EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 14 February 2008
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (30 Points) The continuous-time function x characterized by

$$\forall t \in \mathbb{R}, \quad x(t) = e^{\sigma t} e^{i(\omega t + \phi)}$$

is ubiquitous in the engineering and physical sciences. The parameters σ, ω, and ϕ are all real-valued.

Suppose x is the trajectory of a particle on the complex plane. We may think of $x(t)$ as the instantaneous position of the particle at time t.

Each part below specifies a set of values for σ, ω, and ϕ. For each part, provide a well-labeled sketch of the particle's trajectory x in the time interval $[0, \infty)$; use an arrow to specify the direction of the particle's journey as time moves forward. On each sketch, indicate the location of the particle at time instants $t = 0$, 0.5, 1, 1.5, and 2 seconds, and show where the particle is headed as $t \to \infty$.

Note: Do *not* sketch $\mathsf{Re}(x(t))$, $\mathsf{Im}(x(t))$, $|x(t)|$, or $\angle x(t)$. You may receive credit only if you sketch $x(t)$, $0 \leq t < \infty$, as a trajectory on the complex plane.

(a) $\sigma = 0$, $\omega = -\pi/2$, and $\phi = 0$.

(b) $\sigma = 1$, $\omega = +\pi/2$, and $\phi = 0$.

(c) $\sigma = -1$, $\omega = -\pi/2$, and $\phi = 0$.

(d) $\sigma = 1$, $\omega = 0$, and $\phi = \pi/4$.

(e) $\sigma = -1$, $\omega = 0$, and $\phi = \pi$.

MT1.2 (30 Points) Consider an N^{th}-order polynomial with real coefficients. In particular, let

$$F(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_N z^N,$$

where $a_0, \ldots, a_N$ are all real-valued.

(a) Show that $(z^*)^n = (z^n)^*$, for all $n = 0, 1, \ldots, N$.

(b) Use the result of part (a) to show that $F^*(z) = F(z^*)$.

(c) The solutions of $F(z) = 0$ are called the *zeroes* (roots) of F. Suppose z_0 is a *non-real* zero of F. Show that z_0^* must also be a zero of $F(z)$.

According to this result, if a polynomial has real coefficients, then its zeroes either are real-valued or occur as complex-conjugate pairs.

MT1.3 (25 Points) The continuous-time signal x is characterized by

$$\forall t \in \mathbb{R}, \quad x(t) = A \sin(B \cos t)$$

where $A \in \mathbb{R}$, and $0 < B < 1$.

The two parts of this problem are mutually independent, so you may tackle them in either order.[1]

(a) Is the signal x periodic? If so, determine its fundamental period p. Explain your reasoning succinctly, but clearly and convincingly.

(b) Recall that $\cos(\omega t)$ has frequencies $\pm\omega$. More generally, for a real-valued function $\theta(t)$, the instantaneous frequencies of $\cos[\theta(t)]$ at time t are $\pm\dot{\theta}(t)$.

Determine the range of instantaneous frequencies of the signal x.

[1]Depending how you proceed, you may or not be looking for the identity $\cos(\alpha + \beta) = \cos\alpha\, \cos\beta - \sin\alpha\, \sin\beta$.

MT1.4 (20 Points) George and Harry run at a steady rate around a circular track field of unit radius. They run independently, so neither makes any strategic adjustment based on what the other one is doing.

George completes one lap in T_g seconds. Harry takes T_h seconds to complete a lap. Harry is the slower one, so $T_g < T_h$.

Naturally, George overtakes Harry periodically. How long does it take George to overtake Harry if they start their jog together at time $t = 0$?

<u>Hint</u>: Let $\theta_g(t)$ and $\theta_h(t)$ denote the respective angular positions (in radians) of George and Harry on the track field at time t. Then $\dot{\theta}_g(t) = 2\pi/T_g$ and $\dot{\theta}_h(t) = 2\pi/T_h$ will be their respective angular frequencies.

Let their "phase difference" be $\phi = \theta_g - \theta_h$. By looking at $\dot{\phi}(t)$, determine the time T_0 it takes George to overtake Harry for the first time after their jog begins. Express your answer in terms of T_g and T_h.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

Problem	Points	Your Score
Name	10	
1	30	
2	30	
3	25	
4	20	
Total	**115**	