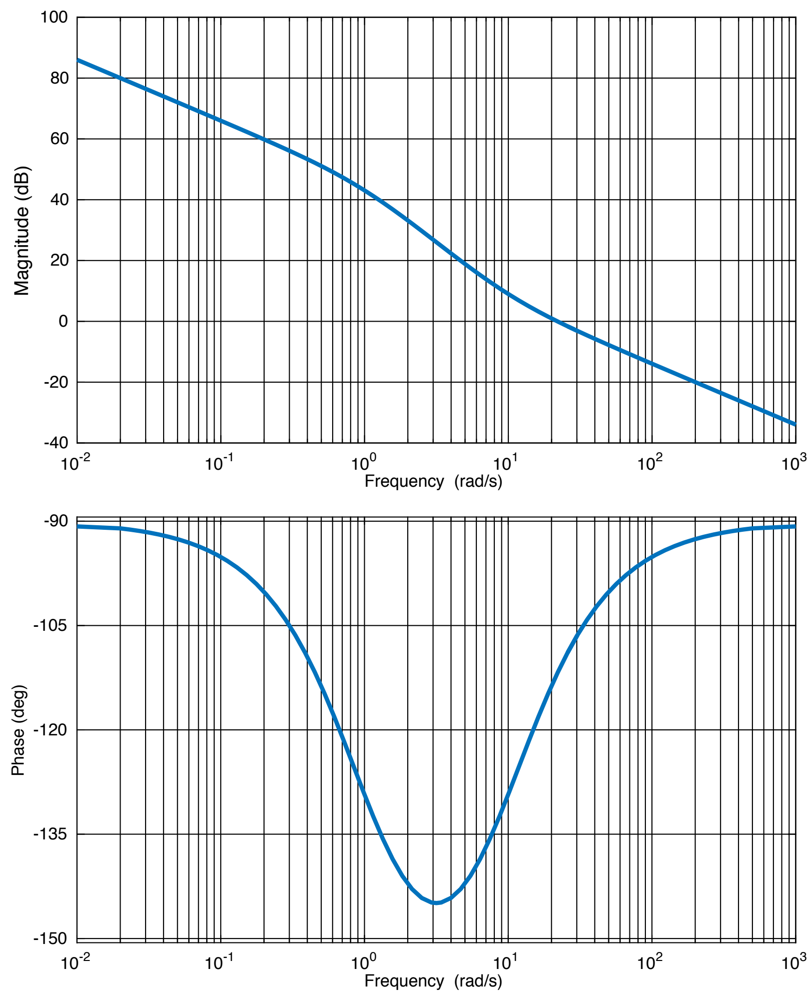



a) Find the gain, *K*, for the uncompensated system to operate with a rise time $t_r$ = 0.5 second.
b) What is the system type?
c) What is the input waveform that yields a constant error?
d) Calculate the appropriate static error constant from the *K* that you get in **(a)**.
e) Calculate the steady-state error for a **unit step input**.
f) Design a **lag** compensator to improve the **steady-state error** from **(d)** by a factor of 30.

**3. [15 points]** The Bode plots for a plant, *G(s)*, used in a unity feedback system are shown below. Determine by hand:

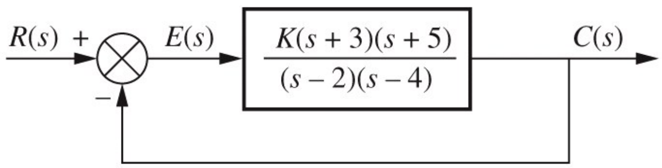
a) The transfer function (approximate value for zeros, poles and gain *K*).
b) The gain margin and phase margin.

Assume that the "y-intercept" of the magnitude line is at 86 dB, i.e. the magnitude of *G(s)* is equal to 86 dB when $\omega = 0.01$ rad/s




Note: show all relevant work on the bode diagram and provide explanations below.

**4. [20 points]** For the unity feedback system shown below:

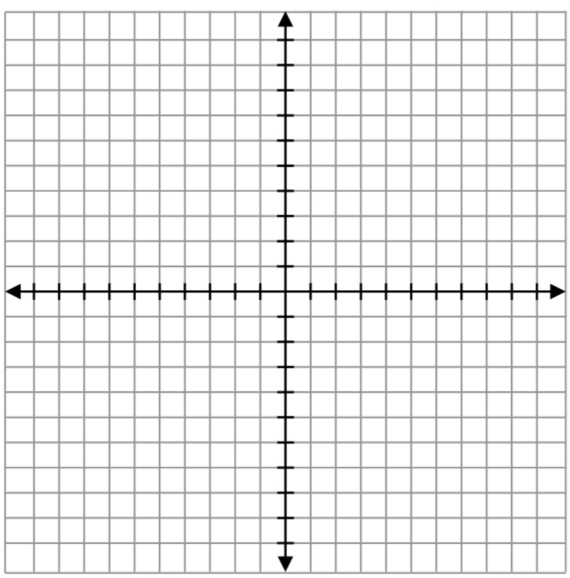


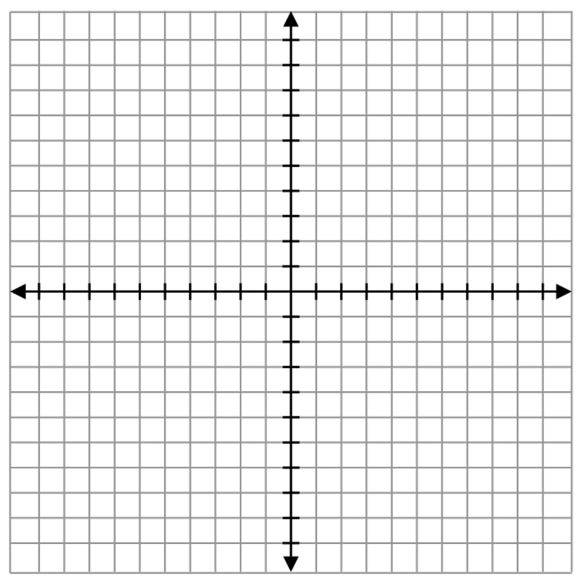

a) Sketch the Nyquist diagram for the open loop system, assuming $K=1$.

b) What is the value of $\boldsymbol{P}$ in the Nyquist criterion $Z = \boldsymbol{P} + N$? (Note: $N$ is the number of Clockwise encirclements of -1)

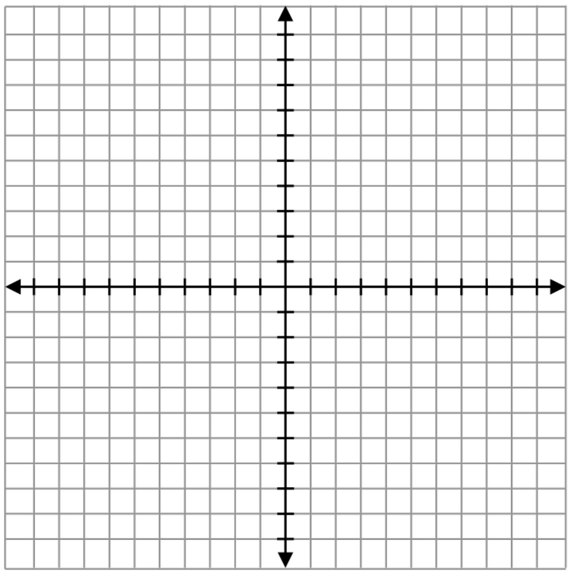
c) Given that the Nyquist diagram crosses the negative real axis at $-4/3$, find the range of values of $K$ ($K > 0$) such that the closed loop system is:

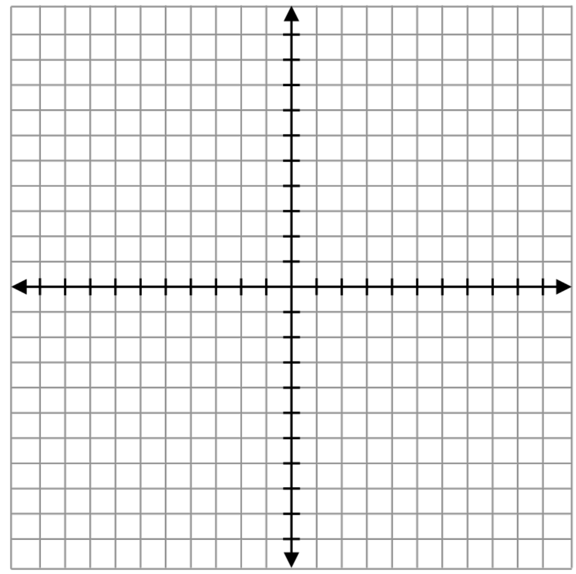
   1) Unstable
   2) Stable
   3) Marginally stable



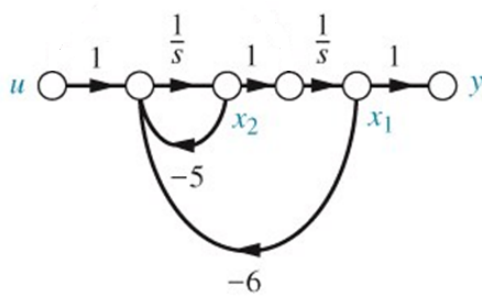


(The following two plots are for you to scratch and won't be graded)



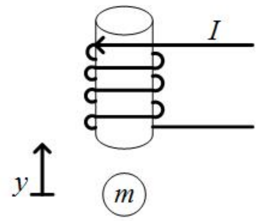


**5. [15 points]** For the plant represented by the signal-flow graph below,




a) Determine the controllability of the system.
b) Determine the observability of the system.
c) Could you have determined the controllability and/or observability by inspection? Justify your answer.
d) Write the state space representation of this system in phase variable form.
e) Assume now that your input is $u = -K\mathbf{x}$
    1) What is the size of $K$?
    2) Design K such that your new system poles are at $-2 \pm 3j$.

**6. [15 points]** Remember from the magnetic levitation system that we did in the lab,




The linearized equation of motion is:

$$m \cdot \ddot{x} = k_I \cdot I + k_x \cdot x$$
$$y = a \cdot x$$

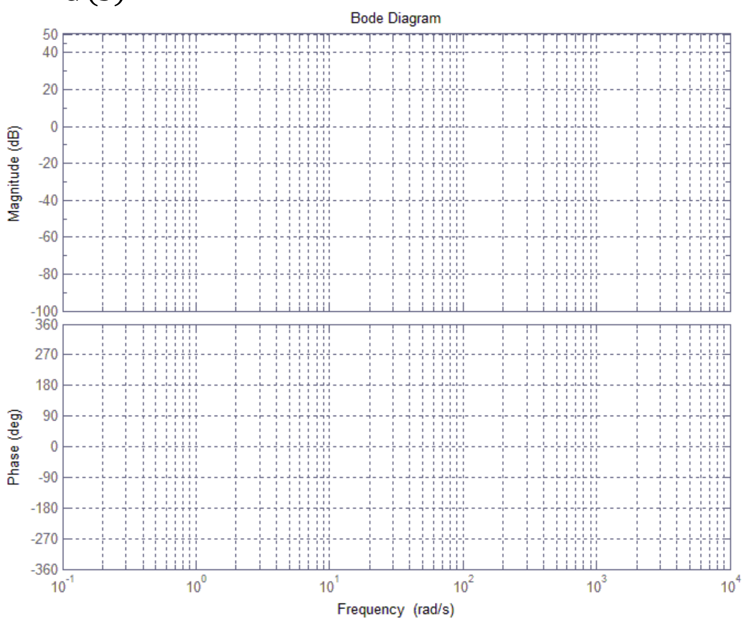
where $m = 1, a = 10, k_x = 100, k_i = 100$. The transfer function for this system is:

$$G(s) = \frac{Y(s)}{I(s)} = \frac{a \cdot k_I}{(ms^2 - k_x)} = \frac{1000}{(s^2 - 100)} = \frac{1000}{(s+10)(s-10)}$$

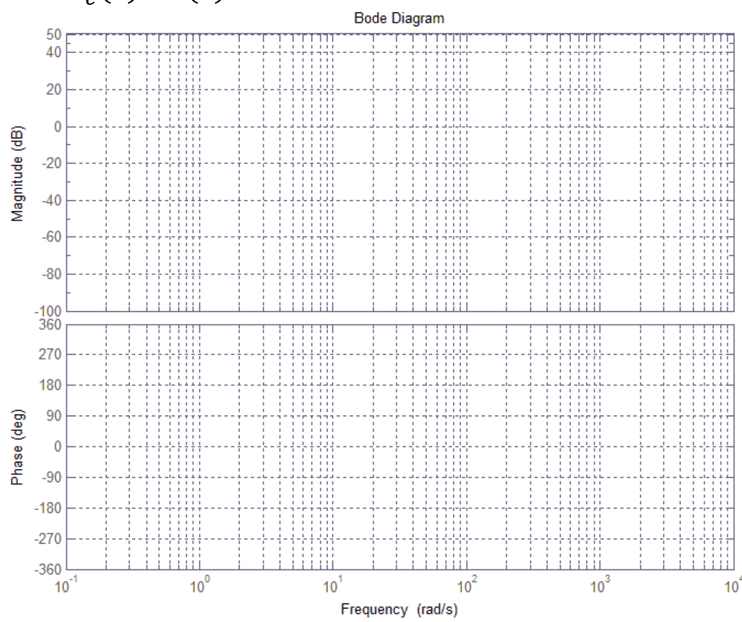
a) Sketch the Bode plot of $G(s)$. (Use the log-scale grid provided on the next page.)
b) Is the open loop system stable? What is the phase margin of the open loop system?
c) Design a controller $G_c(s)$ that stabilizes the system. Explain the reason to choose your controller structure. Sketch the compensated open loop Bode plot for $G_c(s) \cdot G(s)$ on the next page.

**Hint:** What controller did we design in the meg-lev lab?

$G(s)$




$G_c(s) \cdot G(s)$

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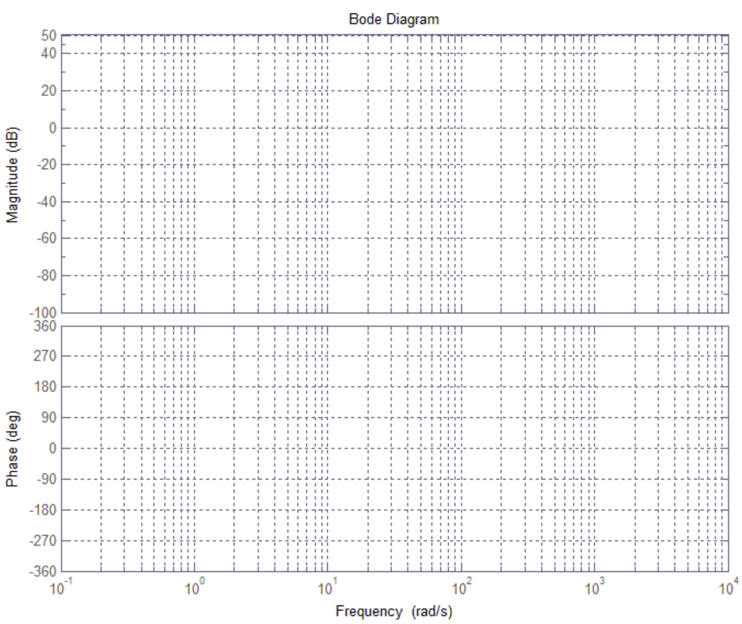
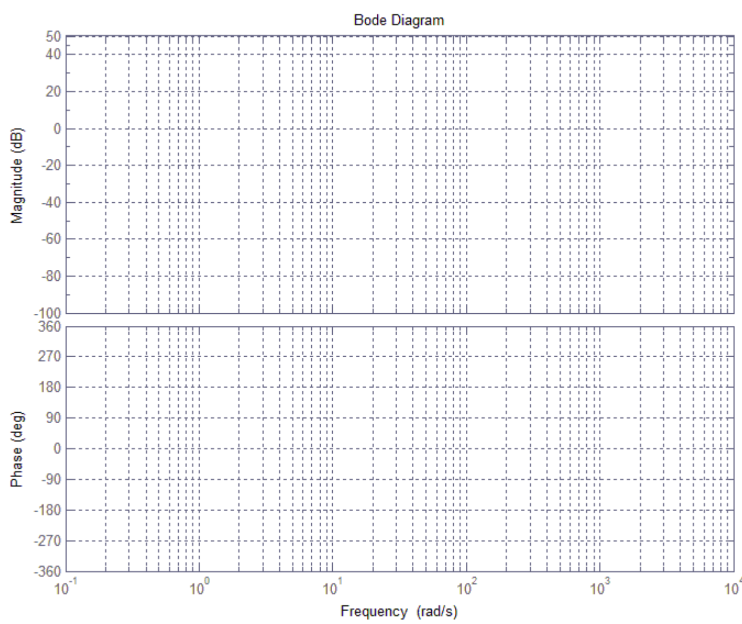


**7. BONUS [10 Bonus Points]** Kailath's (1980) method for optimal control of linear SISO systems states that the control law that minimizes a performance index:

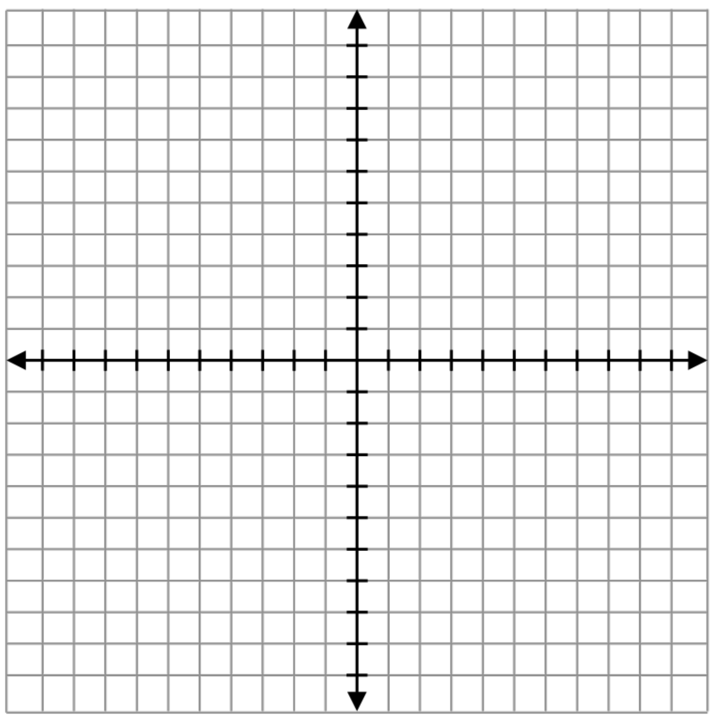
$$J = \int_0^\infty [\rho z^2(t) + u^2(t)]\, dt$$

is given by linear state feedback $u = -Kx$. The optimal value of $K$ is that which places the closed-loop poles at the stable roots of the symmetric root locus (SRL) equation:

$$1 + \rho G(-s)G(s) = 0\,.$$

For a given system with plant: $G(s) = \frac{K}{(s+8)(s+14)(s+20)}$,

a) Sketch the SRL and indicate if this is a $0^o$ or $180^o$ locus.
b) Indicate the part of the SRL that provides with optimal (with respect to J) locations for the closed-loop poles, and briefly discuss the SRL properties with respect to gain, phase margin, and BIBO stability.



# Chi-Chi

(Not all of the equations here are required to answer the questions of this exam)

**TABLE 2.3** Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

| Component | Voltage-current | Current-voltage | Voltage-charge | Impedance $Z(s) = V(s)/I(s)$ | Admittance $Y(s) = I(s)/V(s)$ |
|---|---|---|---|---|---|
| Capacitor | $v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$ | $i(t) = C\frac{dv(t)}{dt}$ | $v(t) = \frac{1}{C}q(t)$ | $\frac{1}{Cs}$ | $Cs$ |
| Resistor | $v(t) = Ri(t)$ | $i(t) = \frac{1}{R}v(t)$ | $v(t) = R\frac{dq(t)}{dt}$ | $R$ | $\frac{1}{R} = G$ |
| Inductor | $v(t) = L\frac{di(t)}{dt}$ | $i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$ | $v(t) = L\frac{d^2q(t)}{dt^2}$ | $Ls$ | $\frac{1}{Ls}$ |

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), $C$ – F (farads), $R$ – Ω (ohms), $G$ – Ω (mhos), $L$ – $H$ (henries).

<u>Final value theorem:</u>

If all poles of $sY(s)$ are in the left half s-plane, then:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

<u>Second-order transfer function parameters and system type</u>

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n \cdot s + \omega_n^2}$$

$$s_{1,2} = -\sigma \pm j\omega_d$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_r \approx \frac{2}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$t_s = \frac{4}{\sigma}$$

Errors as a function of system type

| | Input | | |
|---|---|---|---|
| Type | Step | Ramp | Parabola |
| 0 | $\frac{1}{1+K_p}$ | $\infty$ | $\infty$ |
| 1 | 0 | $\frac{1}{K_v}$ | $\infty$ |
| 2 | 0 | 0 | $\frac{1}{K_a}$ |

Error constants:

$$K_p = \lim_{s\to 0} G(s),$$

$$K_v = \lim_{s\to 0} sG(s),$$

$$K_a = \lim_{s\to 0} s^2 G(s),$$

<u>180° Root Locus</u>

On real axis to the left of odd pole + zero

$$\theta_a = \frac{(2k+1)\cdot 180}{\#finite\ poles - \#finite\ zeros}$$

$$\sigma_a = \frac{\Sigma finite\ poles - \Sigma finite\ zeros}{\#\ finite\ poles - \#\ finite\ zeros}$$

<u>0° Root Locus</u>

On real axis to the left of even pole + zero

$$\theta_a = \frac{360°\cdot k}{\#finite\ poles - \#finite\ zeros}$$

$$\sigma_a = \frac{\Sigma finite\ poles - \Sigma finite\ zeros}{\#\ finite\ poles - \#\ finite\ zeros}$$

State-Space Equations

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$$
$$\boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u}$$

Controllability Matrix

$$\boldsymbol{C_M} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \quad \ldots \quad \boldsymbol{A^{n-1}B}]$$

Observability Matrix

$$\boldsymbol{O_M} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \boldsymbol{CA^2} \\ \ldots \\ \boldsymbol{CA^{n-1}} \end{bmatrix}$$

Phase Variable Form

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} \\ -\boldsymbol{a_0} & -\boldsymbol{a_1} & -\boldsymbol{a_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{1} \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y} = [\boldsymbol{b_0} \quad \boldsymbol{b_1} \quad \boldsymbol{b_2}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Controllable Canonical Form

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{a_2} & -\boldsymbol{a_1} & -\boldsymbol{a_0} \\ \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y} = [\boldsymbol{b_2} \quad \boldsymbol{b_1} \quad \boldsymbol{b_0}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Observable Canonical Form

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{a_2} & \boldsymbol{1} & \boldsymbol{0} \\ -\boldsymbol{a_1} & \boldsymbol{0} & \boldsymbol{1} \\ -\boldsymbol{a_0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{b_2} \\ \boldsymbol{b_1} \\ \boldsymbol{b_0} \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y} = [\boldsymbol{1} \quad \boldsymbol{0} \quad \boldsymbol{0}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Compensators

$$G_c(s) = \frac{s + z_c}{s + p_c}$$

Where, $z_c, p_c > 0$

Lag compensator: $z_c > p_c$

$G_c(s) = K_c \cdot \frac{\frac{s}{z_c}+1}{\frac{s}{p_c}+1}$, $G_c(0) = K_c$

Lead Compensator: $z_c < p_c$

For the desired phase $\phi$, $\alpha = \frac{1-\sin\phi}{1+\sin\phi}$, $\mathrm{T} = \frac{1}{\omega_{PM}\sqrt{\alpha}}$

$z_c = \frac{1}{T}$, $p_c = \frac{1}{\alpha \cdot T}$