Midterm #2 Solutions – EECS 145L Fall 2001

1a Technical requirements of a ground fault interrupter circuit:

(i) open both current carrying power conductors (hot and neutral) when the the difference in their currents exceeds 5 mA

(ii) provide a reset button

(iii) provide a test button

[5 points off for describing the circuit breaker]

[1 points off each for missing the reset or test features]

1b How the ground fault interrupter circuit functions:

(i) the difference in currents is converted into a 60 Hz voltage using a differential transformer

(ii) this voltage is rectified and amplified

(iii) the resulting dc voltage trips two relays which hold both conductors open until reset

(iv) the test button sends a 5 mA current through differential transformer

[1 point off for no test button]

2a Since $R_2/(R_1 + R_2) = R_3/(R_T + R_3)$, $R_2 = R_3$, and R_T = 10 kΩ at 20°C the solution is R_1 = 10 kΩ.

2b $P = V_T^2/R$ = (0.5 volts)^2/(10 kΩ) = 25 µW

2c Amplifier output of 0.05 volts means a bridge output $V_+ - V_-$ = 0.01 volts. Using the bridge equation (supplied on the equation sheets), we have RT = (10000 Ω)*(10000 Ω – 0.01*20,000 Ω)/(10000 Ω + 0.01*20,000 Ω)=9608 Ω

[1 point off for not dividing by the amplifier gain]

[3 points off for assuming a linear response from 0 °C and 0 Ω to 20 °C and 10 kΩ]

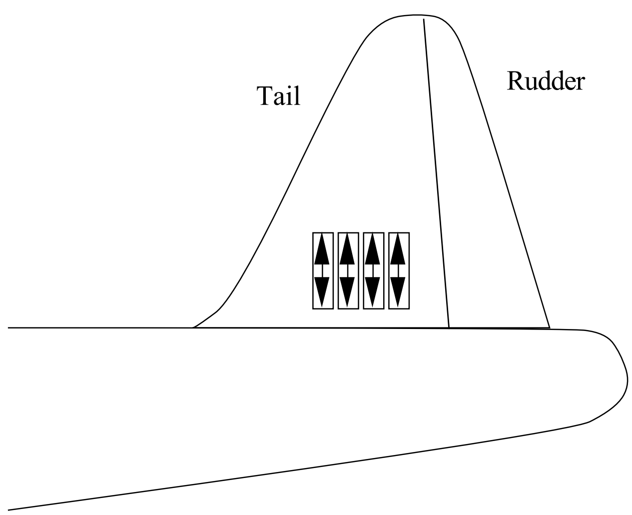
2d T = 20°C + (9608 Ω – 10000 Ω)/(–300 Ω/C°) = 21.3°C

2e V_T = 1 – 10000 Ω /(10000 Ω + 9608 Ω) = 0.490 volts

P = (0.490 volts)^2/(9608 Ω) = 24.99 µW (25 µW was accepted for full credit)

2f Dissipation coefficient = 25 µW/(21.3°C – 20°C) = 19 µW/°C

3a



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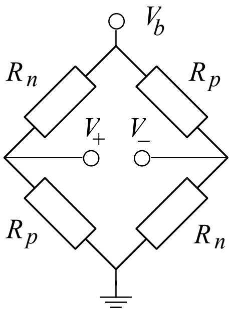
3b



[3 points off if bridge is drawn but gauges are reversed]

3c

$$V_0 = \frac{R_P}{R_P + R_N} - \frac{R_N}{R_P + R_N} = \frac{(R + \Delta R_P) - (R + \Delta R_N)}{2R + \Delta R_P + \Delta R_N}$$

$$V_0 = \frac{\Delta R_P / R - \Delta R_N / R}{2 + \Delta R_P / R + \Delta R_N / R} = \frac{200 \Delta L / L}{2 + 20{,}000 (\Delta L / L)^2} = \frac{100 \Delta L / L}{1 + 10{,}000 (\Delta L / L)^2}$$

[a common error was to write down the bridge equation and then plug in terms like

$R_p = R_0(100\ \Delta L/L + 10{,}000\ (\Delta L/L)^2)$]

3 d For V_s = 1 volt, bridge sensitivity is 0.1 mV per µstrain

3 e The Johnson noise in a single 10 kΩ resistor is given by

$V_{Jrms} = 1.29 \times 10^{-10}\ \mathrm{V\Omega^{-1/2}\ Hz^{-1/2}}$ sqrt(10 kΩ 16 Hz) = $1.29 \times 10^{-10} \times 400$ V = 5.16×10^{-8} V

The bias and ground points are at a fixed voltage, so the Johnson noise in two of the resistors adds a Johnson noise voltage in quadrature to V– and the Johnson noise in the other two resistors adds a Johnson noise in quadrature to V+. (Note that the bridge equation describes how external average voltages are distributed to produce V– and V+)

$$V_{-\,\mathrm{rms}} = \sqrt{V_{J\mathrm{rms}}^2 + V_{J\mathrm{rms}}^2} \qquad V_{+\,\mathrm{rms}} = \sqrt{V_{J\mathrm{rms}}^2 + V_{J\mathrm{rms}}^2}$$

The Johnson noise in $V_0 = V_+ - V_-$ is given by adding the noise of the individual components in quadrature:

$$V_{0\mathrm{rms}} = \sqrt{V_{+\mathrm{rms}}^2 + V_{-\mathrm{rms}}^2} = \sqrt{4V_{J\mathrm{rms}}^2} = 2V_{J\mathrm{rms}}$$

This is 0.103 µV rms, which corresponds to $\Delta L/L \approx 10^{-9}$ rms.

[2 points off for giving the rms equivalent strain due to the Johnson noise in only one resistor]

[3 points off for giving the rms voltage noise from one resistor but not relating it to rms strain]

[4 points off for writing down the Johnson noise equation and using it improperly or incompletely]

145L midterm #2 undergraduate grade distribution:

Problem	
1	13.0 (20 max)
2	34.9 (40 max)
3	29.6 (40 max)
total	77.4 (100 max)

31-40		
41-50	1	D
51-60	1	C
61-70	3	B–
71-80	8	B
81-90	4	A
91-100	3	A+