MAKEUP

EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2

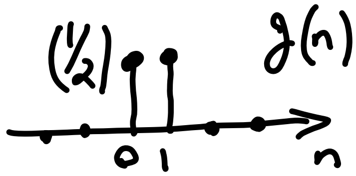
Department of Electrical Engineering and Computer Sciences 15 March 2012

University of California Berkeley

LAST Name Varian FIRST Name Laurent T.

Lab Time Dawn

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.



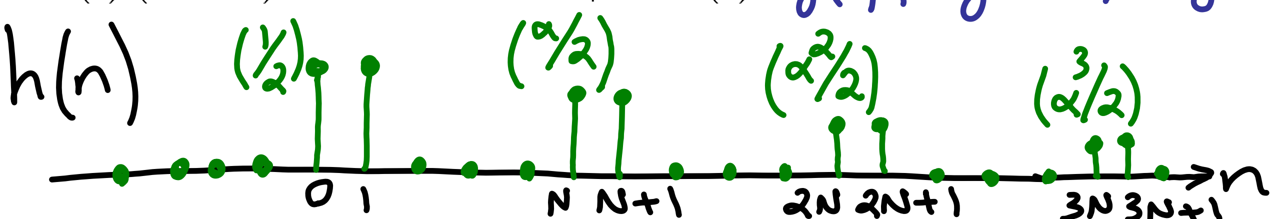
$$\longrightarrow G(\omega) = \frac{1-e^{-i\omega}}{2} = \cos(\omega/2)\, e^{-i\omega/2}$$

MT2.1 (30 Points) The impulse response h of a discrete-time LTI system H is

$$\forall n \in \mathbb{Z}, \quad h(n) = \sum_{\ell=0}^{+\infty} \alpha^{\ell} g(n-\ell N),$$

where $0 < \alpha < 1$; N is a positive integer greater than 2; and g is the impulse response of the two-point moving-average filter G. That is, $g(n) = \dfrac{\delta(n) + \delta(n-1)}{2}$, $\forall n \in \mathbb{Z}$.

(a) (5 Points) Provide a well-labeled plot of $h(n)$ $= g(n) + \alpha g(n-N) + \alpha^2 g(n-2N) + \cdots$



(b) (10 Points) Determine a reasonably simple expression for the frequency response $H(\omega)$ of the filter H; your expression must be in terms of the parameters α and N. You may receive partial credit if you express $H(\omega)$ in terms of the frequency response $G(\omega)$ of the moving-average filter G.

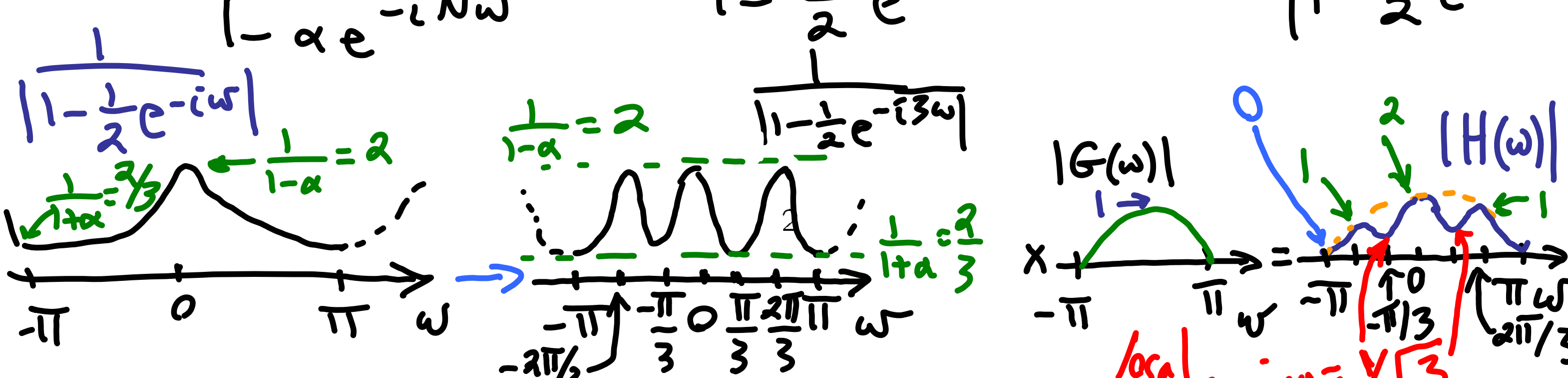
$$H(\omega) = \sum_{n=\infty}^{\infty} h(n) e^{-i\omega} = \sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{\infty} \alpha^{\ell} g(n-\ell N)\, e^{-i\omega n}$$

$$= \sum_{\ell=0}^{\infty} \alpha^{\ell} \sum_{n=-\infty}^{\infty} g(n-\ell N) e^{-i\omega n} \underset{\boxed{\text{let } m = n-\ell N}}{=} \sum_{\ell=0}^{\infty} \alpha^{\ell} \sum_{m=-\infty}^{\infty} g(m) e^{-i\omega(m+\ell N)}$$

$$= \sum_{\ell=0}^{\infty} \alpha^{\ell} e^{-iN\omega\ell}\, G(\omega) = \frac{G(\omega)}{1-\alpha e^{-iN\omega}} = \frac{1}{2}\,\frac{1-e^{-i\omega}}{1-\alpha e^{-iN\omega}}$$

(c) (15 Points) Provide a well-labeled plot of the magnitude response $|H(\omega)|$ of the filter H. Assume $\alpha = 1/2$ and $N = 3$.

$$H(\omega) = \frac{\cos(\omega/2)\, e^{-i\omega/2}}{1-\alpha e^{-iN\omega}} = \frac{\cos(\omega/2)\, e^{-i\omega/2}}{1-\frac{1}{2} e^{-i3\omega}} \Rightarrow |H(\omega)| = \frac{|\cos(\omega/2)|}{|1-\frac{1}{2}e^{-i3\omega}|}$$

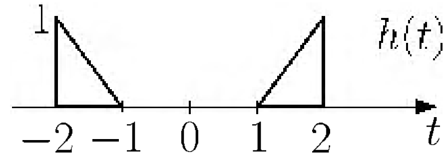


local minima = 1/√3

Local maxima at $\pm\frac{2\pi}{3} = 2\cos(\pi/3) = 1$

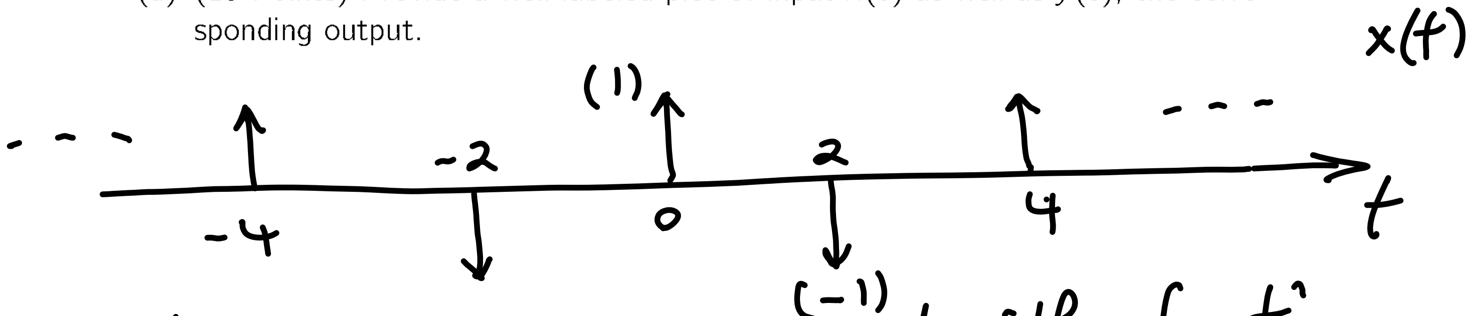
local minima @ $\pm\frac{\pi}{3} = (2/3)\cos(\pi/6) = \frac{2}{3}\cdot\frac{\sqrt{3}}{2} = 1/\sqrt{3}$

MT2.2 (35 Points) The impulse response h of a continuous-time system H is shown below.

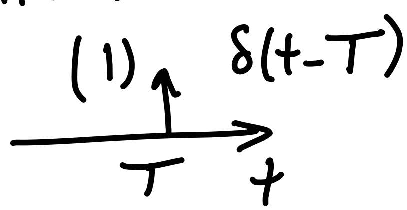


We apply to the system the input signal described by $x(t) = \sum_{\ell=-\infty}^{+\infty} (-1)^\ell \delta(t - 2\ell)$.

(a) (15 Points) Provide a well-labeled plot of input $x(t)$ as well as $y(t)$, the corresponding output.



A shifted impulse, when convolved with a function, shifts the function to its own location. For example,

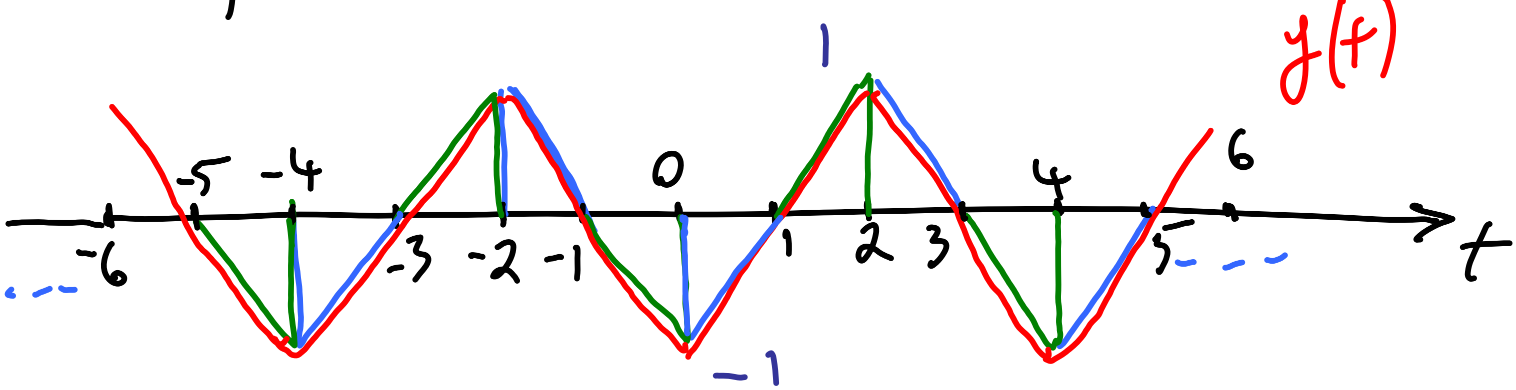


$$\star\ g(t) = g(t-T)$$

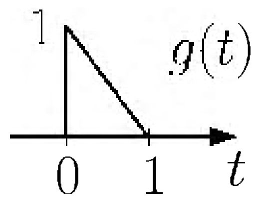
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = (x \star h)(t) = \sum_{\ell=-\infty}^{\infty} (-1)^\ell h(t-2\ell)$$

$$y(t) = \dots + h(t+4) - h(t+2) + h(t) - h(t-2) + h(t+4) - \dots$$

This is plotted below:



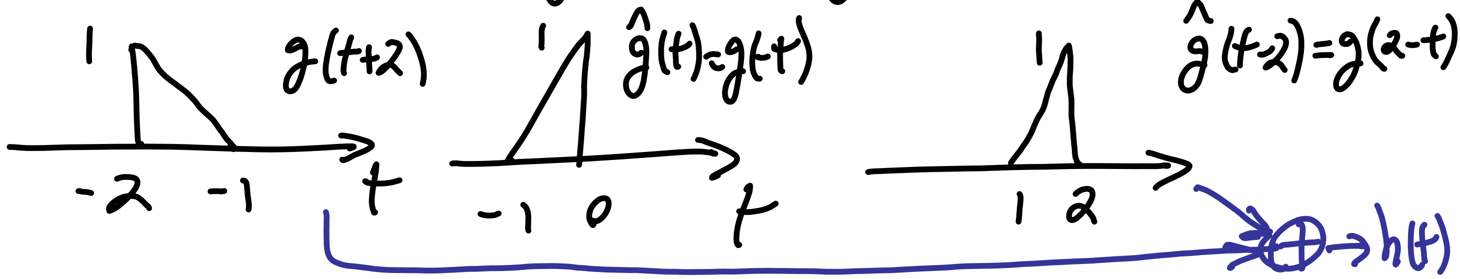
(b) (20 Points) Consider a related LTI system G whose impulse response g is shown below:



Express $H(\omega)$, the frequency response of the system H, in terms of $G(\omega)$, the frequency response of the system G. Provide a succinct, but clear and convincing, explanation.

Note that $H(\omega) \triangleq \int_{-\infty}^{+\infty} h(t)\, e^{-i\omega t}\, dt$, and $G(\omega)$ is defined similarly.

Note that $h(t) = g(t+2) + g(2-t)$



$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} [g(t+2) + g(2-t)] e^{-i\omega t} dt$$

$$= \int_{-\infty}^{\infty} g(t+2) e^{-i\omega t} dt + \int_{-\infty}^{\infty} g(2-t) e^{-i\omega t} dt$$

let $\tau = t+2 \rightarrow dt = d\tau$ let $\lambda = 2-t \rightarrow dt = -d\lambda$

$$= \int_{-\infty}^{\infty} g(\tau) e^{-i\omega(\tau-2)} d\tau - \int_{+\infty}^{-\infty} g(\lambda) e^{i\omega(\lambda-2)} d\lambda = \underbrace{\left[\int_{-\infty}^{\infty} g(\tau) e^{-i\omega\tau} d\tau\right]}_{G(\omega)} e^{i2\omega} + \underbrace{\left[\int_{-\infty}^{\infty} g(\lambda) e^{i\lambda\omega} d\lambda\right]}_{G(-\omega)} e^{-i2\omega}$$

$$H(\omega) = G(\omega) e^{i2\omega} + G(-\omega) e^{-i2\omega}$$

Note: $g(t) \in \mathbb{R} \; \forall t \Rightarrow G^*(\omega) = \int_{-\infty}^{\infty} g(t) e^{i\omega t} dt = G(-\omega) \Rightarrow$

$$H(\omega) = G(\omega) e^{i2\omega} + [G(\omega) e^{i2\omega}]^* = 2\,\mathrm{Re}[G(\omega) e^{i2\omega}]$$

$$\Rightarrow H(\omega) = 2\,\mathrm{Re}\left[|G(\omega)| e^{i\measuredangle G(\omega)} e^{i2\omega}\right] = 2|G(\omega)| \cos(\measuredangle G(\omega) + 2\omega)$$

MT2.3 (35 Points) Consider a linear (but possibly *time-varying*) discrete-time system G. If the input to the system is $x(n) = \delta(n-m)$, an impulse shifted by m samples, then the corresponding output is $y(n) = g_m(n)$. In general, the discrete-time function g_m depends on m.

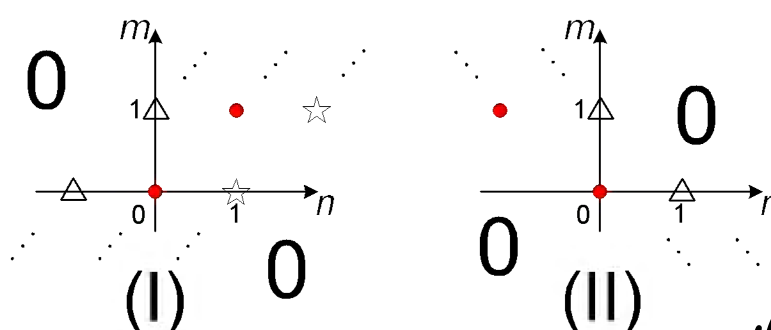
The output of the system is given by

$$y(n) = \sum_{m=-\infty}^{+\infty} x(m)\, g_m(n), \quad \text{for all integers } n.$$

(a) (10 Points) Each of the figures below depicts—by a combination of circles, stars, and triangles—the non-zero values $g_m(n)$ of a possible function g_m. Determine which one corresponds to a time-invariant system.

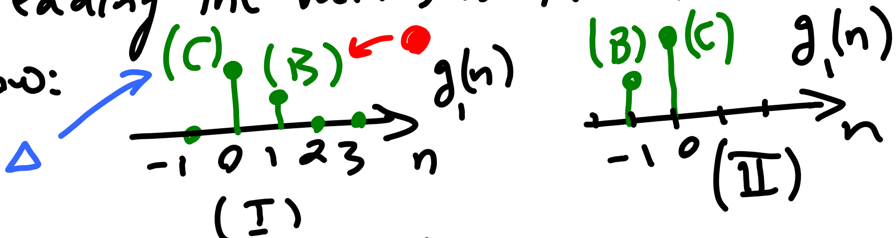
All the stars correspond to the same numerical value A of $g_m(n)$, all the circles correspond to the same numerical value B of $g_m(n)$, and all the triangles correspond to the same numerical value C of $g_m(n)$.

Time Invariant



Time Varying

$\delta(n-m) \longrightarrow \boxed{G} \longrightarrow g_m(n)$ To determine the response to any impulse $\delta(n-m)$, read the row corresponding to that m. For example, we obtain $g_1(n)$ for each system (I) and (II) by reading the values of the row at $m=1$. These are shown below:



Clearly, in system (I), as we increase m by 1, the corresp. response shifts to the right by 1 sample. Alternatively, $g_m(n)$ depends only on the difference $n-m$. All value along each $n-m=K$ diagonal line are the same $\Rightarrow$ (I) is Time Invariant. System (II) is Time Varying. To see this, note that $g_1(n)=g_0(n+1)$ whereas it should have been $g_1(n)=g_0(n-1)$.

(b) (25 Points) Prove that the system G is BIBO stable if, and only if,

$$\sum_{m=-\infty}^{+\infty} |g_m(n)| < \infty, \quad \text{for every integer } n.$$

Hint: Show the necessity and sufficiency separately, just as you saw it done for LTI systems in class. In particular, show that if $\sum_{m=-\infty}^{+\infty} |g_m(n)| < \infty$, then every bounded input produces a bounded output. This is the sufficiency condition, and it's easier to prove than the necessity condition.

Afterward, show that if $\sum_{m=-\infty}^{+\infty} |g_m(n)| = \infty$ for even a single value of n, a bounded input signal x exists such that a bound for the output y does not exist. You must find such a bounded input and show how the output becomes unbounded.

Sufficiency: $\sum_m |g_m(n)| = G_0 < \infty \ \forall n \Rightarrow$ Every bounded input produces a bounded output.

$y(n) = \sum_m x(m) g_m(n)$. Let $|x(n)| \leq B_x, \forall n$.

$|y(n)| = \left|\sum_m x(m) g_m(n)\right| \leq \sum_m |x(m)||g_m(n)| \leq B_x \sum_m |g_m(n)| = \underbrace{B_x G_0}_{\text{let be } B_y}$

We've shown that $|y(n)| \leq B_y \quad \forall n$.

Necessity: $\sum_m |g_m(n)| = \infty \ \exists n \Rightarrow$ A bounded input exists such that the output is infinite at the n for which $\sum_m |g_m(n)| = \infty$.

$$\text{Let } x(m) = \operatorname{sgn}\{g_m(n)\} = \begin{cases} 1 & \text{if } g_m(n) > 0 \\ 0 & \text{if } g_m(n) = 0 \\ -1 & \text{if } g_m(n) < 0 \end{cases}$$

Then

$$y(n) = \sum_m x(m) g_m(n) = \sum_m \operatorname{sgn}(g_m(n))\, g_m(n) = \sum_m |g_m(n)| = \infty$$

Note: If $g_m(n)$ is specified on the 2D plane, akin to how it was done in part (a), then every column must be absolutely summable. For a TI system we don't need to test every column (ie, every n), because each column is a shifted version of any other column.

MT2.4 (5 Points) Which Exam Should We Grade?

Select one, and *only one*, option below, and **indicate your choice on the <u>back cover</u>.**

Definitions:

Original Midterm 2 (OrigMT2): The midterm administered in EE 20 on Tuesday, 13 March 2012.

S_{Orig} Your score on the Original Midterm 2, normalized to be out of 115 points.

Makeup Midterm 2 (MkupMT2): The midterm administered in EE 20 on Thursday, 15 March 2012.

S_{Mkup} Your score on the Makeup Midterm 2, out of 115 points.

S_2**:** Your overall Midterm 2 score (out of 115 points) counted toward your final course grade.

(A) Grade only my MkupMT2 (i.e., let it count for 100% of my overall Midterm 2 score). That is, disregard my OrigMT2 so that $S_2 = S_{\text{Mkup}}$.

(B) Grade only my OrigMT2 (i.e., let it count for 100% of my Midterm 2 score). That is, disregard my MkupMT2 so that $S_2 = S_{\text{Orig}}$.

(C) Grade *both* my OrigMT2 and MkupMT2. Compute my overall score for Midterm 2 according to the following weighted linear combination:

$$S_2 = 0.25\, S_{\text{Orig}} + 0.75\, S_{\text{Mkup}}.$$

That is, let my OrigMT2 score count for 25% and my MkupMT2 score for 75% of my overall Midterm 2 score.

(D) Grade *both* my OrigMT2 and MkupMT2. Compute my overall score for Midterm 2 according to the following weighted linear combination:

$$S_2 = 0.75\, S_{\text{Orig}} + 0.25\, S_{\text{Mkup}}.$$

That is, let my OrigMT2 score count for 75% and my MkupMT2 score for 25% of my overall Midterm 2 score.

(E) Grade *both* my OrigMT2 and MkupMT2, and count them equally toward my overall Midterm 2 score. That is,

$$S_2 = \frac{S_{\text{Orig}} + S_{\text{Mkup}}}{2}.$$

LAST Name Varian FIRST Name Laurent T.

Lab Time Dawn

The Grading Option You've Chosen for Your Midterm 2 (See Prob. 4 on p. 7): _____

Problem	Points	Your Score
Name	10	10
1	30	30
2	35	35
3	35	35
4	5	5
Total	**115**	115