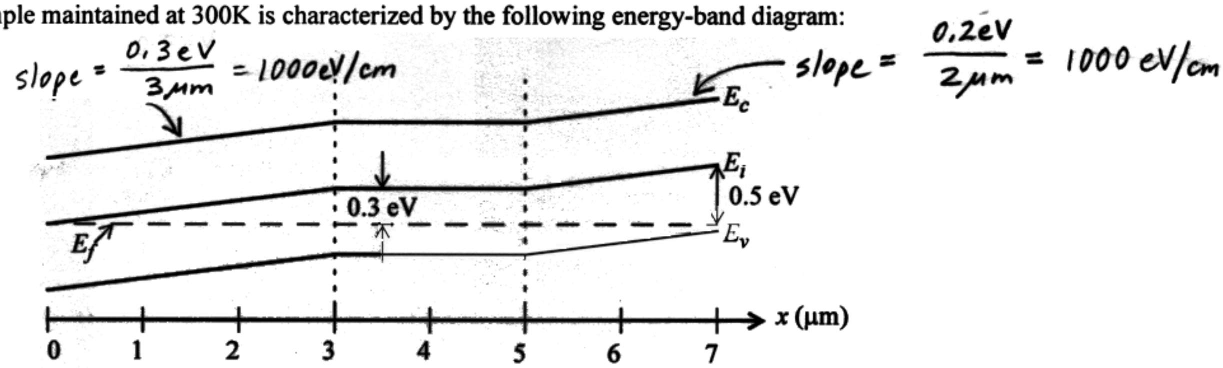
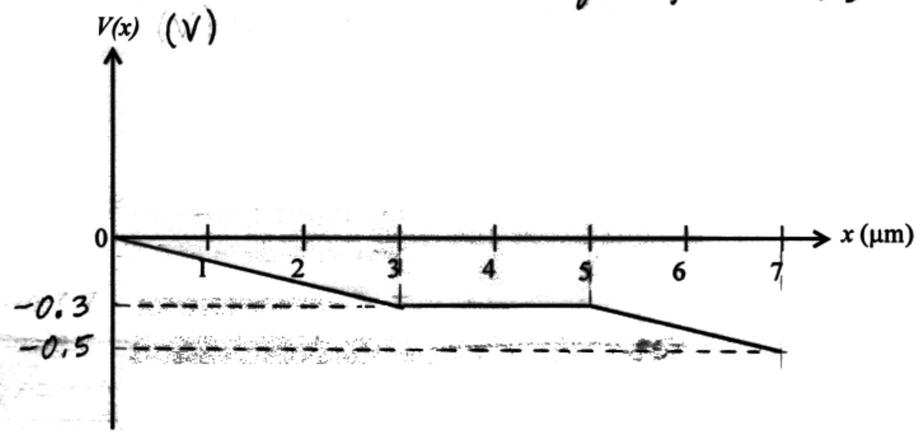
Problem 1: Semiconductor Fundamentals [30 points]

A silicon sample maintained at 300K is characterized by the following energy-band diagram:



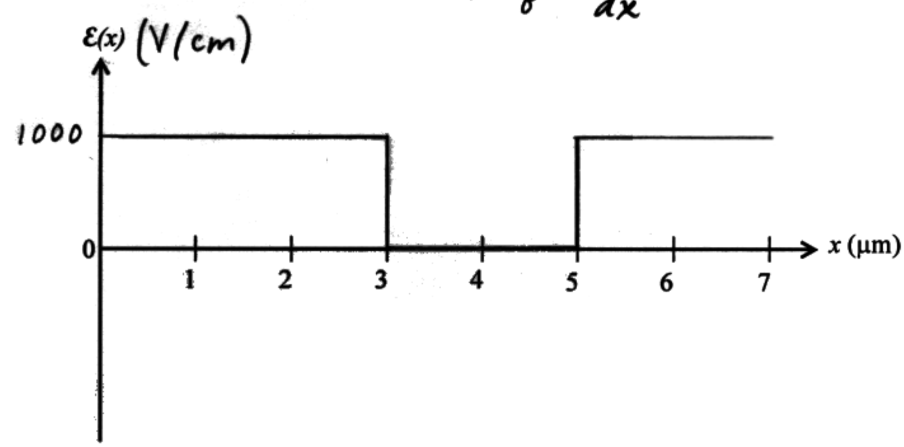
a) Draw the electric potential $V(x)$. [6 pts]

$$V(x) = \frac{1}{q}\left[E_f - E_i(x)\right]$$



b) Draw the electric field $\mathcal{E}(x)$. [6 pts]

$$\mathcal{E}(x) = \frac{1}{q}\frac{dE_i(x)}{dx}$$



c) Do equilibrium conditions prevail? Explain briefly. [3 pts]

Yes; the Fermi level is constant throughout the sample.

(There is no net current flow; diffusion current is balanced exactly by drift current.)

d) What are the electron and hole concentrations at $x = 4$ μm? **[6 pts]** $E_i - E_f = 0.3\,eV$

$$p = n_i e^{(E_i - E_f)/kT} = 10^{10} e^{(0.3/0.026)} \simeq \boxed{10^{15}\ cm^{-3}}$$

$$n = \frac{n_i^2}{p} = \frac{10^{20}}{10^{15}} = \boxed{10^5\ cm^{-3}}$$

e) What is the electron drift current density flowing at $x = 0$? **[5 pts]**

At $x = 0$: $E_i = E_f$ so the sample is intrinsic

$\Rightarrow n = n_i$ and $\mu_n = 1400\ cm^2/V \cdot s$

From part (b), $\mathcal{E} = 1000\ V/cm$

$$J_{n,drift} = q \mu_n n \mathcal{E} = (1.6 \times 10^{-19})(1400)(10^{10})(10^3) = \boxed{2.24\ mA/cm^2}$$

f) What is the mean free path of a hole at $x = 4$ μm? **[4 pts]**
Assume that the hole thermal velocity is 2×10^7 cm/s, and $m_p = 0.39 m_o$.

At $x = 4\ \mu m$: $p = N_a = 10^{15}\ cm^{-3}$

$$\mu_p \cong 450\ cm^2/V \cdot s = \frac{q \tau_{mp}}{m_p}$$

$$l = \tau_{mp} v_{th} = \left(\frac{\mu_p m_p}{q}\right) v_{th} = \frac{(450)(0.39 \times 9.1 \times 10^{-31})}{1.6 \times 10^{-19}} (2 \times 10^7)$$

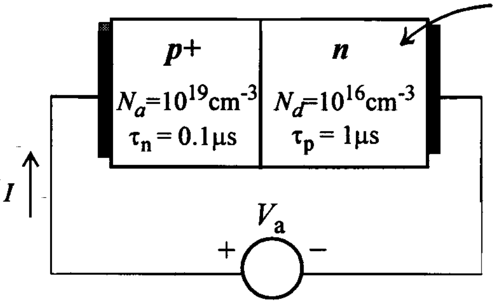
$$0.02 \times 10^{-4}\ cm = \boxed{20\ nm}$$

(Recall that $kg \cdot cm^2/V \cdot s / C = 10^{-4}\ s$)

Problem 2: **p-n Junction Diode [35 points]**

Consider the silicon pn junction diode below, maintained at 300K with a forward bias $V_a = 0.6$ V. The cross-sectional area of the diode is 100 μm^2.

(Long-base diode)



$\mu_p \cong 400 \; cm^2/V \cdot s$

$D_p = \frac{kT}{q} \mu_p = 0.026(400)$
$\cong 10 \; cm^2/s$

a) Draw the energy-band diagram (showing E_c, E_i, E_v, E_{fn} E_{fp}), indicating the values of $|E_f - E_i|$ in the quasi-neutral regions, as well as the width of the depletion region. **[12 pts]**

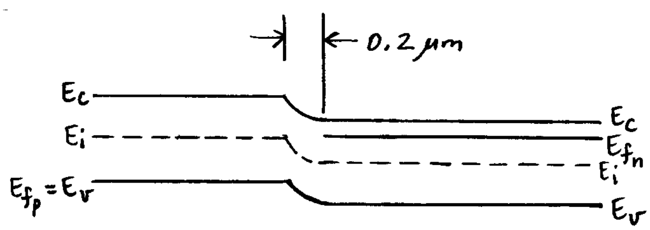
The p+ side is degenerately doped $\rightarrow E_{f_p} = E_v$

On the n side, $E_{f_n} - E_i = kT \ln\left(\frac{N_d}{n_i}\right) = kT \ln\left(\frac{10^{16}}{10^{10}}\right) = 6(\underbrace{kT \ln 10}_{0.060 \, eV}) = 0.36 \, eV$

The built-in potential $\phi_{bi} = \frac{1}{2}\left(\frac{E_g}{q}\right) + \frac{kT}{q} \ln\left(\frac{10^{16}}{10^{10}}\right) = 0.56 + 0.36 = 0.92$ V

This is a one-sided junction, so $W_{dep} = \sqrt{\frac{2\epsilon_s}{qN_d}(\phi_{bi} - V_a)} = \sqrt{\frac{2 \times 10^{-12}}{(1.6 \times 10^{-19})(10^{16})}(0.92 - 0.6)}$

$= 2 \times 10^{-5}$ cm $= 0.2 \; \mu m$

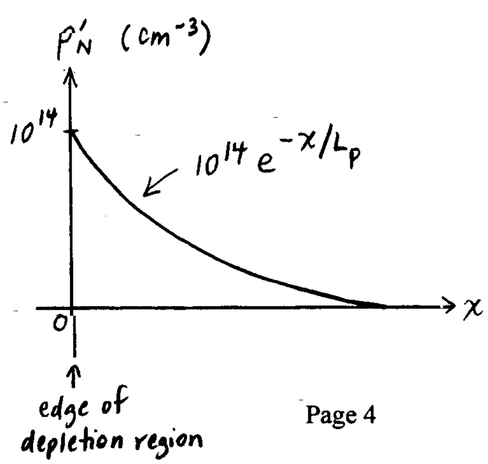


$E_i - E_{f_p} = 0.56 eV$ $\quad$ $E_{f_n} - E_i = 0.36 eV$

b) Sketch the excess minority carrier profile $p_N'(x)$ in the quasi-neutral n-type region. **[5 pts]**

At the edge of the depletion region, $p_N' = \frac{n_i^2}{N_d}\left(e^{qV_a/kT} - 1\right)$

$\cong \frac{10^{20}}{10^{16}} e^{(0.6/0.026)} \cong 10^{14} \; cm^{-3}$



$L_p = \sqrt{D_p \tau_p} = \sqrt{10(10^{-6})} \cong 3.2 \times 10^{-3} cm$
$= 32 \; \mu m$

c) What is the current I flowing through the diode? [10 pts]

One-sided junction under strong forward bias ($V_a >> \frac{kT}{q}$):

$$I = qAn_i^2 \frac{D_p}{L_p N_d} e^{qV_a/kT}$$

$$= (1.6\times10^{-19})(100\times10^{-8})(10^{10})^2 \frac{10}{(32\times10^{-4})(10^{16})} e^{(0.6/0.026)}$$

$$= 5.3\times10^{-8}\ A = \boxed{53\ nA}$$

d) What is the stored excess minority charge (in Coulombs) in the diode? [4 pts]

$$Q_p = I_p \tau_p = 53\times10^{-9}(10^{-6}) = 53\times10^{-15} = \boxed{53\ fF}$$

e) Calculate the small-signal capacitance of the diode. [4 pts]

$$C_{dep} = A\frac{\epsilon_{si}}{W_{dep}} = 100\times10^{-8}\frac{10^{-12}}{0.2\times10^{-4}} = 5\times10^{-14}\ F$$ ← much smaller than C_{diff}

$$C_{diff} = \frac{q}{kT}Q_p = \frac{53\times10^{-15}}{0.026} \cong 2\times10^{-12}\ F$$

$$C = C_{dep} + C_{diff} \approx \boxed{2\ pF}$$

Problem 3: Bipolar Junction Transistor [40 points]

a) Provide brief answers to the following questions regarding a typical BJT:

i) Why is the collector less heavily doped than the base? Give 2 reasons. [6 pts]

The collector doping is relatively light, in order to achieve the following:

1. Low collector-junction depletion capacitance C_{jc}
   $\Rightarrow$ reduced Early effect, and
   improved transient response time and cutoff frequency

2. High collector-junction breakdown voltage V_{CBO}
   $\Rightarrow$ high collector-to-emitter breakdown voltage V_{CEO}

ii) Why is it desirable to make the base width small? Give 2 reasons. [6 pts]

A small base width W_B is needed to

1. ensure that the base transport factor α_T is close to 1
   $\Rightarrow$ high current gain $\beta_F \propto \frac{1}{W_B}$

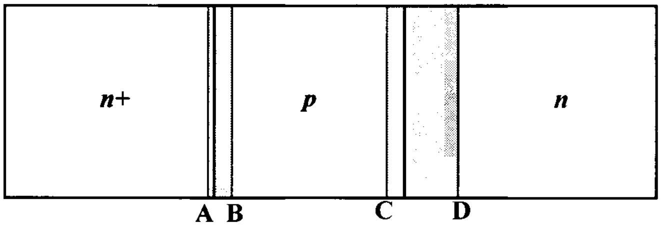
2. minimize the excess minority-carrier charge stored in the quasi-neutral base region
   $\Rightarrow$ faster switching speed

iii) Would it be desirable to increase the band-gap of the emitter relative to that of the base? Explain. [3 pts]

Yes! $n_i^2 \propto e^{-E_g/kT}$ so an increase in E_g $\Rightarrow$ decrease in n_i^2

$\beta_F \propto \frac{1}{n_{iE}^2}$ so it is desirable to decrease n_{iE}^2 in order to achieve high gain

b) Consider an npn silicon BJT of area $A = 10^{-6}$ cm^2 maintained at 300K, operating in the active region with $V_{BE} =$ 0.6 V and $V_{CB} = 1$ V, so that $W_B = 0.6$ µm. Assume that the emitter and collector regions are long.



$L_B = \sqrt{D_B \tau_B} = \sqrt{20(10^{-6})}$

$= 44.7\,\mu m$

$W_B << L_B$

Each region of the BJT is uniformly doped: $N_E = 10^{19}$ cm^{-3}, $N_B = 10^{17}$ cm^{-3}, $N_C = 10^{15}$ cm^{-3}.
The minority-carrier diffusion constants are $D_E = 2$ cm^2/s, $D_B = 20$ cm^2/s, $D_C = 12$ cm^2/s.
The minority carrier lifetimes are $\tau_E = 10^{-7}$ s and $\tau_B = \tau_C = 10^{-6}$ s.

i) What is the common-emitter d.c. current gain, β_F, of this transistor? **[5 pts]**

Ignoring band-gap narrowing due to heavy doping $\rightarrow n_{iE} = n_{iB}$.

$$= \sqrt{D_E \tau_E} = \sqrt{2 \times 10^{-7}} = 4.5\,\mu m$$

$$= \frac{D_B N_E L_E}{D_E N_B W_B} = \frac{20\,(10^{19})(4.5 \times 10^{-4})}{2\,(10^{17})(0.6 \times 10^{-4})} = \boxed{7500}$$

Note: In an actual BJT, band-gap narrowing in the emitter decreases β_F. Also, the emitter is usually short.

ii) What are the excess minority-carrier concentrations at the edges of the depletion regions (locations **A**, **B**, **C** and **D** in the diagram above)? **[8 pts]**

Emitter region:

$$p_{E0} = \frac{n_{iE}^2}{N_E} = \frac{10^{20}}{10^{19}} = 10\ cm^{-3}$$

$$p_E(A) = p_{E0}\left(e^{qV_{BE}/kT} - 1\right) \simeq 10\, e^{(0.6/0.026)} \cong 10^{11}\ cm^{-3} \quad (A)$$

Base region:

$$n_{B0} = \frac{n_{iB}^2}{N_B} = \frac{10^{20}}{10^{17}} = 10^3\ cm^{-3}$$

$$n_B(B) = n_{B0}\left(e^{qV_{BE}/kT} - 1\right) \simeq 10^3\, e^{(0.6/0.026)} \simeq 10^{13} cm^{-3} \quad (B)$$

$$n_B(C) = n_{B0}\left(e^{qV_{BC}/kT} - 1\right) \simeq -n_{B0} = -10^3 cm^{-3} \quad (C)$$

Collector region:

$$p_{C0} = \frac{n_{iC}^2}{N_C} = \frac{10^{20}}{10^{15}} = 10^5\ cm^{-3}$$

$$p_C(D) = p_{C0}\left(e^{qV_{BC}/kT} - 1\right) \simeq -p_{C0} = -10^5\ cm^{-3} \quad (D)$$

iii) What is the collector current, I_C? [4 pts]

$$I_c = qA \frac{D_B}{W_B} n_{B0} \left(e^{qV_{BE}/kT} - 1\right)$$

$$\simeq (1.6\times10^{-19})(10^{-6})\left(\frac{20}{0.6\times10^{-4}}\right)(10^{3})\, e^{(0.6/0.026)}$$

$$= 5.6\times10^{-7}\ A = \boxed{0.56\ \mu A}$$

iv) Calculate the Early voltage, V_A. [8 pts]

Collector junction: $\phi_{bi} = \frac{kT}{q}\ln\frac{N_B}{n_i} + \frac{kT}{q}\ln\frac{N_C}{n_i}$

$$= 7(0.06) + 5(0.06) = 0.72\ V$$

$$W_{dep} = \sqrt{\frac{2\epsilon_s}{q}(\phi_{bi} - V_{BC})\left(\frac{1}{N_B} + \frac{1}{N_C}\right)}$$

$$= \sqrt{\frac{2(10^{-12})}{1.6\times10^{-19}}(0.72+1)\left(\frac{1}{10^{17}} + \frac{1}{10^{15}}\right)}$$

$$\cong 1.5\times10^{-4}\ cm = 1.5\ \mu m$$

$$C_{jc} = \frac{\epsilon_{Si}}{W_{dep}} = \frac{10^{-12}}{1.5\times10^{-4}} = 6.7\times10^{-9}\ F/cm^2$$

$$V_A = \frac{qN_BW_B}{C_{jc}} = \frac{(1.6\times10^{-19})(10^{17})(0.6\times10^{-4})}{6.7\times10^{-9}} = \boxed{143\ V}$$

Problem 4: Metal-Oxide-Semiconductor Capacitor [25 points]
Consider a p+ poly-Si gated capacitor of area 10^{-4} μm^2 maintained at 300K.
The Si substrate is uniformly doped with $N_d = 10^{16}$ cm^{-3}, the oxide thickness $T_{ox} = 10$ nm, and the oxide charge density at the Si-SiO$_2$ interface $Q_{ox} = 10^{11}$ q/cm^2.

a) Calculate the flatband voltage, V_{fb}. [8 pts]

$$C_{ox} = \frac{\epsilon_{ox}}{T_{ox}} = \frac{3.45\times10^{-13}}{10\times10^{-7}} = 3.45\times10^{-7}\ F/cm^2$$

$$\chi_{Si} + \frac{E_{g,Si}}{q} = 4.05 + 1.12 = 5.17\ V$$

$$\frac{kT}{q}\ln\left(\frac{N_d}{n_i}\right) = \frac{kT}{q}\ln\left(\frac{10^{16}}{10^{10}}\right) = 6\,(0.06) = 0.36\ V$$

$$\chi_{Si} + \frac{1}{2}\frac{E_{g,Si}}{2} - \phi_B = 4.05 + 0.56 - 0.36 = 4.25\ V$$

$$V_{fb} = \Psi_M - \Psi_s - \frac{Q_{ox}}{C_{ox}} = 5.17 - 4.25 - \frac{10^{11}\times1.6\times10^{-19}}{3.45\times10^{-7}} = \boxed{0.87\ V}$$

b) Calculate the threshold voltage, V_t. [6 pts]

This is a PMOS device.

$$V_t = V_{fb} - 2\phi_B - \frac{\sqrt{4\epsilon_{Si}\phi_B q N_d}}{C_{ox}}$$

$$= 0.87 - 2(0.36) - \frac{\sqrt{4(10^{-12})(0.36)(1.6\times10^{-19})(10^{16})}}{3.45\times10^{-7}}$$

$$= 0.87 - 0.72 - 0.14 \cong \boxed{0\ V}$$

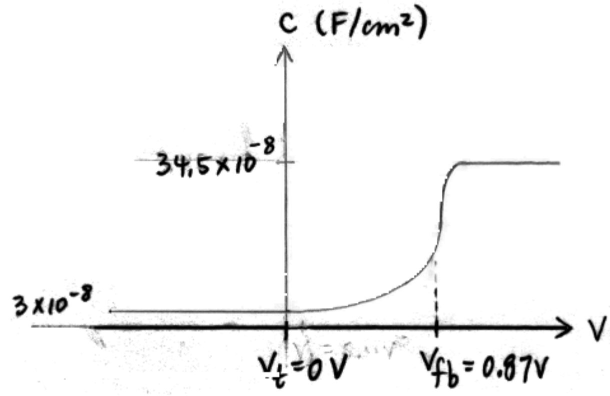
voltage dropped across oxide

Voltage dropped in Si

c) Draw the high-frequency *C-V* curve for this capacitor, indicating the maximum and minimum capacitance values on your plot. [8 pts]

$$W_{dmax} = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_d}} = \sqrt{\frac{4(10^{-12})(0.36)}{(1.6\times10^{-19})(10^{16})}} = 3\times10^{-5}\ \text{cm}$$

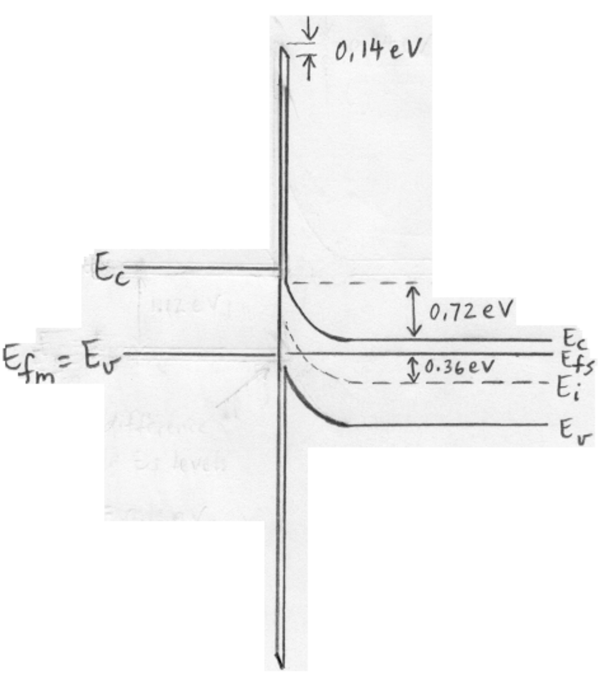
$$C_{min} = \left[\frac{1}{C_{ox}} + \frac{W_{dmax}}{\epsilon_{si}}\right]^{-1} = \left[\frac{1}{3.45\times10^{-7}} + \frac{3\times10^{-5}}{10^{-12}}\right]^{-1} = 3.0\times10^{-8}\ \text{F/cm}^2$$



(multiply by $A = 10^{-12}\ \text{cm}^2$ to obtain C in units of Farads)

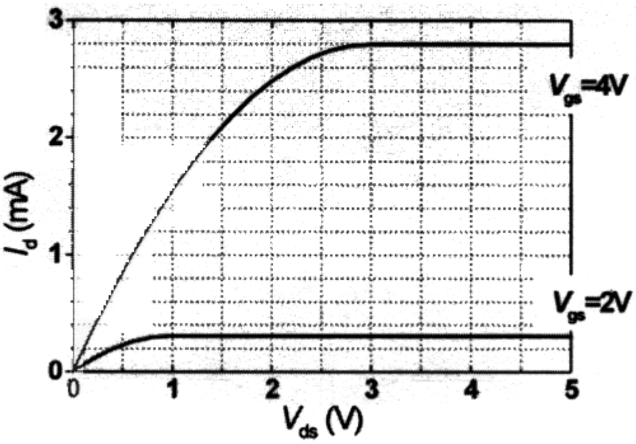
d) Draw the energy-band diagram of the MOS structure at threshold. Indicate the amount of band bending in the Si ($q\phi_s$) as well as the band bending across the oxide (qV_{ox}). [8 pts]

At threshold, $V_g = 0.11\text{ V} = V_{fb} + \underset{-0.72}{\phi_s} + \underset{-0.14}{V_{ox}}$



Problem 5: MOS Field-Effect Transistor [40 points]

a) The figure below shows the I_{ds} vs. V_{ds} characteristic of a long-channel nMOSFET maintained at 300K. The gate-oxide thickness $T_{oxe} = 10$ nm and the channel length $L = 2$ µm. Assume the body-effect factor $m = 1.2$.



$$V_{dsat} = \frac{V_{gs} - V_t}{m}$$

$$\Rightarrow V_t = V_{gs} - m V_{dsat}$$

i) Estimate V_t. [3 pts]

From the curve for $V_{gs} = 4V$, $V_{dsat} \simeq 3V \Rightarrow V_t = 4 - (1.2)(3) = 0.4V$

From the curve for $V_{gs} = 2V$, $V_{dsat} \cong 1V \Rightarrow V_t = 2 - (1.2)(1) = 0.8V$

ii) Estimate the electron surface mobility for $V_{gs} = 4$ V, using the universal effective mobility model. [3 pts]

Effective vertical electric field is $\dfrac{V_{gs} + V_t + 0.2}{6T_{oxe}} = \dfrac{4 + 0.4 + 0.2}{6(10 \times 10^{-7})} = 0.77$ MV/cm

From the plot on Page 1, $\boxed{\mu_{ns} \simeq 300 \text{ cm}^2/\text{V·s}}$

iii) Based on your answers to parts (i) and (ii), determine the channel width W. [4 pts]

For $V_{gs} = 4V$, $I_{dsat} = 2.8$ mA

$$C_{oxe} = \frac{\varepsilon_{ox}}{T_{oxe}} = \frac{3.45 \times 10^{-13}}{10 \times 10^{-7}} = 3.45 \times 10^{-7} \text{ F/cm}^2$$

$$I_{dsat} = \frac{W}{2mL} \mu_{ns} C_{oxe} (V_{gs} - V_t)^2$$

$$\Rightarrow W = \frac{2mL I_{dsat}}{\mu_{ns} C_{oxe} (V_{gs} - V_t)^2} = \frac{2(1.2)(2 \times 10^{-4})(2.8 \times 10^{-3})}{(300)(3.45 \times 10^{-7})(4 - 0.4)^2} = 1.0 \times 10^{-3} \text{ cm} = \boxed{10 \text{ µm}}$$

iv) For what channel lengths will the effect of velocity saturation be significant (*i.e.* resulting in a reduction in I_{dsat} by more than a factor of 2)? [5 pts]

Velocity saturation will be significant for L such that $\mathcal{E}_{sat} L \lesssim \dfrac{V_{gs} - V_t}{m}$

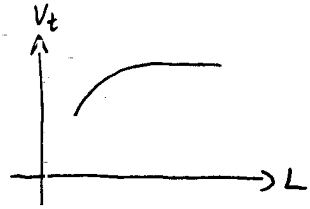
$$\mathcal{E}_{sat} = \frac{2 v_{sat}}{\mu_{ns}} = \frac{2(8 \times 10^6)}{300} = 5.3 \times 10^4 \text{ V/cm}$$

$$\Rightarrow \frac{(V_{gs} - V_t)}{m \mathcal{E}_{sat}} = \frac{3}{5.3 \times 10^4} = 5.7 \times 10^{-5} \text{ cm} = 0.57 \text{ µm}$$

$\therefore$ velocity saturation important for $\boxed{L \lesssim 0.57 \text{ µm}}$

b) Short-Answer Questions:

i) What is the "short-channel effect" (why does it occur)? Describe 3 ways to reduce this effect. **[6 pts]**

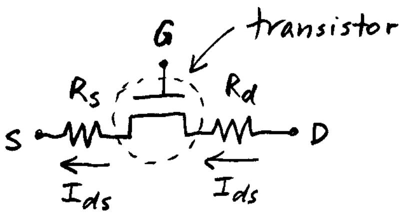


Reduction in V_t with decreasing L as a larger percentage of the depletion charge in the channel is supported by the source and drain pn junctions, rather than by the gate.

Methods of reducing "V_t roll-off".

1. Reduce
2. Reduce S/D junction depth r_j
3. Increase channel doping concentration

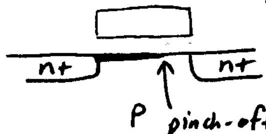
ii) How do parasitic source resistance R_s and parasitic drain resistance R_d degrade MOSFET performance? **[3 pts]**



When the transistor is on, voltage is dropped across R_s and R_d. Thus, the gate-to-source bias applied to the transistor is reduced by $I_{ds} R_s$. Likewise, the drain-to-source bias applied to the transistor is reduced by $I_{ds}(R_s + R_d)$. These reductions result in lower drive current I_{ds} (as compared to the case where $R_s = R_d = 0$).

Decreasing T_{oxe} $\Rightarrow$ increasing C_{oxe}

c) Indicate in the table below (by checking the appropriate box for each line) the effect of **decreasing the gate-oxide thickness (T_{oxe})** on various MOSFET parameters. **[16 pts]**

MOSFET parameter	**increases**	**decreases**	**remains the same**	**Brief Explanation**
Transconductance (g_m)	✓			$g_m \propto C_{oxe}$
Body effect parameter (γ)		✓		$\gamma = \dfrac{\sqrt{qN_a 2\varepsilon_s}}{C_{oxe}} \propto \dfrac{1}{C_{oxe}}$
Channel-length modulation parameter (λ)			✓	λ is not directly affected by T_{oxe}, but can be affected if a change in V_t results.  pinch-off point located where $V_g - V_c = V_t$
Subthreshold swing (S)		✓		$S = \left[\frac{kT}{q}\ln 10\right]\left(1 + \frac{C_{dep}}{C_{oxe}}\right)$ this term decreases with increasing C_{oxe}

Problem 6: Metal-Semiconductor Contact [25 points]

a) Short-Answer Questions:

i) Explain why a Schottky-barrier diode switches very rapidly (as compared to a pn-junction diode). **[4 pts]**

There is no storage of excess minority carriers in a Schottky diode; thus, there is no time delay associated with the build-up or removal of minority carriers when a Schottky diode is turned on or off, respectively.

ii) Describe 2 ways to achieve an ohmic metal-semiconductor contact. **[6 pts]**

In an ohmic contact, carriers can tunnel into and out of the semiconductor very easily.

The contact resistance is dependent on the Schottky barrier height ϕ_B and the semiconductor doping concentration:

$$R_C \propto e^{H\phi_B/\sqrt{N}}$$

Therefore, to achieve low R_C (for good ohmic contact), we can

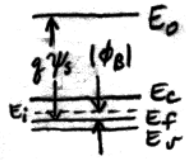
1. Increase the semiconductor doping concentration at the contact
2. Select an appropriate metal such that ϕ_B is low.

p-type semiconductor

b) Consider an **ideal** Schottky-barrier diode maintained at 300K, made by depositing tungsten ($\psi_M = 4.6$ eV) onto a silicon substrate which is doped uniformly with 10^{15} cm^{-3} Boron. The minority-carrier lifetime in the Si is τ.

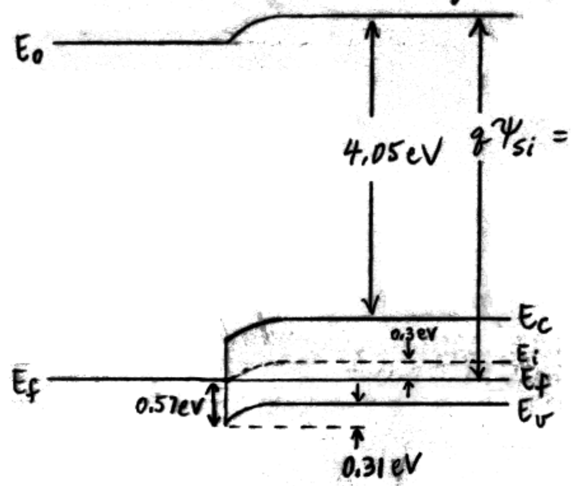
i) Draw the equilibrium energy-band diagram. Label , $q\Psi_{Si}$, $q\chi_{Si}$, $q\phi_B$, and $q\phi_{bi}$, as well as E_c, E_v, E_i and E_f in the Si. **[12 pts]**

$$\phi_B = -\frac{kT}{q}\ln\left(\frac{10^{15}}{10^{10}}\right) = -5\,(0.06) = -0.3\text{ V}$$



$$\phi_{Bp} = \chi_{Si} + \left(\frac{E_g}{q}\right) - \Psi_M = 4.05 + 1.12 - 4.6 = 0.57\text{ V}$$

$$\phi_{bi} = \phi_{Bp} - \left(\frac{E_f - E_v}{q}\right) = \phi_{Bp} - \left[\frac{E_g}{2q} - |\phi_B|\right] = 0.57 - [0.56 - 0.3]$$
$$= 0.31\text{ V}$$



$$\Psi_{Si} = \chi_{Si} + \frac{E_g}{2q} + |\phi_B| = 4.05 + 0.56 + 0.3 = 4.91\text{ V}$$

ii) Suppose the Si is uniformly illuminated with light, resulting in a photogeneration rate G_L electron-hole pairs per cm^3-sec. How will the *I-V* characteristic of the Schottky diode be affected? **[3 pts]**

Minority carriers generated within a diffusion length of the depletion region will be collected: once an electron reaches the depletion region, it will "roll downhill" – see band diagram above – to the metal; holes in the depletion region "float uphill" into the semiconductor. The result is an added current component (proportional to G_L) in the "reverse" diode direction.