EECS 20. Midterm No. 1 Solution October 8, 2001.

1. **40 points.** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $[\{1,2,3\} \to \{a,b\}] \subset [\textit{Naturals} \to \{a,b\}]$
   Solution: false

   (b) $\{g \mid g = \textit{graph}(f) \wedge f\colon X \to Y\} \subset X \times Y$
   Solution: false

   (c) $F\colon [\textit{Reals} \to \textit{Reals}] \to [\textit{Reals} \to \textit{Reals}]$, such that $\forall\ t \in \textit{Reals}$, and $\forall\ x \in [\textit{Reals} \to \textit{Reals}]$,

   $$(F(x))(t) = \sin(2\pi \cdot 440t)$$

   is a memoryless system.
   Solution: false

   (d) Let $f\colon \textit{Reals} \to \textit{Reals}$ and $g\colon \textit{Reals} \to \textit{Reals}$, where g is obtained by delaying f by $\tau \in \textit{Reals}$. That is,

   $$\forall\ t \in \textit{Reals}, \quad g(t) = f(t - \tau).$$

   Then $\textit{graph}(g) \subset \textit{graph}(f)$.
   Solution: false

2. **30 points.** Consider a state machine where

   $$\textit{Inputs} = \{1, \textit{absent}\},$$

   $$\textit{Outputs} = \{0, 1, \textit{absent}\},$$

   $$\textit{States} = \{a, b, c, d, e, f\},$$

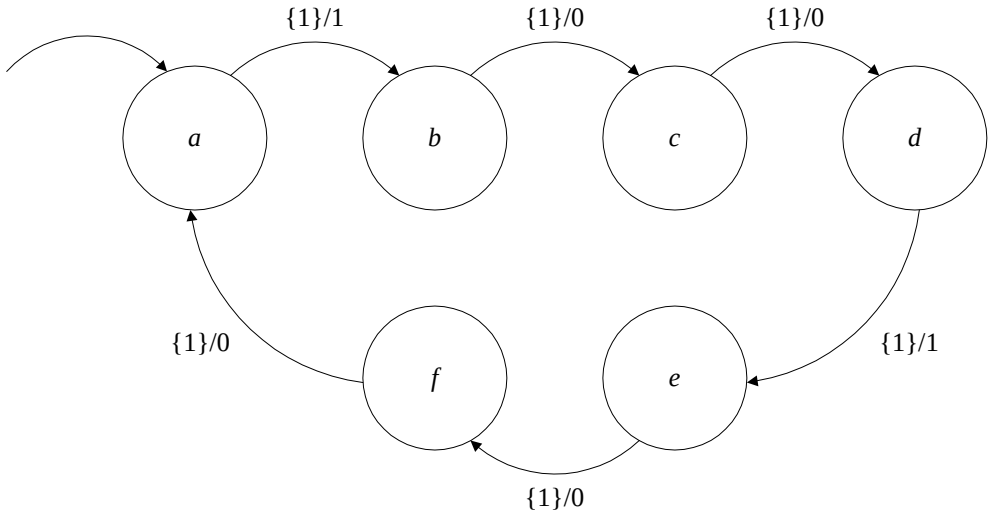
   $$\textit{initialState} = a,$$

   and the *update* function is given by the following table (ignoring stuttering):

$(\textit{currentState}, \textit{inputSymbol})$	$(\textit{nextState}, \textit{outputSymbol})$
$(a, 1)$	$(b, 1)$
$(b, 1)$	$(c, 0)$
$(c, 1)$	$(d, 0)$
$(d, 1)$	$(e, 1)$
$(e, 1)$	$(f, 0)$
$(f, 1)$	$(a, 0)$

(a) Draw the state transition diagram for this machine.

Solution:



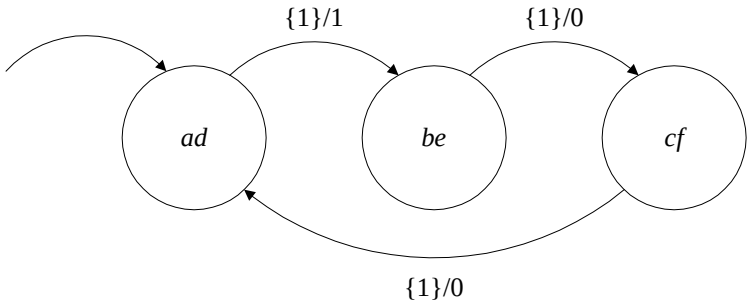
(b) Ignoring stuttering, give the *Behaviors* relation for this machine.

Solution:

$$\textit{Behaviors} = \{(x, y) \mid x = (1, 1, 1, \cdots) \ \wedge \ y = (1, 0, 0, 1, 0, 0, 1, 0, 0, \cdots)\}.$$

(c) Find a state machine with three states that is bisimilar to this one. Draw that state machine, and give the bisimulation relation.

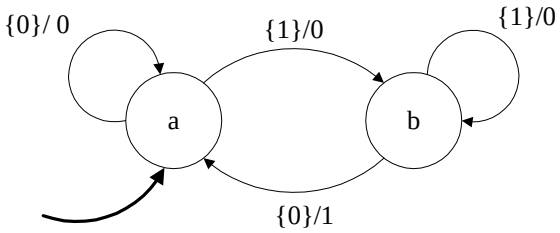
Solution:



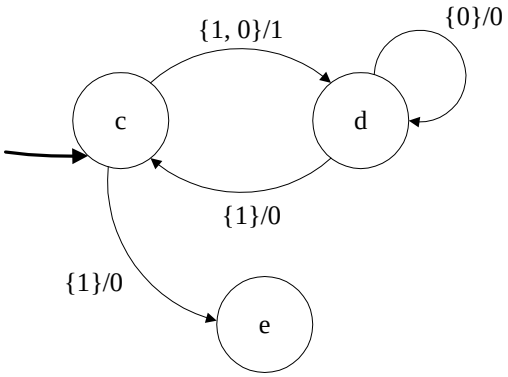
The bisimulation relation is

$$\{(a, ad), (b, be), (c, cf), (d, ad), (e, be), (f, cf)\}$$

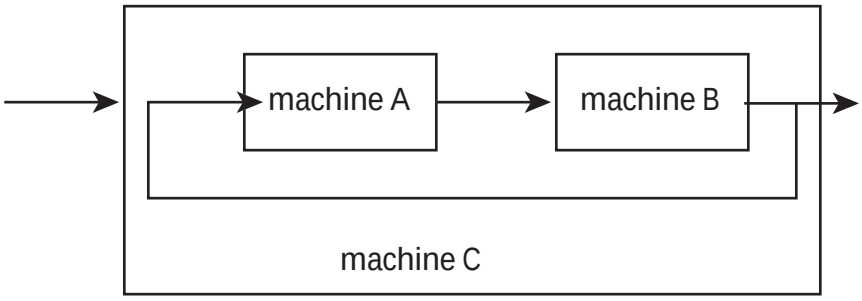
3. **30 points.** Consider the following three state machines:



machine A



machine B



Machines A and B have input and output alphabets

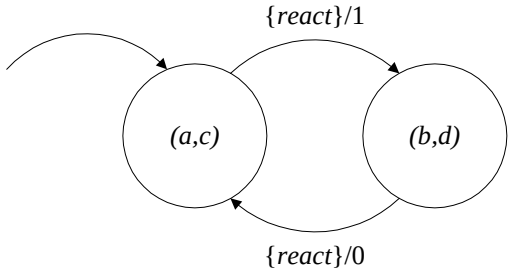
$$\mathit{Inputs} = \mathit{Outputs} = \{0, 1, \mathit{absent}\}.$$

Machine C has the same output alphabet, but input alphabet $\mathit{Inputs}_C = \{\mathit{react}, \mathit{stutter}\}$.

(a) Which of these machines is deterministic?
Solution: A and C are deterministic.

(b) Draw the state transition diagram for the composition (machine C), showing only states that are reachable from the initial state.

Solution:



(c) Give the $Behaviors_C$ relation for the composition of machine C, ignoring stuttering.

Solution:

$$Behaviors_C = \{(a, b) \mid a = (react, react, \cdots) \ \wedge \ b = (1, 0, 1, 0, \cdots)\}$$