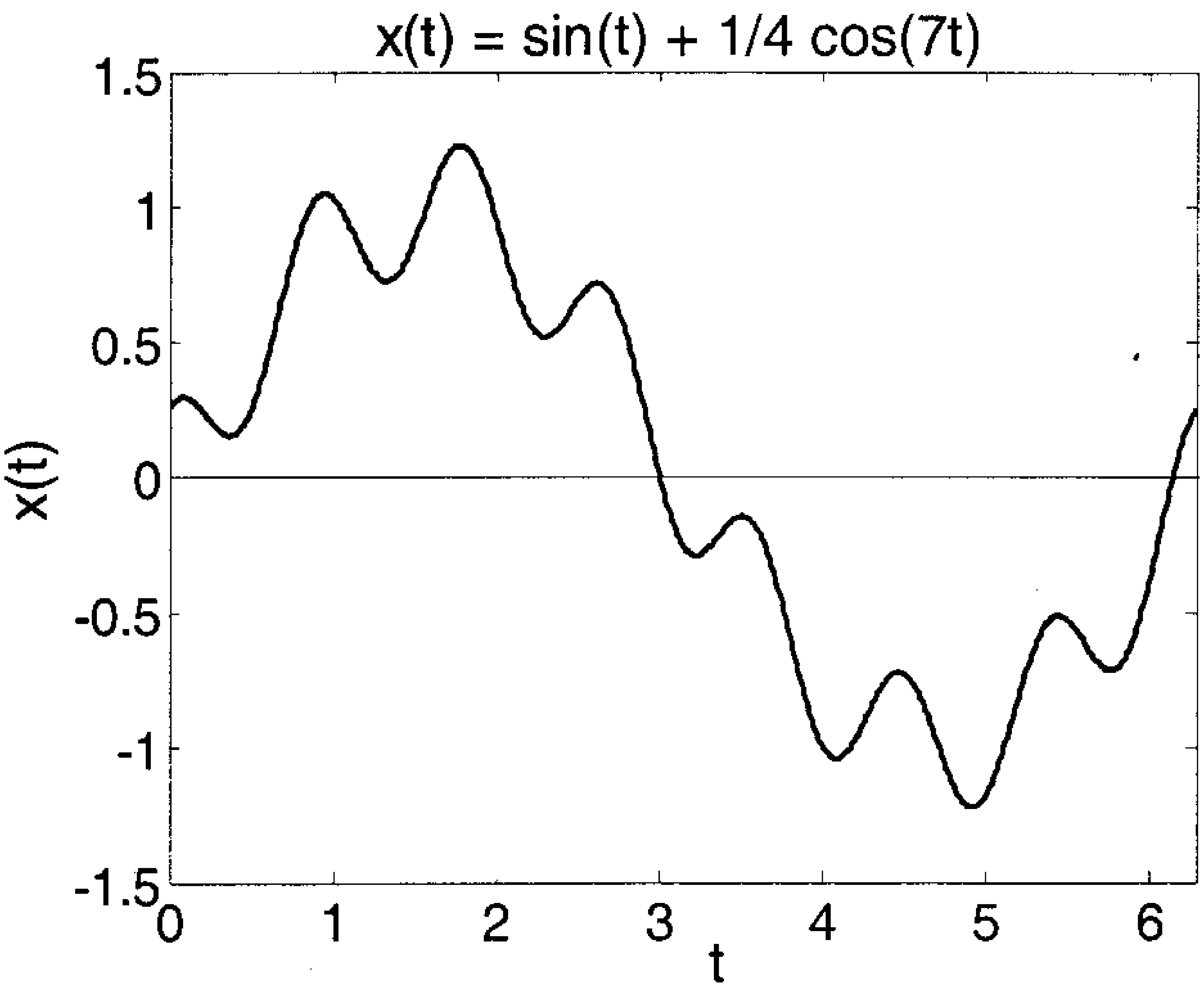
Name: ______________________

Problem 1: 9 Points Possible

Consider the following signal: x(t) = sin(t) + ¼ cos(7t) for all t in Reals

This signal is shown below.



a) What is the fundamental frequency ω_0 for this signal?

$$P = 2\pi \left(\begin{matrix} 2\pi \text{ for } \sin t \\ 2\pi/7 \text{ for } \cos 7t \end{matrix} \right) \quad \text{so} \quad \omega_0 = \frac{2\pi}{P} = 1$$

b) Of the graphs of A_k and φ_k on the next page, only one pair of graphs (one A_k graph and its corresponding φ_k graph) shows the correct trigonometric Fourier series for this signal.

Which is the correct graph for A_k? Which is the corresponding correct graph for φ_k? Write your choices here. Justify your answer!

$$X(t) = \cos(t - \pi/2) + 1/4 \cos 7t$$

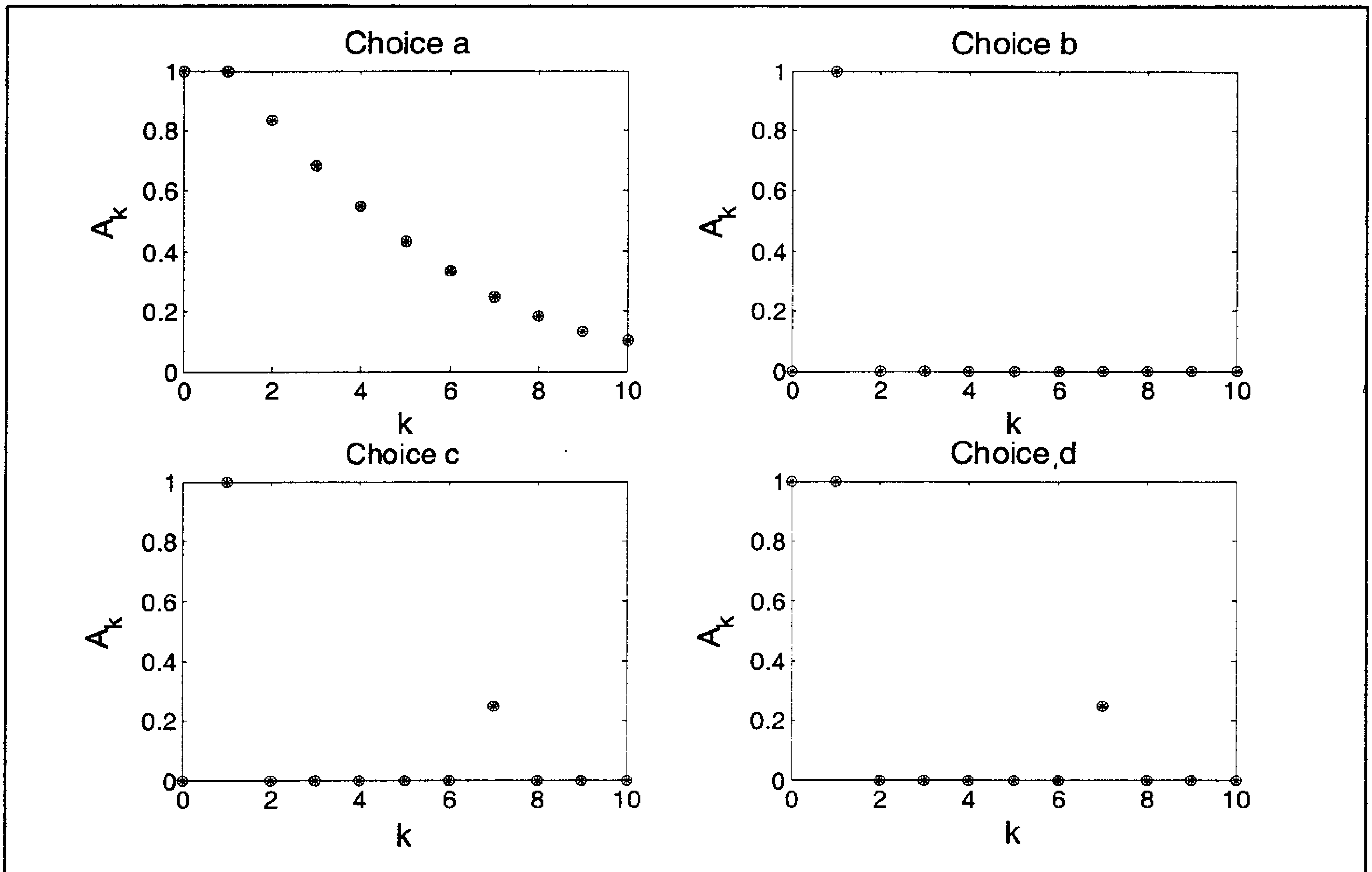
$$\Downarrow \qquad\qquad \Downarrow$$

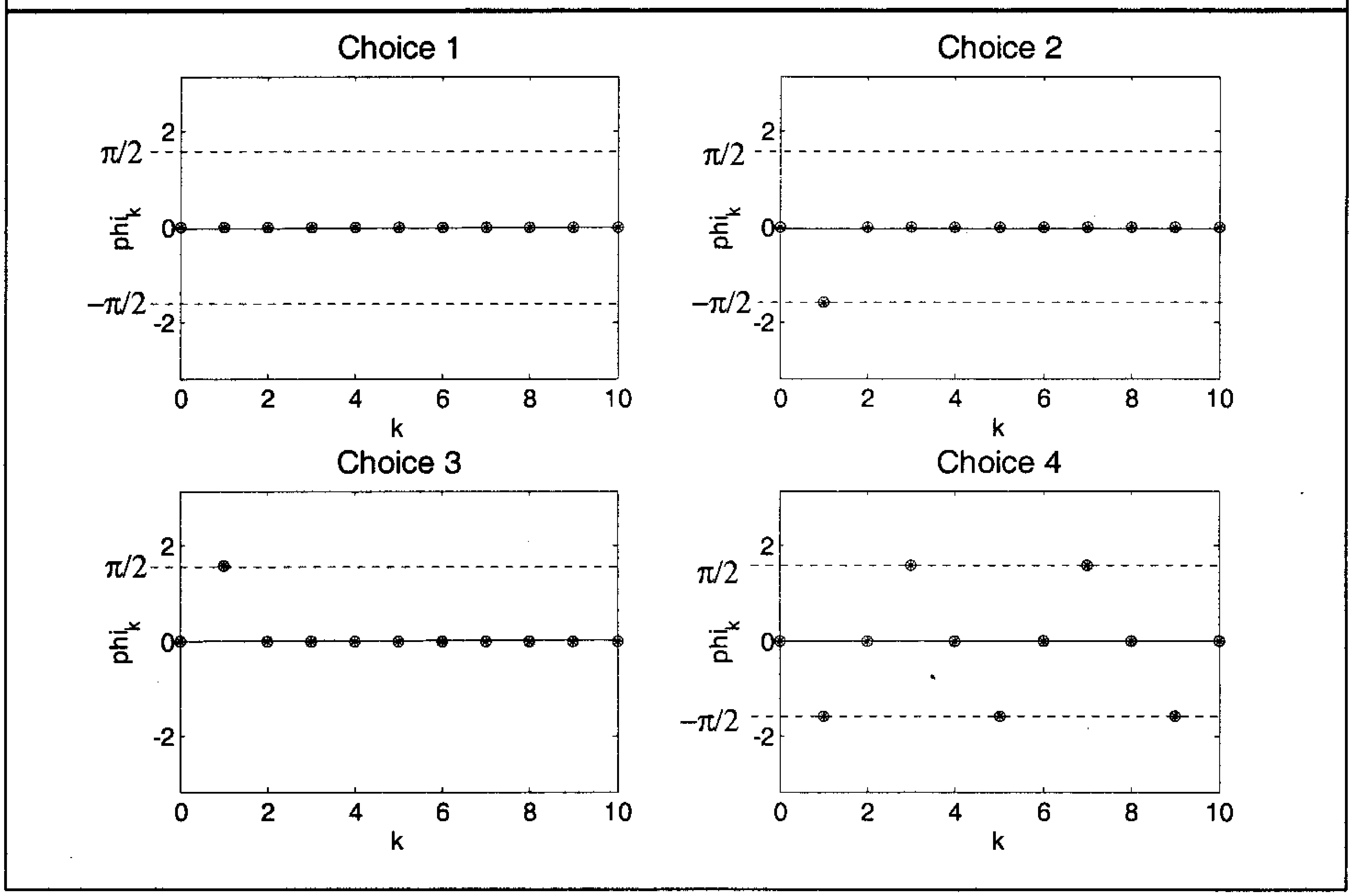
$$A_1 = 1 \quad \phi_1 = -\pi/2 \qquad A_7 = 1/4 \quad \phi_7 = 0$$

all other terms zero

c, 2

Name: ______________________________


Choice a
Choice b
Choice c
Choice,d
A_k
k

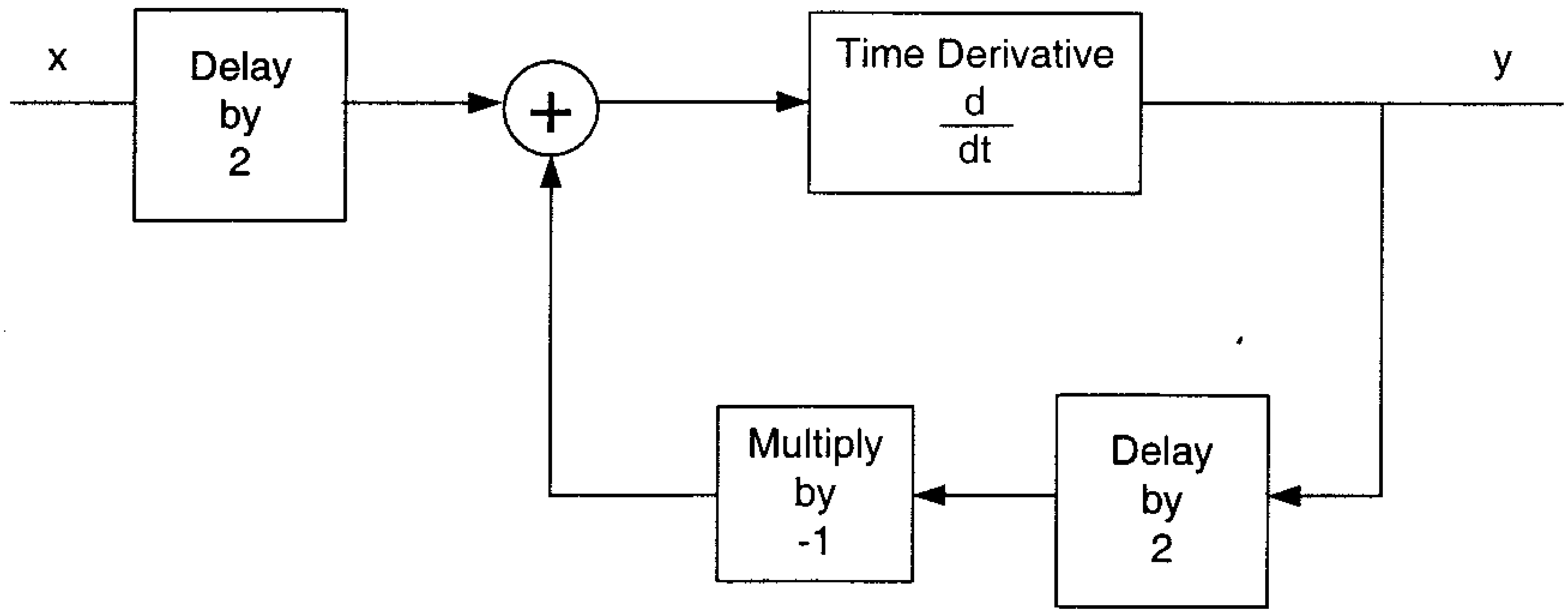

Choice 1
Choice 2
Choice 3
Choice 4
$\pi/2$
$-\pi/2$
phi_k
k

Name: ______________________

Problem 2: 12 Points Possible

Consider the continuous-time system with input x and output y defined by the diagram below.

Find the frequency response H(ω) for this system. <u>Clearly indicate your final answer</u>.



Delay by 2 has $H(\omega) = e^{-i\omega 2}$ $\left(e^{i\omega(t-2)} = e^{i\omega t} e^{-i\omega 2}\right)$

Time derivative has $H(\omega) = i\omega$ $\left(\frac{d}{dt} e^{i\omega t} = i\omega e^{i\omega t}\right)$

Multiply by -1 has $H(\omega) = -1$

Feedback loop may be replaced with $\dfrac{i\omega}{1 - (-i\omega e^{-i\omega 2})}$

This is cascaded with $e^{-i\omega 2}$, hence overall $H(\omega)$ is

$$\boxed{H(\omega) = e^{-i\omega 2}\left(\frac{i\omega}{1 + i\omega e^{-i\omega 2}}\right)}$$

Name: ______________________

Problem 3: 12 Points Possible

Consider the continuous-time LTI system described by the impulse response

$h(t) = \delta(t) + 2\,\delta(t-2) + 3\,\delta(t+3)$

a) Is this a FIR system or an IIR system? Justify your answer.

FIR. The impulse response is only nonzero for a finite amount of time ($t=0$, $t=2$, and $t=-3$).

b) Is this system causal? Justify your answer.

No. $h(t)$ is not zero for all negative t due to the $\delta(t+3)$ term. The output will depend on a future value of the input.

c) For a general input x, give a simple expression for the output y. Justify your answer.

$$y(t) = h * x = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \big(\delta(t) + \delta(t-2) + \delta(t+3)\big)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau + 2\int_{-\infty}^{\infty} \delta(\tau-2)\, x(t-\tau)\, d\tau + 3\int_{-\infty}^{\infty} \delta(\tau+3)\, x(t-\tau)\, d\tau$$

$$\boxed{y(t) = x(t) + 2\,x(t-2) + 3\,x(t+3)}$$

(sifting property)

Name: ______________________

Problem 4: 12 Points Possible

Indicate whether the following continuous-time systems are linear, time invariant, and/or causal by writing yes or no in the spaces provided. You are not required to show your reasoning.

a) $S(x)(t) = e^{i2\pi t} x(t)$

Linear? yes

Time-invariant? no

Causal? yes

b) $S(x)(t) = x(-t - 2)$

Linear? yes

Time-invariant? no

Causal? no

c) $S(x)(t) = (x(t-2))^2$

Linear? no

Time-invariant? yes

Causal? yes

d) $S(x)(t) = x(t^2-2)$

Linear? yes

Time-invariant? no

Causal? no

Name: ______________________

Problem 5: 18 Points Possible

Consider the discrete-time system given by

y(n) + 2 y(n-2) = x(n)

a) Find the frequency response $H(\omega)$ for this system. Write your final answer here.

$$H(\omega) e^{i\omega n} + 2 H(\omega) e^{i\omega(n-2)} = e^{i\omega n}$$

$$H(\omega) e^{i\omega n}(1 + 2e^{-i\omega 2}) = e^{i\omega n}$$

$$\boxed{H(\omega) = \frac{1}{1 + 2e^{-i\omega 2}}}$$

b) Provide matrices A, B, C, and D and a state s(n) leading to the equivalent description

s(n+1) = A s(n) + B x(n) y(n) = C s(n) + D x(n)

Write your final answer here.

$$S(n) = \begin{bmatrix} y(n-1) \\ y(n-2) \end{bmatrix} \quad A = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = [0 \quad -2] \quad D = 1$$

c) Find the impulse response h(n) for this system. Write your final answer here.
Hint: Is this system causal? What does that tell you about h(n)?

$h(n) = 0$ for all $n < 0$ since causal

$$h(n) = \delta(n) - 2h(n-2)$$

$h(0) = 1$

$h(1) = 0$

$h(2) = -2$

$h(3) = 0$

$h(4) = 4$

$$h(n) = \begin{cases} (-2)^{n/2} & \text{for } n \text{ even and nonnegative} \\ 0 & \text{otherwise} \end{cases}$$

Name: ______________________

Problem 6: 12 Points Possible

Consider the continuous-time system with magnitude response and phase response given by

$$|H(\omega)| = \begin{cases} 10 & \text{for } \omega \in [-\pi/2, \pi/2] \\ 0 & \text{otherwise} \end{cases} \qquad \angle H(\omega) = \begin{cases} \omega & \text{for } \omega \in [-\pi/2, \pi/2] \\ 0 & \text{otherwise} \end{cases}$$

and the continuous-time input

$$x(t) = 4 + 3\sin(\frac{\pi}{3}t) - 2\cos(\frac{\pi}{2}t) - \sin(\pi t)$$

a) What is the period of the input x?

$P_1 = \frac{2\pi}{\pi/3} = 6 \quad P_2 = \frac{2\pi}{\pi/2} = 4 \quad P_3 = \frac{2\pi}{\pi} = 2 \quad \text{LCM} = \boxed{P = 12}$

b) What is the output y corresponding to the input x? Express your answer <u>without using imaginary numbers</u>. <u>Clearly indicate your final answer in the space below</u>.

For each sinusoid, the individual output is

$y(t) = |H(\omega)| \cos(\omega t + \angle H(\omega))$ (for a pure cosine, adjust for others)

We can add the individual outputs due to linearity.

$4 = 4\cos(0t) \Rightarrow 10 \cdot 4$

$3\sin \pi/3\, t \Rightarrow 10 \cdot 3\sin(\pi/3\, t + \pi/3)$

$-2\cos \pi/2 \Rightarrow 10 \cdot -2 \cdot \cos(\pi/2\, t + \pi/2)$.

$-\sin \pi t \Rightarrow 0$ (frequency outside $[-\pi/2, \pi/2]$)

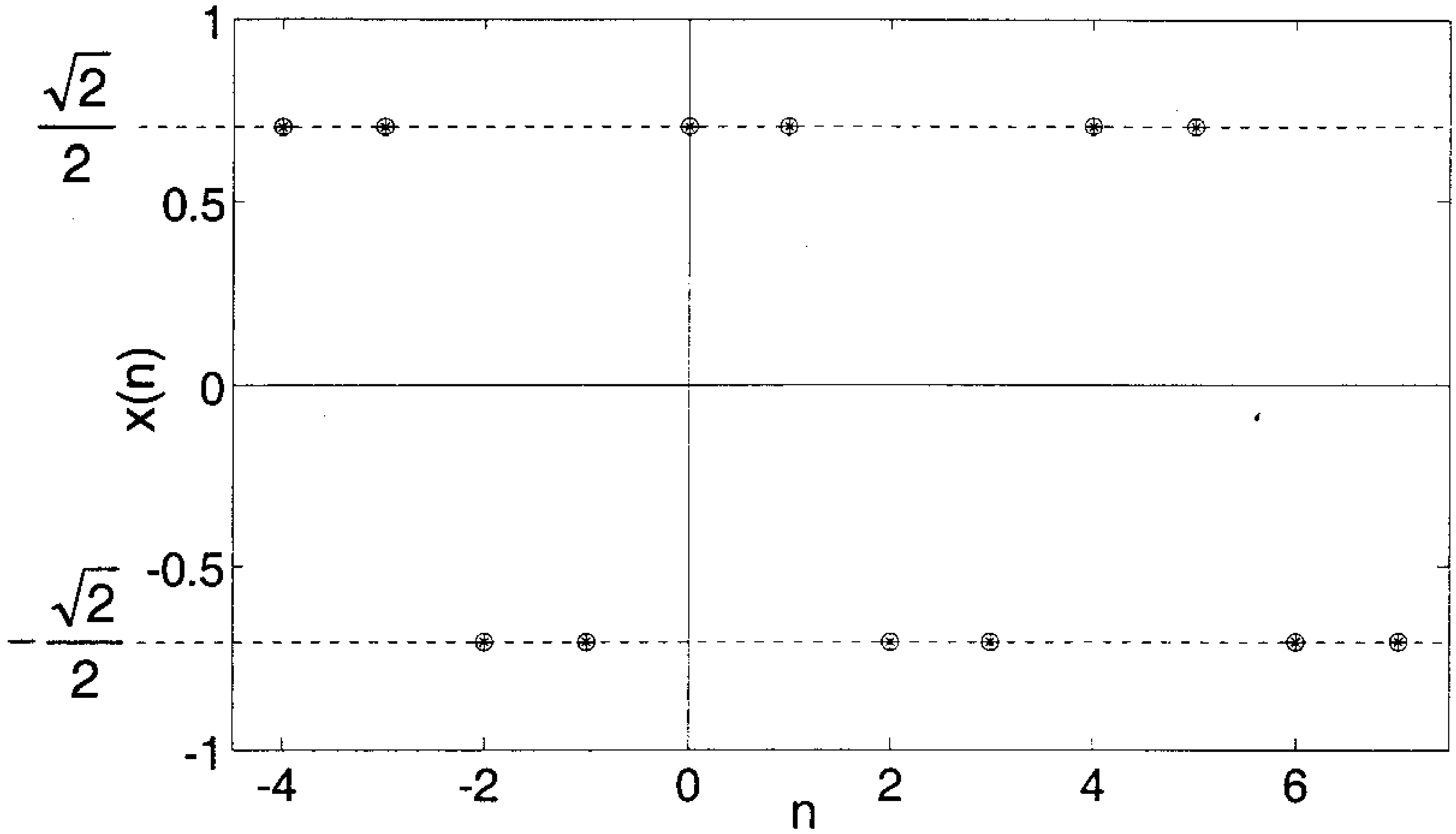
$$\boxed{y(t) = 40 + 30\sin(\pi/3(t+1)) - 20\cos(\pi/2(t+1))}$$

$\cos(\pi/3\, t - \frac{\pi}{6})$

Name: ______________________

Problem 7: 16 Points Possible

Consider the discrete-time signal x depicted below over three periods:



Find both the trigonometric and complex exponential Fourier coefficients for this signal.

The simpler your final answer is, the more credit you will receive. Clearly indicate your final answers in the space below.

$X(n) = \cos(\pi/2\, n - \pi/4)$ $\quad P = 4 \quad \omega_0 = \pi/2$

$\boxed{A_1 = 1 \quad \phi_1 = -\pi/4 \quad \text{all other terms zero}}$

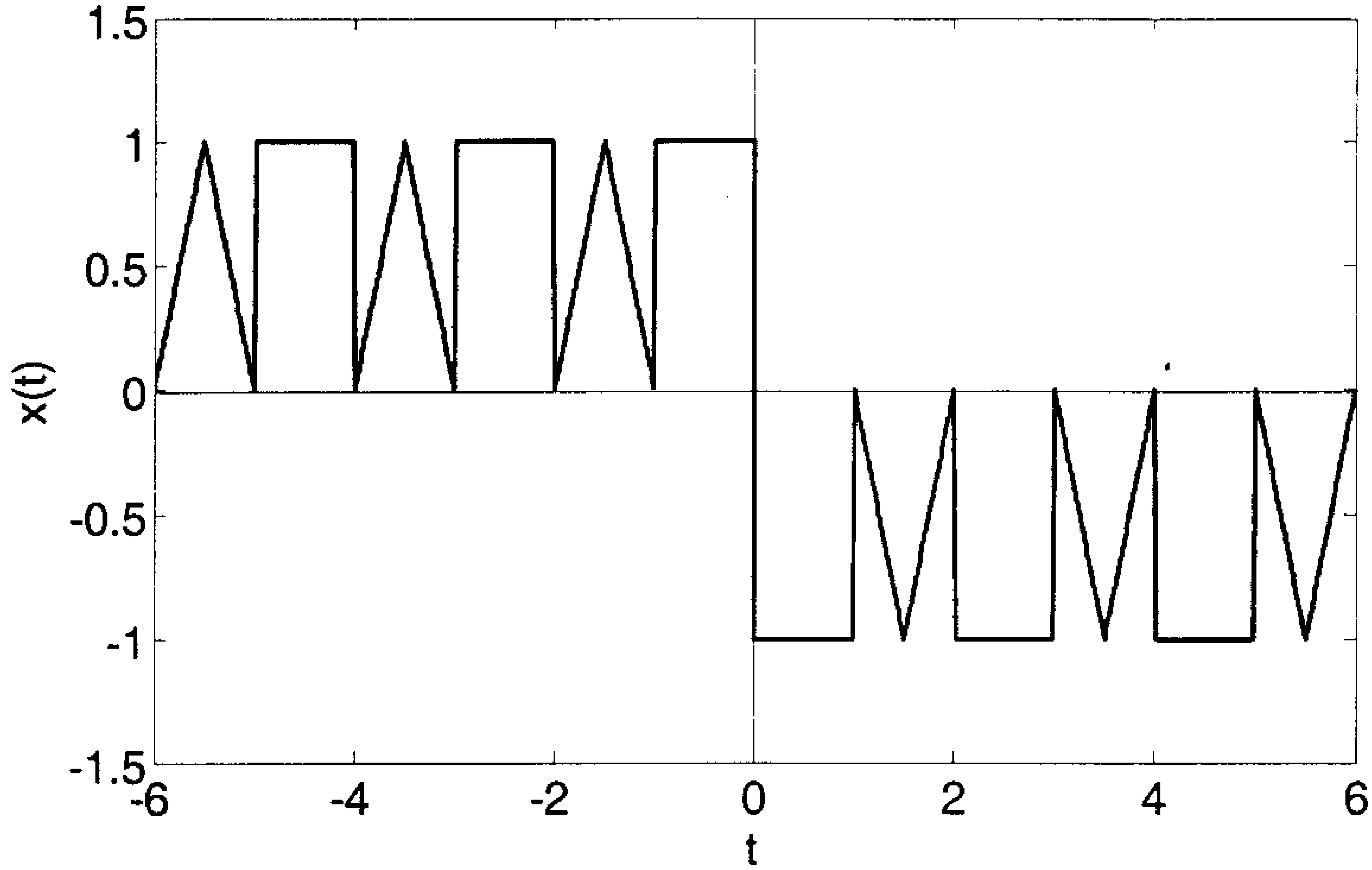
Using the conversion formula from the text

$\boxed{X_1 = 1/2\, e^{-i\pi/4} \quad X_3 = 1/2\, e^{i\pi/4} \quad \text{all other terms zero}}$

Name: ______________________

Problem 8: 9 Points Possible

Consider the continuous-time real-valued "mystery signal" illustrated below for one period:



a) What is the fundamental frequency ω_0 for this signal?

Above graph is for one period $\Rightarrow p=12 \Rightarrow \omega_0 = \frac{2\pi}{12} = \boxed{\pi/6}$

b) Of the graphs of $|X_k|$ and $\angle X_k$ on the next page, only one pair of graphs (one $|X_k|$ graph and its corresponding $\angle X_k$ graph) shows the correct complex exponential Fourier series for this signal.

Which is the correct graph for $|X_k|$? Which is the corresponding correct graph for $\angle X_k$? Write your choices here. Justify your answer.

The function has equal positive and negative area; the mean value over one period is zero. $X_0 = 0$

Choices c & d eliminated.

The function is not a pure sinusoid. Choice b eliminated.

The function is real. Choice 2 eliminated (it has $X_k \neq X_{-k}^*$)

The function is odd: all X_k have angle $\pm \pi/2$ (except X_0).

Choices 1 & 4 eliminated.

a, 3

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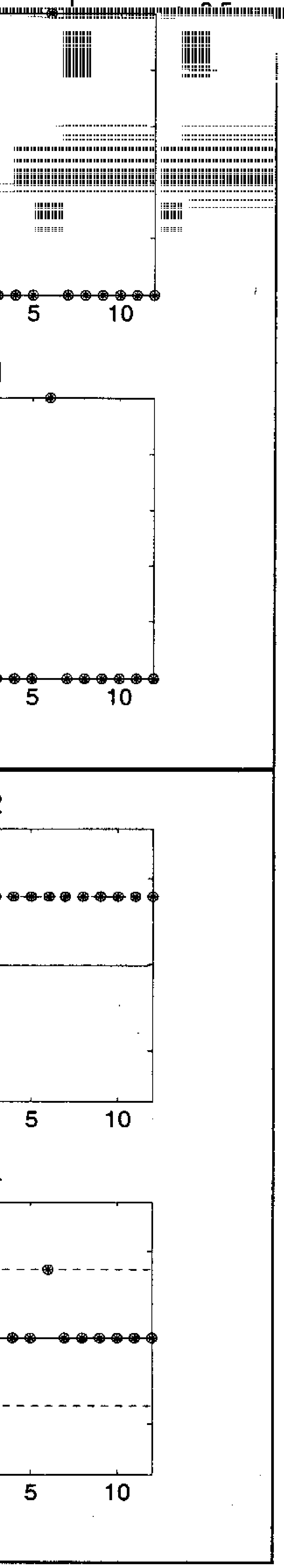

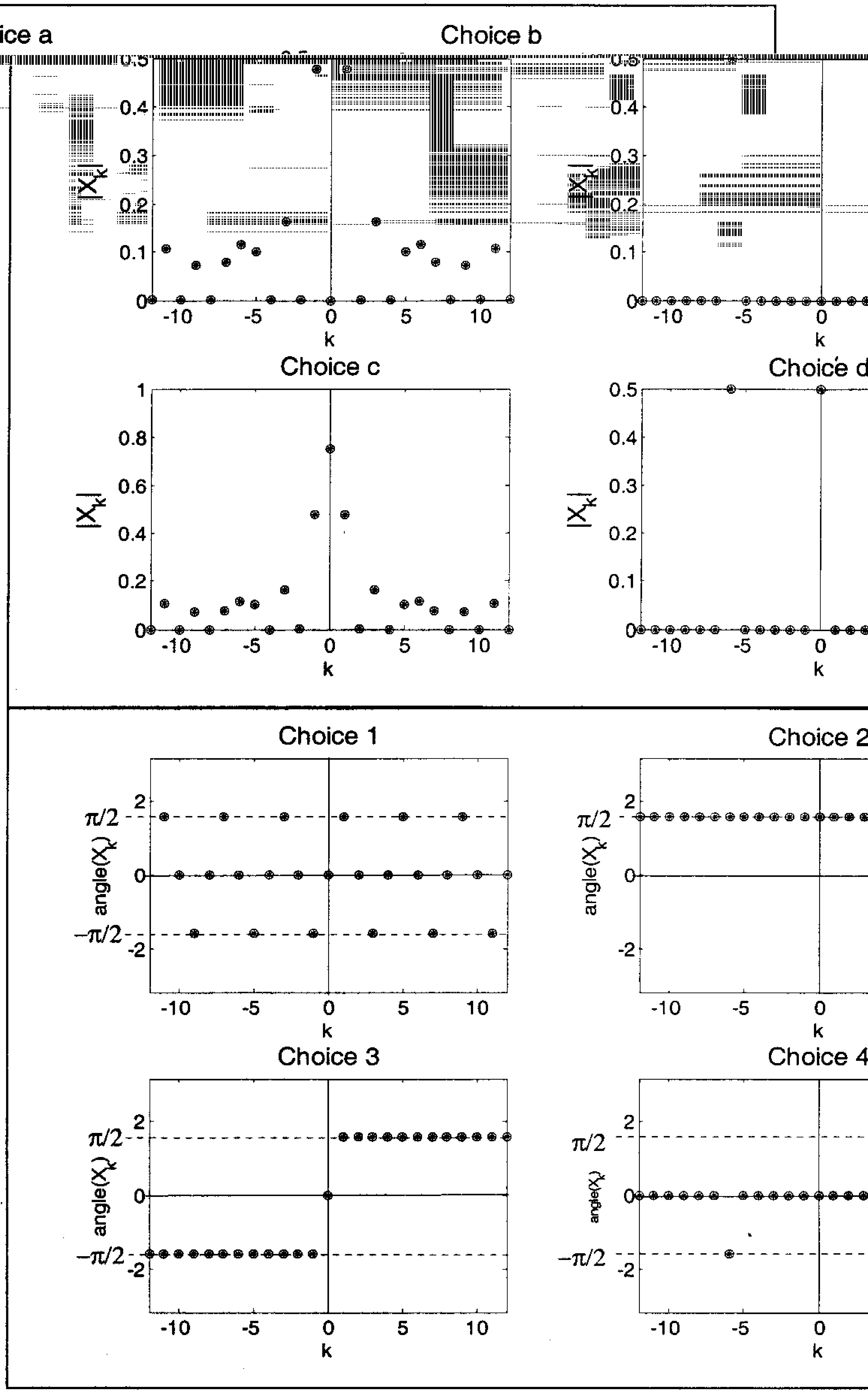

Choice a
Choice b
Choice c
Choice d
Choice 1
Choice 2
Choice 3
Choice 4
$|X_k|$
angle(X_k)
k
$\pi/2$
$-\pi/2$