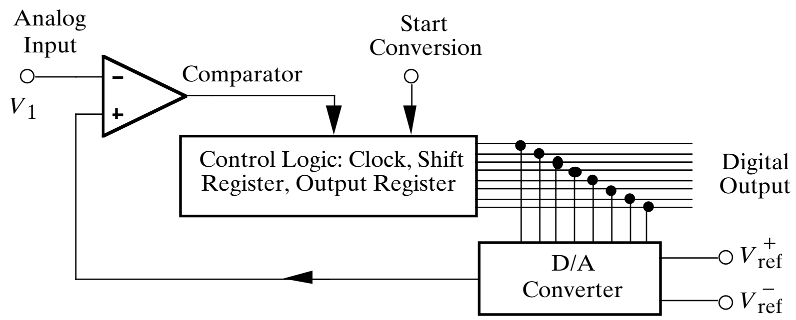
Solutions for Midterm #2 - EECS 145M Spring 2001

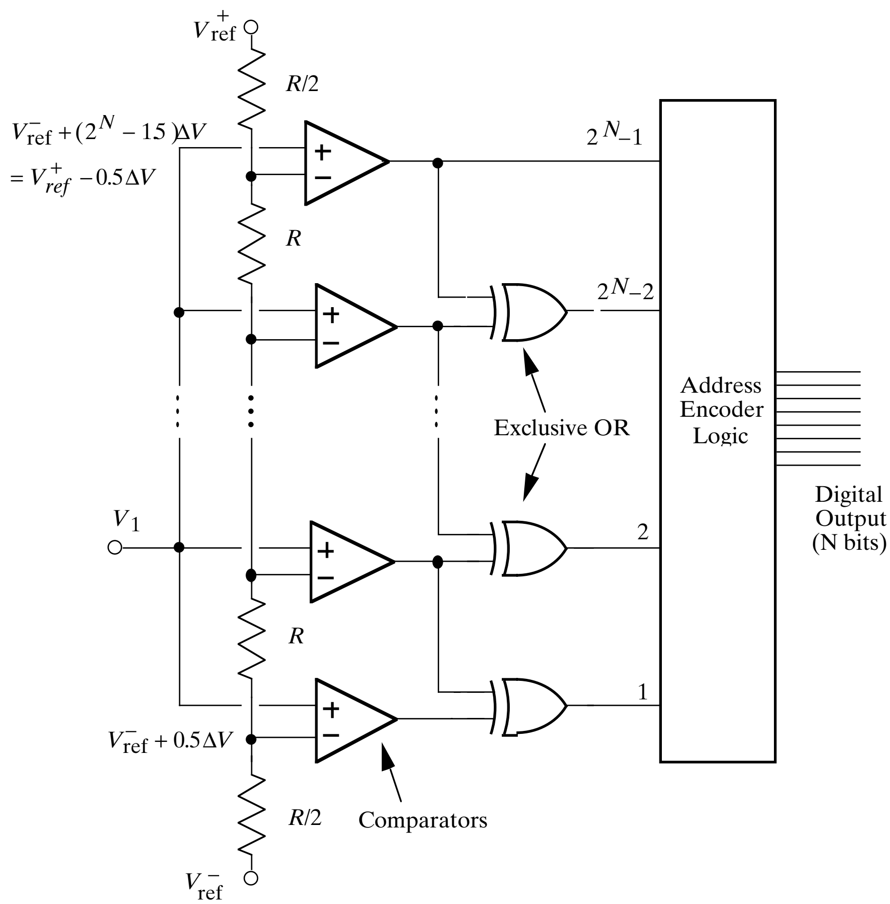
1a Successive approximation A/D



1b

1 set all bits to zero
2 set index i = N (MSB)
3 set bit i to one
4 send bit pattern to D/A
5 if analog input is less than D/A output, set bit i to zero
6 i = i -1
7 return to step 3 (quit if i = zero)

2a Flash A/D



2b

1 Analog input is sent to the (+) inputs of 2^N-1 comparators

2 (–) inputs of comparators connected to points between resistors connected in series

3 comparator outputs are sent to a circuit that determines the N-bit address of the highest comparator whose output is one

4 the N-bit address is the converted output

3a

An infinite periodic series of square pulses of width T_0 and period T_r is the convolution of the square wave $h(t)$ with an infinite periodic series of delta functions:

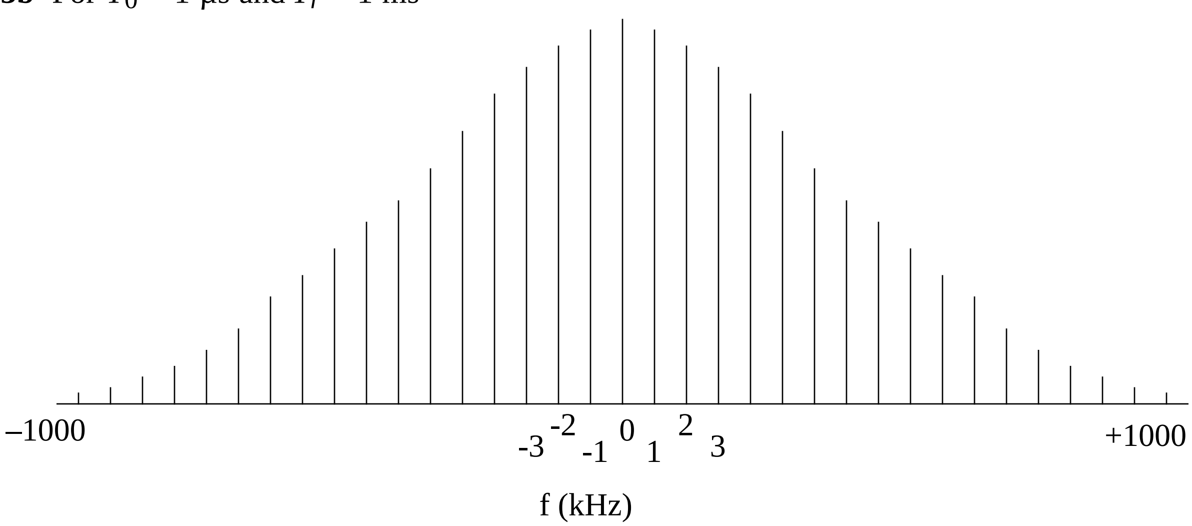
$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_r)$$

By the Fourier convolution theorem, the Fourier transform of $h(t)$ convolved with $g(t)$ is the simple product of the individual Fourier transforms $H(f)$ and $G(f)$:

$$G(f)H(f) = \sum_{n=-\infty}^{\infty} \frac{\sin(\pi T_0 f)}{\pi T_0 f} f_r \, \delta(f - nf_r) \qquad f_r = 1/T_r$$

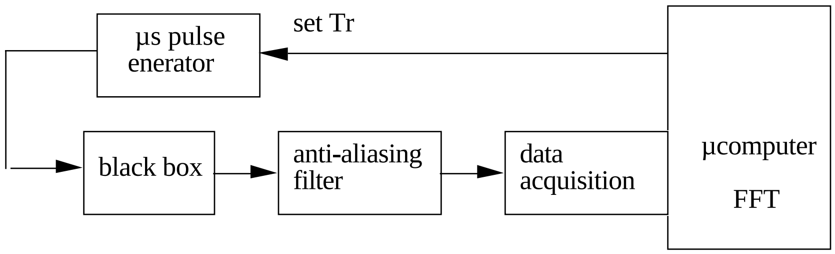
This Fourier transform has the envelope of $H(f)$ but is non-zero only at integer multiples of the repeat frequency f_r.

3b For T_0 = 1 µs and T_r = 1 ms



The Fourier transform is non-zero only at integer multiples of the repeat frequency f_r = 1 kHz

4a



4b Want filter gain $G_1 > 0.999$ for frequencies $f_1 < 100$ kHz.
From equation sheet, an 8-pole filter has a gain of 0.999 at $f/fc = 0.678$
Solve for $f_c = f_1/0.678 = 147.5$ kHz

Want filter gain $G_2 < 0.01$ at the lowest frequency f_2 that could alias below $f_1 = 100$ kHz
From equation sheet, an 8-pole filter has a gain of 0.01 at $f/f_c = 1.778$
Solve for $f_2 = 1.778\ f_c = 262$ kHz

f_2 aliases to f_1 when $f_s = f_1 + f_2$
to avoid aliasing we want $f_s > 100$ kHz + 262 kHz = 362 kHz

[the requirement that $f_s > 2\ f_2 = 524$ kHz is more conservative than necessary but was accepted with no deduction]

4c Since we only need Fourier magnitudes at multiples of 100 Hz, the series of 1 μs pulses needs to contain harmonic frequencies only at multiples of 100 Hz. By choosing a pulse repetition period ***Tr* = 0.01 seconds**, the series of 1 μs pulses contains a fundamental frequency of 100 Hz and higher harmonic multiples of 100 Hz.

Since the number of samples M is equal to the number of Fourier magnitudes, the lowest M is achieved when the frequency spacing is $\Delta f = 100$ Hz. Since $S = 1/\Delta f$, $S = 0.01$ seconds. By increasing the sampling frequency in part 4b from $f_s = 362$ kHz to $\boldsymbol{f_s} = $ **409.6 kHz**, we will have ***M* = 4096 samples** (and Fourier magnitudes) in 0.01 seconds.

4d H_n is the Fourier coefficient at the frequency $f_n = n$ 100 Hz

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{1}{H_0} \frac{\sqrt{[\text{Re}(H_n)]^2 + [\text{Im}(H_n)]^2}}{\sin(\pi \mu s\ f_n)/(\pi \mu s\ f_n)}$$

Note 1: The gain is computed as the output amplitude (Fourier magnitude) divided by the input magnitude of the 1 μs pulses at that frequency. The response of the Butterworth anti-aliasing filter does not enter because its gain is >0.999 below 100 kHz.

Note 2: The gain is normalized to 1 at zero frequency

Midterm #2 class statistics:

Problem	max	average	rms
1	20	15.1	5.2
2	20	15.3	5.3
3	20	14.4	5.6
4	40	26.8	6.1
total	100	71.5	14.4

Grade distribution:

Range	number	*approximate* letter grade
46-50	2	C–
51-55	0	
56-60	2	C+
61-65	1	B–
66-70	2	B
71-75	2	B+
76-80	1	A–
81-85	2	A
86-90	2	A
91-95	1	A+
96-100	0	

6 A’s; 5 B’s; 4 C’s