Midterm 1

October 13, 2010

YOUR NAME:

Instructions:

This exam is open-book, open-notes. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 80 minutes. There are 5 questions worth a total of 140 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page. *You can use without proof any result proved in class but clearly state the result you are using.*

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total	

Problem 1: [True or False, with justification] (40 points)

For each of the following questions, state TRUE or FALSE. Justify your answer in brief, indicating only the "proof idea" or counterexample, drawing a diagram if needed.

(a) Consider the following Boolean function in sum-of-products (SOP) representation:

$$F_1(a, b, c) = ab\bar{c} + a\bar{b} + \bar{a}c$$

Then $ab\bar{c}$ is a prime implicant of F_1.

(b) The Boolean function $G_2 = a + b$ is a Boolean factor of $F_2 = b$.

(c) Suppose you try to solve $g(x) = x^3$ using the (standard, unmodified) Newton-Raphson algorithm. Is the following statement TRUE or FALSE? Justify your answer (using a diagram if you like). Assume perfect numerics.

Newton-Raphson will succeed in converging to the solution regardless of the initial

starting guess.

(d) Suppose you solve the differential equation $\dot{x} = x$. Is the following statement TRUE or FALSE? Justify your answer.

Backward Euler method with a fixed timestep $h = 10$ will succeed in finding a solution that is at least qualitatively accurate.

Problem 2: (20 points)

Consider the circuit given below.

Delays of gates are indicated in the picture. There are no edge (wire) delays.
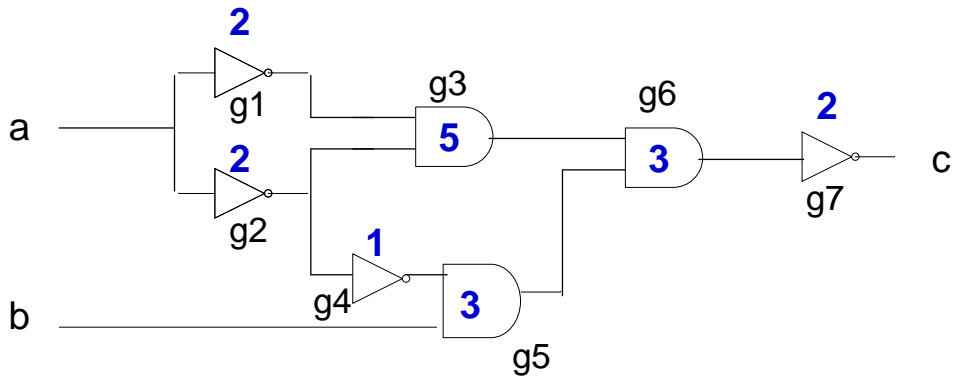
(a) Ignoring whether paths are true or false, what is the longest path in the circuit? What is its length?

(b) Is there a statically sensitizable (SS) path in the circuit? If yes, which is the longest SS path and what is its length? Justify your answer.

Problem 3: (30 points)

Consider the finite-impulse response (FIR) filter shown below:

Assume that the triangles are multipliers (by a constant $h(i)$), and the circles are adders. Suppose that the multipliers and adders each have a delay of one time unit, and that there are no other delays (i.e., no propagation delays, setup times, etc.). Model the environment in which this circuit operates as a feedback path from $y(n)$ to $x(n)$ with negligible delay.

(a) Sketch a graph abstraction where nodes represent the adders and multipliers, and any other nodes you need, (weighted with their delays), and the arcs represent connections (weighted by the number of registers). Be sure to include the environment, so that your graph is fully connected. Show the weights of all nodes and arcs.

(b) Give the W and D matrices for the retiming problem for your graph abstraction. Be sure to clearly number the vertices of your graph, so that the rows and columns of your matrices correspond to these numbers.

(c) Use your matrices to determine whether there is a retiming that has clock period 1. If there is such a retiming, sketch it. If not, explain why not.

Problem 4: (30 points)

Figure 1 shows a simple inverter circuit, where $u(t)$ is the input voltage source. The parameters are: $R = C = 1$.

The inverter is characterized by its V_{in} vs. V_{out} transfer characteristic, given by:

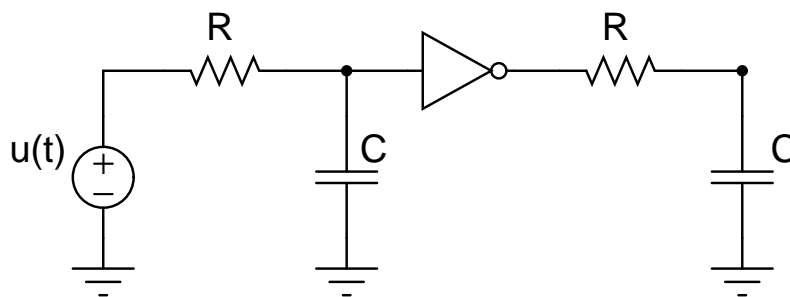$$V_{out} = -\tanh(5V_{in}) = \frac{e^{-5V_{in}} - e^{5V_{in}}}{e^{-5V_{in}} + e^{5V_{in}}}. \tag{1}$$
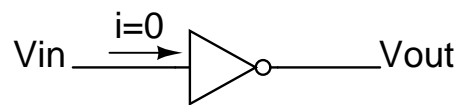

Figure 1: A simple inverter circuit.

Figure 2: Inverter input-output characteristic.

Note that the current flowing into the inverter at the V_{in} node is always 0, as shown in 2.

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(a) Write out differential equations for this simple circuit, in canonical form, in such a way that the forward Euler method can be applied successfully for its transient solution. (10 points)

(b) Suppose that at time $t = 0$, $u(t) = -1$ and the circuit is at its quiescent (DC) steady state. At time $t = 0+$, the input switches to $u(t) = 1$.

    1 Apply FE using time step $h = 0.1$ to compute the voltages at $t = 0.1$. Is the FE result correct? (12 points) (Hint: you may find the approximation $\tanh(5) \simeq 1$ useful.)

2 Write out the TRAP equations using $h = 0.1$. Describe briefly how you will solve the TRAP equations. (8 points)

Problem 5: (20 points)

Computing reciprocal square roots ($\frac{1}{\sqrt{x}}$) has many uses in numerics (*e.g.*, vector normalization). Instead of using the `sqrt` function in the system math library and computing its reciprocal, a smart engineer decided to write her own routine to compute $\frac{1}{\sqrt{x}}$, hoping for a speed advantage with acceptable compromise in accuracy.

Assuming $1 \leq x < 2$, she wrote the following MATLAB code[1]:

```
1 function y = InvSqrt(x)
2   xhalf = 0.5 * x;
3   y = 0.8;
4   y = y * (1.5 - xhalf * y * y);
5 end
```

   (a) Explain what line 3 and line 4 mean and explain why the above code computes a good approximation of $\frac{1}{\sqrt{x}}$. (10 points)

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[1]Note that $0.8 \simeq 1/\sqrt{1.5}$.

(b) The smart engineer tested this code with $x = 9$. The code returns $y = -1.1040$, which is not accurate (the correct solution is $1/3$). Explain why. (5 points)

(c) The smart engineer was not satisfied with this code. She decided to improve it so that it works for all real numbers.

She noticed that floating point numbers are stored in the form of $x = m \times 2^e$ where e is an integer called the *exponent*, and $m \in [1, 2)$ is a real number called the *mantissa*.

To improve the above code to apply for all floating numbers, the engineer modified the code to take m and e as inputs to the `InvSqrt` function, as follows.

```
1 function y = InvSqrt2(m, e)
2    x = m * 2^e;
3    xhalf = 0.5 * x;
4    y = _____;
5    y = y * (1.5 - xhalf * y * y);
6 end
```

Fill in line 4 of the above code to make it work. Explain your answer. (5 points)