# Midterm #1 Solutions – EECS 145L Fall 2003

## 1a

To measure common mode gain, connect both inputs of the instrumentation amplifier to a sine wave generator and measure $V_{\mathrm{in}}$ and $V_{\mathrm{out}}$ vs frequency. $G_c = V_{\mathrm{out}}/V_{\mathrm{in}}$.

To measure differential gain, ground one input and connect the other to a sine wave generator and measure $V_{\mathrm{in}}$ and $V_{\mathrm{out}}$ vs frequency. The differential input is $V_{\mathrm{in}}$ and the common mode is $V_{\mathrm{in}}/2$. From $V_{\mathrm{out}} = G_\pm V_{\mathrm{in}} + G_c V_{\mathrm{in}}/2$ and $G_c$ measured above, compute $G_\pm$.

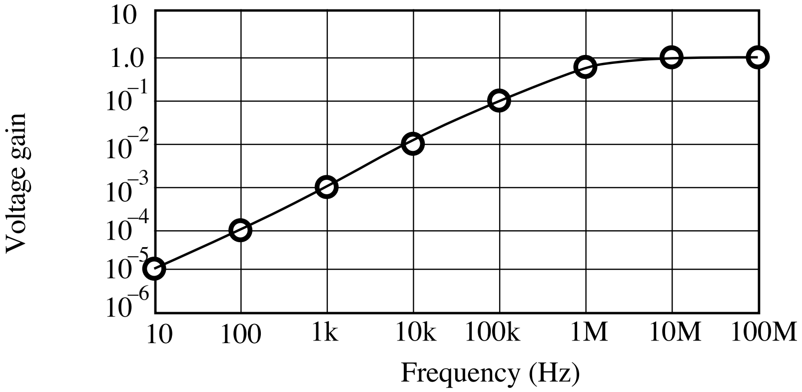
[1 point off for connecting one input to a wave generator and the other to ground, and then using $G_\pm = V_{\mathrm{out}}/V_{\mathrm{in}}$ This ignores the contribution of the small common mode input.]

[3 points off for $G_\pm = V_0/(V_+ - V_-)$ without providing that $V_+ + V_- = 0$]

Note: In Lab 5 you first measured the common mode gain vs. frequency. Then you used an input with both differential and common modes and used the general equation to solve for the differential input.

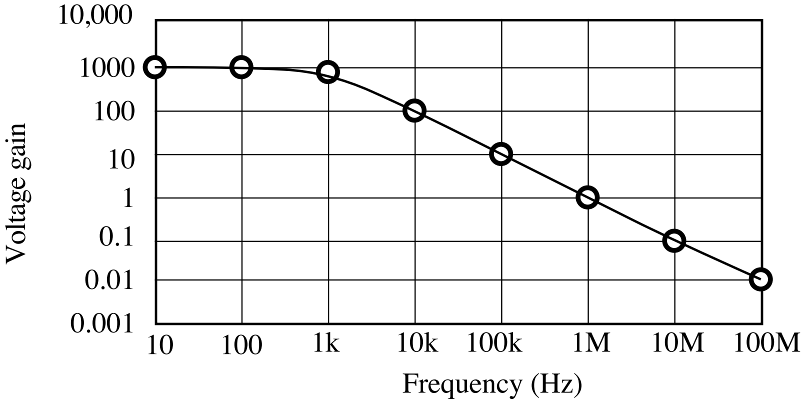
## 1b

$$G_c = \frac{f/10^6\,\mathrm{Hz}}{\sqrt{1+\left(f/10^6\,\mathrm{Hz}\right)^2}}$$



## 1c

$$G_\pm = \frac{1000}{\sqrt{1+\left(f/10^3\,\mathrm{Hz}\right)^2}}$$



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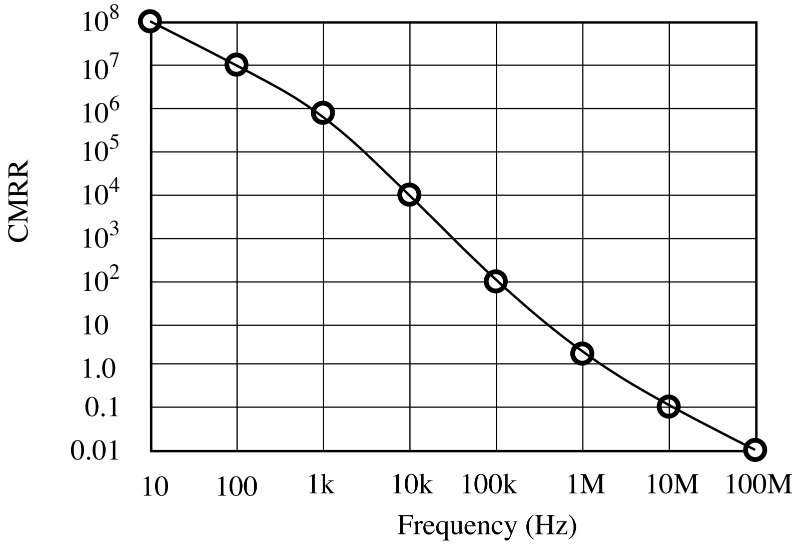
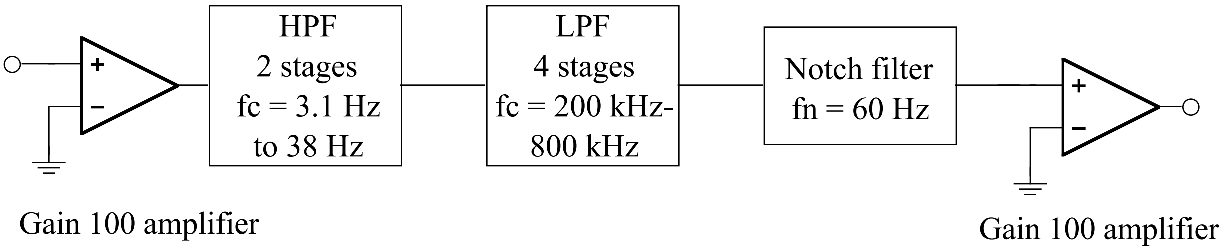
**1d** CMRR = $G_{\pm}/G_c$



**2a**



**Differential gain:**

The 1 mV signal in the 100 to 100 kHz range must be amplified by a factor of 10,000 to bring it to the desired 10 V amplitude.

If a gain of 10,000 is used before filtering, the 10 mV interferences will be amplified to 100V and cause saturation. Some amplification before filtering is desirable to minimize the effect of additive noise in the filter stages. The final amplification is done after the filters. Putting the 10,000x amplifier after the filter stages was also acceptable.

Since the signal is to be amplified with an accuracy of 1%, any filters used to remove unwanted frequencies must have gains >0.99 over the 100 to 100 kHz frequency range.

**5 MHz interference:**

The unwanted 5 MHz interference has an amplitude of 10 mV and must be amplified by < 10 to produce an output <0.1 V. A notch filter is not recommended for a frequency this high.

A low pass filter is needed to drop the system gain from 10,000 at 100 kHz to <10 at 5 MHz. Therefore the LPF gain must be >0.99 at 100 kHz and <0.001 at 5 MHz, a factor of 50 in frequency.

Looking at the Butterworth LPF gain table:

The n=2 row has f1/fc1 = 0.377 at G = 0.99 and f2/fc2 = 31.623 at G = 0.001. Since f1 = 100 kHz and f2 = 5 MHz, these give fc1 = 265 kHz and fc2 = 158 kHz, respectively. This does not satisfy f1 < fc1 < fc2 < f2, so we must increase n.

The n=4 row is what we are looking for, and f1/fc1 = 0.614 at G = 0.99 and f2/fc2 = 5.623 at G = 0.001. These give fc1 = 162 kHz and fc2 = 889 kHz, respectively. So fc = 200 to 600 kHz gives G > 0.99 for f ≤ 100 kHz and G < 0.001 for f ≥ 5 MHz.

[3 points off for using a 5 MHz notch filter, since it is beyond the limits of what is practical (stray capacitance)] [A 5 MHz notch filter was accepted if an n =2 LPF was also used]

**0 to 0.1 Hz temperature drift:**

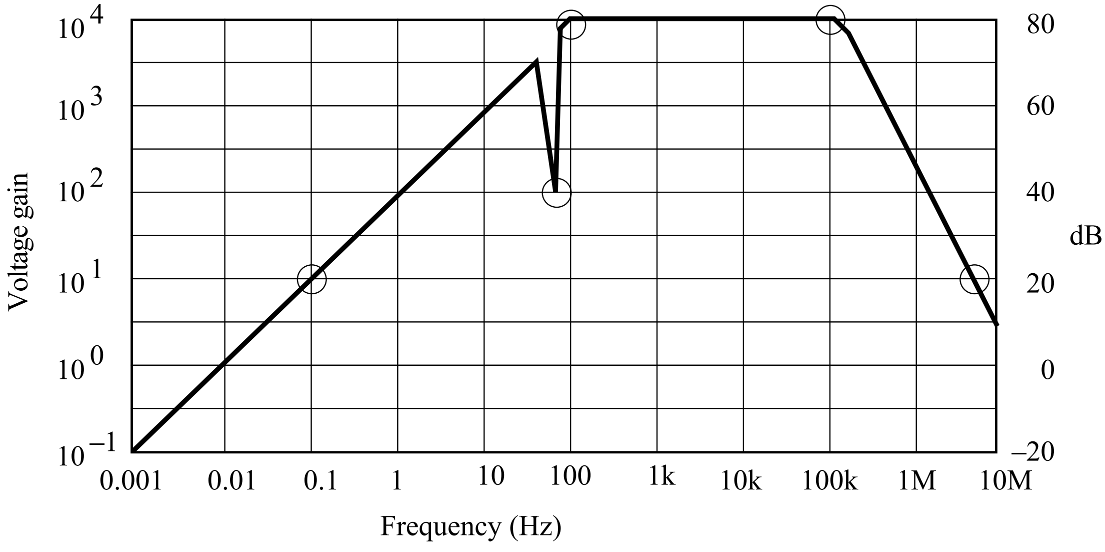
The unwanted 0 to 0.1Hz temperature drift has an amplitude of 10 mV and must be amplified by < 10 to produce an output <0.1 V.

A high pass filter is needed to drop the system gain from 10,000 at 100 Hz to <10 at 0.1 Hz. Therefore the HPF gain must be >0.99 at 100 Hz and <0.001 at 0.1 Hz, a factor of 1000 in frequency.

Looking at the Butterworth HPF gain table, the n=2 row has f1/fc1 = 2.65 at G = 0.99 and f2/fc2 = 0.032 at G = 0.001, where f1 = 100 Hz and f2 = 0.1 Hz.. So fc1 = 38 Hz and fc2 = 3.1 Hz.

[-2 points for each missing corner or notch frequency]
[-2 points for each missing filter order]
[-2 points for gain of 10,000 before filtering]
[-2 points for using a HPF with a large number of poles for 60 Hz, rather than a notch filter]
[-2 points for using an excessive value of n for the LPF or HPF, e.g. n>6]
[-1 point for HPF n =1 (need n = 2)]
[-1 point for HPF fc = 100 Hz (gain >0.99 at 100 Hz requires fc < 100 Hz)]
[-1 point for LPF n = 2 (need n = 4)]
[-1 point for LPF fc = 100 kHz (gain >0.99 at 100 kHz requires fc > 100 kHz)]
[-1 point for LPF fc = 5 MHz (gain < 0.001 at 5 MHz requires fc << 5MHz)]
[-2 points for LPF fc in the 100 Hz to 100 kHz range]
[-6 points for missing an entire element in the system, such as amplification or one of the three required filters]

**2b**



[-2 points if the gain is not $10^4$ in the 100 Hz to 100 kHz band]
[-2 points each if the gains at 0.1 Hz, 60 Hz, and 5 MHz are too high]
[-2 for a gain vs. frequency plot with square wave shape (vertical sides means n = ∞)]
[-3 for an inadequate vertical scale]

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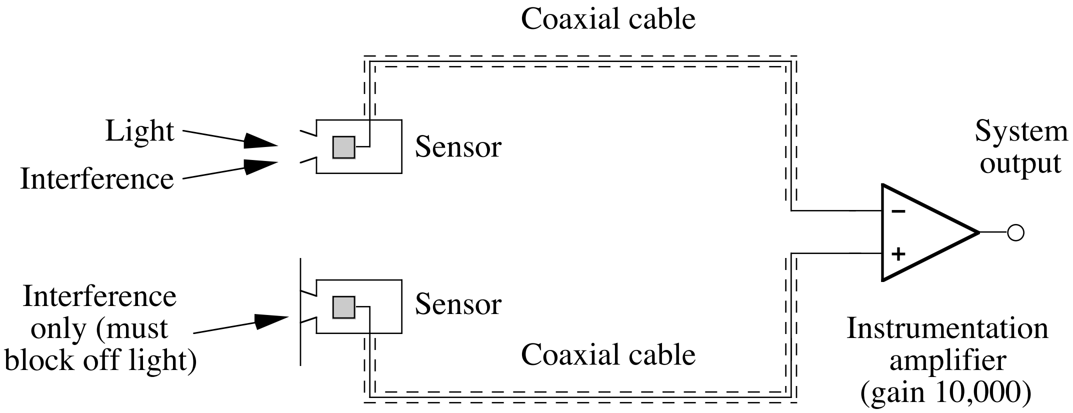
**3a**



[-10 points if both sensors see the same signal- differential amplification will then yield zero]
[-10 points if one sensor is blocked from both light and interference]

**3b**

Differential gain $G_\pm = 10V/1\ mV = 10{,}000$.
Common mode gain $G_c < 0.1\ V/10\ mV = 10$ at 0 to 0.1 Hz
Common mode gain $G_c < 0.1\ V/1\ mV = 100$ at 60 Hz
Common mode gain $G_c < 0.1\ V/10\ mV = 10$ at 1 MHz
So Common Mode Rejection requirements of the instrumentation amplifier are

0 to 0.1 Hz $10^3$ or 60 dB

60 Hz $10^2$ or 40 dB

5 MHz $10^3$ or 60 dB

The most difficult requirement will be at 5 MHz, because stray capacitive coupling makes $G_c$ large at high frequency and limited amplifier gain-bandwidth product makes $G_\pm$ smaller at high frequency.

[-8 points for only providing the definition of CMR]

**145L midterm #1 grade distribution:**

| Problem | |
|---|---|
| 1 | 31.2 (4.7 rms) (35 max) |
| 2 | 27.7 (7.6 rms) (35 max) |
| 3 | 19.0 (9.0 rms) (30 max) |

maximum score = 100
average score = 77.9 (rms = 17.8)

| | | |
|---|---|---|
| 30-39 | 1 | F |
| 40-49 | 0 | D |
| 50-59 | 4 | C- |
| 60-69 | 4 | C |
| 70-79 | 4 | B- |
| 80-89 | 5 | B |
| 90-99 | 4 | A |
| 100 | 4 | A+ |

(4 graduate student average = 83.5)