

Solutions for Midterm #2 - EECS 145M Spring 2004

- 1a** Frequency aliasing is caused by sampling at a frequency f_s that is less than twice the highest frequency f_{\max} in the waveform.

It may be avoided by (i) increasing the sampling frequency so that $f_s > 2 f_{\max}$ or (ii) using an analog low-pass filter to eliminate frequencies above $f_s/2$.

[either were accepted for full credit]

- 1b** Spectral leakage is caused when frequency components are not sampled for a whole number of cycles, which results in a discontinuity between the last sample and the “next sample”, which is also the first sample.

It is avoided by multiplying the sampled values by a windowing function that has zero value and zero slope at the ends of the sampling window.

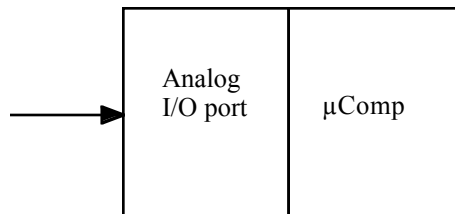
[3 points off if the only answer is to sample for an integer number of cycles- most waveforms contain many harmonics]

- 2** $n = 8$; $f_1/f_c = 0.784$; $f_c = 25.5$ kHz; $f_2/f_c = 1.778$; $f_2 = 45.4$ Hz; $f_s = f_1 + f_2 = 65.4$ kHz

$n = 6$; $f_1/f_c = 0.723$; $f_c = 27.7$ kHz; $f_2/f_c = 2.154$; $f_2 = 59.6$ Hz; $f_s = f_1 + f_2 = 79.6$ kHz

$n = 6$, $f_c = 27.7$ kHz best meets the requirements

- 3a** The system is quite simple, since most of the control and data transfer lines are internal. An anti-aliasing filter is not needed since we know that the highest frequency in the signal is 100 kHz.



- 3b** Sample at 262,144 Hz for 1 second to take 262,144 samples (a power of 2)

- 3c** The successive approximation, flash, and half-flash would have the necessary speed and accuracy

[2 points off for tracking, which would be too slow]

- 3d**

- 1 program sets value[M] = 0
- 2 program tells the analog I/O board to take $M = 262,144$ samples at 262,144 Hz and store in vector value[]
- 3 program loops until value[M] = -1 while analog I/O board acquires samples
- 4 multiply sampled values by Hann window
- 5 take fast Fourier transform to get complex frequency values H_n
- 6 Convert complex values to magnitude values

$$F_n = \sqrt{[\operatorname{Re}(H_n)]^2 + [\operatorname{Im}(H_n)]^2}$$

- 7 scan though the F_n values to identify the two peaks at n_1 and n_2

- 8 the two frequencies are $f_1 = n_1$ Hz and $f_2 = n_2$ Hz
- 9 because of short-range spectral leakage, there will be side lobes at n_1-1 , n_1+1 , n_2-1 , and n_2+1 . But these side lobes will have amplitudes proportional to the main peak so F_2/F_1 is a good estimate of the amplitude ratio.

[2 points off for omitting the Hann window- there was no guaranty that both $f_1 \times S$ and $f_2 \times S$ are integers.]

- 3e** The two amplitude peaks will occur at F_{3000} and F_{3001} . Because of the short-range spectral leakage, there will be non-zero values at F_{2999} , F_{3000} , F_{3001} , and F_{3002} . The design using $S = 1$ sec does not work, because if one harmonic is much stronger than the other, the weaker harmonic will be lost in the $\pm 1/S$ side lobes of the stronger harmonic. In addition, the result is the same as a single harmonic of a frequency between 3000.0 and 3001.0 Hz, depending on the relative amplitudes.

To see the 3000 and 3001 Hz components as separate peaks in spite of the $\pm 1/S$ side lobes, sample for 4 seconds (1,048,576 samples), use the Hann window and take the fft. The result will be two peaks centered at F_{12000} and F_{12004} .

[full credit if the design sampled for 4 seconds in part 3b; then 3e works without modification]

- 4a** In this problem we are only interested in the Fourier magnitude at 180 Hz. There is no need to perform the fft and get the magnitudes at all frequencies. Note that this problem was the subject of lecture slide 298.

1 sample 3072 values h_k over 1/60 sec.

2 use finite impulse response filters to compute the real and imaginary Fourier components

$$\text{Re}(H_3) = \sum_{k=0}^{3071} h_k \cos(2\pi k 3 / 3072)$$

$$\text{Im}(H_3) = \sum_{k=0}^{3071} h_k \sin(2\pi k 3 / 3072)$$

(Since the sampling window is $S = 1/60$ seconds, the 180 Hz Fourier coefficient is H_3 .)

3 compute and display the magnitude value at 180 Hz

$$F_3 = \sqrt{[\text{Re}(H_3)]^2 + [\text{Im}(H_3)]^2}$$

4 shift all the h_k values down, delete the old h_0 , and sample a new value h_{1023} .

5 go back to step 2

(Note that to perform steps 2-5 at 184 kHz. the multiplications and additions in step 2 must be performed at 368 Mhz; this requires high-speed processors)

[2 points off for not sampling waveform]

[2 points off for sampling for only 1/180 sec; spectral leakage from the 1/3 cycle of 60 Hz would overwhelm]

[5 points off for using a FIR digital filter but not giving an equation for it]

[6 points off for sampling for 1/60 sec and doing the full FFT]

- 4b** Note that this problem was the subject of lecture slides 299 and 300.

1 sample 3072 values h_k over 1/60 sec.

2 use finite impulse response filters to compute the real and imaginary Fourier components

$$\text{Re}(H_3) = \sum_{k=0}^{3071} h_k \cos(2\pi k/3072)$$

$$\text{Im}(H_3) = \sum_{k=0}^{3071} h_k \sin(2\pi k/3072)$$

3 compute and display the magnitude value at 180 Hz

$$F_3 = \sqrt{[\text{Re}(H_3)]^2 + [\text{Im}(H_3)]^2}$$

4 Sample a new value h_{3072}

5 Compute the new value H_3' as

$$H_3' = e^{j2\pi/3072} [H_3 - h_0 + h_{3072}]$$

6 compute and display the magnitude value at 180 Hz

$$F_3' = \sqrt{(\text{Re } H_3')^2 + (\text{Im } H_3')^2}$$

7 shift all the h_k values down, delete the old h_0 , set $H_3 = H_3'$

8 go back to step 4

(Note performing step 4 at 184 kHz. requires multiplications and additions at < 1 Mhz)

[9 points off for writing the definition of the IIR filter only]

Midterm #2 class statistics:

Problem	max	average	rms
1	16	15.6	1.1
2	25	23.0	4.6
3	35	29.1	5.1
4	24	9.3	4.6
total	100	76.9	9.9

Grade distribution- (range = 57 to 93):

Range	number	<i>approximate</i> letter grade
40-49	0	F
50-59	1	D
60-69	4	C
70-79	4	B
80-89	7	A
90-99	2	A, A+
100	0	