Fall 2005 EE20 Midterm1, Professor Edward Lee, Babak Ayazifar

Problem 1 (42 Points)

Consider the set

$$X= [\{a,b,c\} \rightarrow \{1,2,3\}]$$

(a) (14 Points) How many elements are in the set X? Explicitly specify one element in X.

(b) (14 Points) Of the elements in X, how many define, respectively, a graph of an "onto" function *f*: $\{a,b,c\} \rightarrow \{1,2,3\}$? Explicitly specify one element in X that defines a graph of an onto function.

(c) (14 Points) Of the elements in X, how many define, respectively, a graph of a one-to-one function *f*: $\{a,b,c\} \rightarrow \{1,2,3\}$? Explicitly specify one element in X that defines a graph of a one-to-one function.

Problem 2 (20 Points)

Consider two assertions P and Q. We know that the following compound assertion is true:

$$(P=>Q)\wedge(P=>\sim Q)$$

We do not know whether Q is true, it may or may not be true.

Choose the strongest statement below that is most accurately reflective of the truth or falsehood of P. Circle your choice.

(I) P must be true.
(II) P could be true, but does not have to be true.
(III) P must be false.

Feel free to use the blank space below for scratch work. There is no partial credit here, so your work below will not be considered in grading this problem. Only the choice of your circle will be considered.

Problem 3 (14 Points)

Consider the following system F:

$$F : [\mathbb{R} \to \mathbb{R}] \to [\mathbb{R} \to \mathbb{R}] \text{ such that } \forall x \in [\mathbb{R} \to \mathbb{R}] \text{ and } \forall t \in \mathbb{R},$$

$$(F(x))(t) = (t+1)x(t) ,$$

where R denotes the set of real numbers.

Choose the strongest statement below that most accurately reflects one or more properties of the system F.

(I) The system F is memoryless and causal.
(II) The system F is memoryless, but it is not causal.
(III) The system F is not memoryless, but it is causal.
(IV) The system F is neither memoryless nor causal.

There is no partial credit here. Accordingly, no explanation will be considered in grading your selection among the statements above.

Problem 4 (14 Points)

Determine whether the following statement is true or false:

$$\{g | g = \text{graph}(f) \wedge f : X \to Y\} \subset P(X \times Y) ,$$

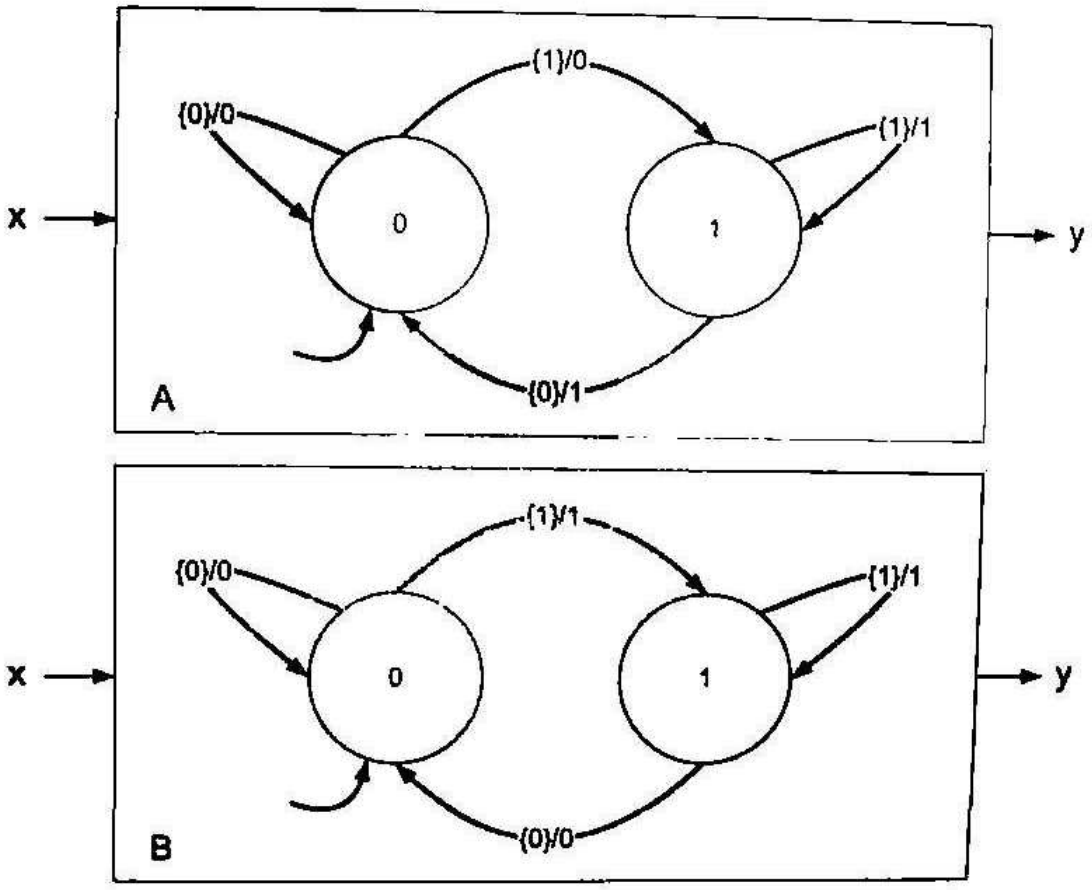
where P(XxY) denotes the power set of XxY.
Circle your answer:

True False

You will receive 0 credit for an incorrect choice, 8 points for no choice (i.e., for not answering the problem), and 14 points (full credit) for a correct choice. This is to discourage selection based on a coin toss!

Problem 5 (30 Points)

Consider two finite-state machines *A* and *B* having input and output alphabets $I_A = O_A = I_B = O_B$ = {0,1,absent} and further characterized, respectively, by the state transition diagrams shown below:

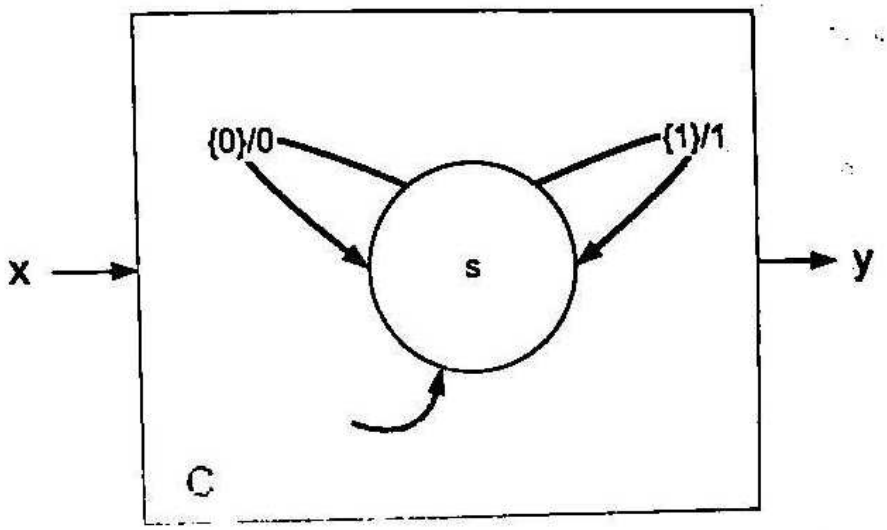


The input and output signal spaces for each machine are identical, i.e., *x,y belong to* $[\mathbb{N}_0 \rightarrow \{0, 1, \text{absent}\}]$, where $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

(a) (15 Points) Which one of the following four statements is true? Circle your choice.

(I) A is not memoryless, and B is not memoryless.
(II) A is not memoryless, but B is memoryless.
(III) A is memoryless, but B is not memoryless.
(IV) A is memoryless, and B is memoryless.

(b) Consider a finite state machine C having input and output alphabets given by $I_C=O_C$ = {0,1,absent}. The state-transition diagram for C is shown below:



Which of the following four statements is true? Circle your choice.

(I) C is bisimilar to A and is bisimilar to B.
(II) C is bisimilar to A but is not bisimilar to B.
(III) C is not bisimilar to A but is bisimilar to B.
(IV) C is bisimilar neither to A nor B.

Problem 6 (20 Points)

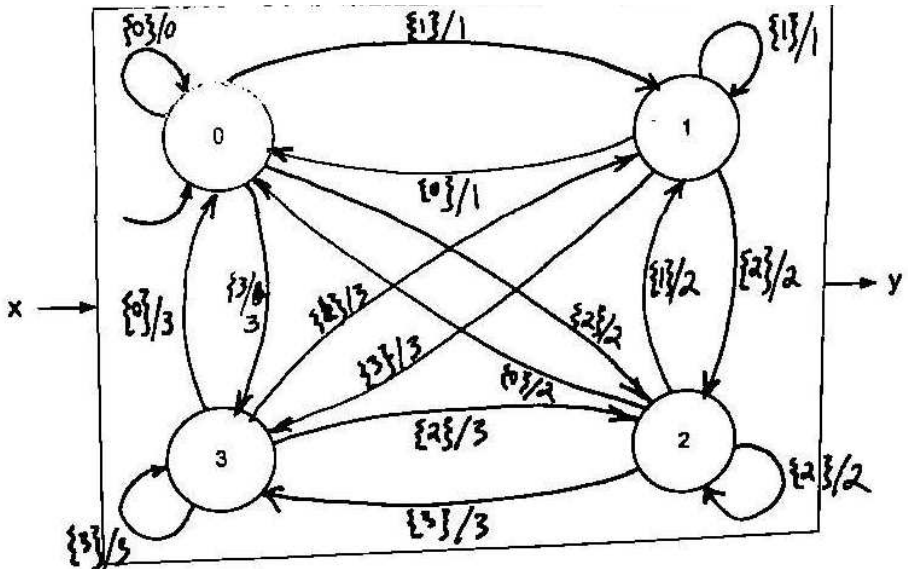
Consider a system with an input signal space X= [N→ {0,1,2,3}] and an output signal space Y identical to the input signal space.

For any input signal x belonging to X, the system produces an output signal y belonging to Y as follows:

$$\forall n \in \mathbb{N}_0, y(n) = \max(x(n), x(n-1)) .$$

The following skeleton finite-state machine, if properly defined, implements the system. Define the finite state machine by labeling all the salient features, including, for example, drawing the appropriate arcs, specifying all state transition parameters (guards and outputs on the arcs, including on self-loops, if any), etc.

For this problem, ignore stuttering.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