EECS 20. Midterm No. 2
April 5, 2000.

Please use these sheets for your answer. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab time below

Name: _____

Lab time: _____

Problem 1:

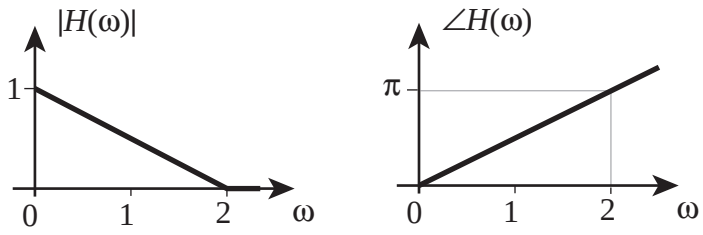
Problem 2:

Problem 3:

Problem 4:

Total:

1. **40 points.** Suppose that a continuous-time LTI system S has the frequency response H shown in the following plots:



Suppose further that the input is

$$x(t) = 1 + \sin(t) + \cos(2t).$$

(a) Find the fundamental frequency and the Fourier series coefficients A_k and ϕ_k for the input.

(b) Find the output y.

(c) Suppose that a new system is constructed by cascading two copies of S as shown below:

Give the frequency response $G(\omega)$, the magnitude response $|G(\omega)|$, and the phase response $\angle G(\omega)$ of the cascade composition $S \circ S$ in terms of the frequency response H of S, and give a sketch.

(d) Find the output of the cascade composition for the same input x.

2. **20 points.** Consider a discrete-time LTI system S whose response to the unit step input u is

$$y(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3),$$

where δ is the Kronecker delta function. Recall that the unit step is u: *Ints* $\to$ *Reals* such that for all $n \in$ *Ints*,

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and the Kronecker delta function is δ: *Ints* $\to$ *Reals* such that for all $n \in$ *Ints*,

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch u, δ, and y.

(b) Find the impulse response h of S (i.e., the output when the input is δ). Give both a formula and a sketch.

3. **30 points** Consider the continuous-time signal

$$x(t) = \cos^2(\pi t/6) + \sin(\pi t/6).$$

(a) Express this as a sum of cosines with frequencies that are integer multiples of $\pi/6$, with suitably chosen phases.

(b) Find the fundamental frequency ω_0 of x and give the Fourier series coefficients A_k and ϕ_k.

(c) Find the Fourier series coefficients X_k in the Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}.$$

4. **20 points** Suppose that a discrete-time system S has input x: *Ints* $\rightarrow$ *Reals* and output y: *Ints* $\rightarrow$ *Reals* that satisfy the difference equation

$$\forall\, n \in \textit{Ints}, \quad y(n) = x(n) - 0.9y(n-1).$$

   (a) Construct a state-machine model for this system. I.e., give the parameters for the state update and output equations. **Hint:** Begin by defining the state $s(n)$. You will lose credit for choosing a state with more dimensions than necessary.

   (b) Find the zero-state impulse response of this system. Give an expression and a well-labeled sketch.

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