EECS 40, Fall 2008
Prof. Chang-Hasnain
Final Exam
Solutions Version A

8:00 am – 11:00 am, Saturday December 13, 2008
Total Time Allotted: 180 minutes
Total Points: 300

1. This is a closed book exam. However, you are allowed to bring 4 pages (8.5" x 11"), double-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. **Remember to put down units. Points will be taken off for missed unit.**

Last (Family) Name:____

First Name: ____

Student ID: _____________________________ Discussion Session: ________________

Signature: ____

Score:	
Problem 1 (116 pts)	
Problem 2 (36 pts)	
Problem 3 (58 pts)	
Problem 4 (90 pts)	
Total (300 pts)	

Problem 1: Transistor Amplifier (116 pts)

Note: The sub-parts are somewhat independent. Read through the entire problem even if you are not able to solve some parts. Good luck!

In this problem you are about to analyze the amplifier circuit shown in the figure below. The threshold voltage V_{t0} = 1 V and the capacitor impedance at signal frequency is negligible.

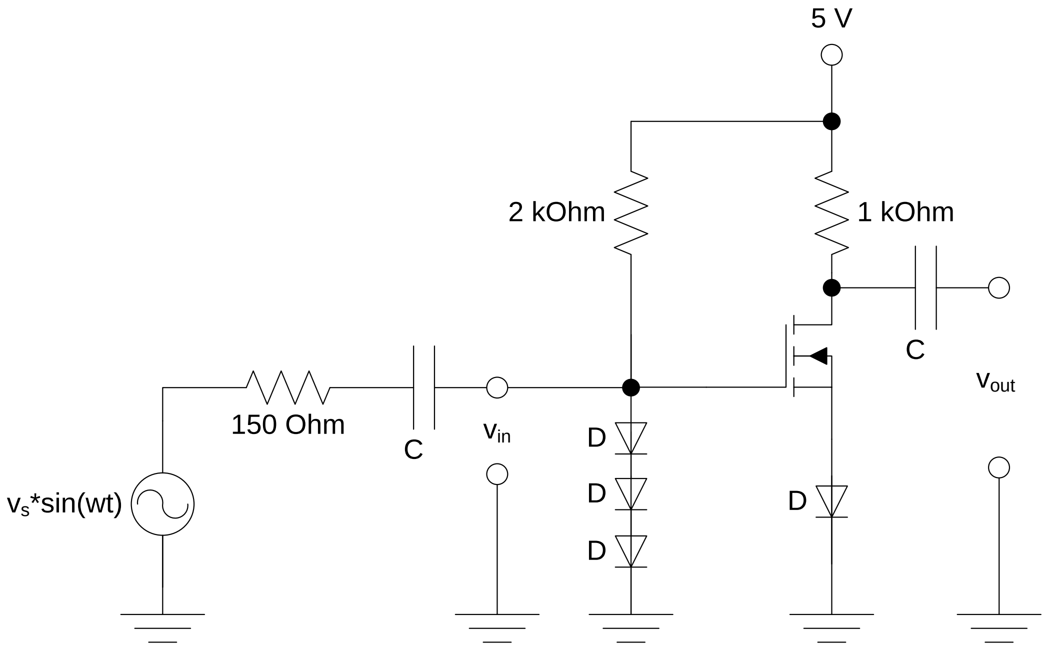


Figure 1: Amplifier circuit for Problem 1

The diodes D in figure 1 have the following I-V characteristic:

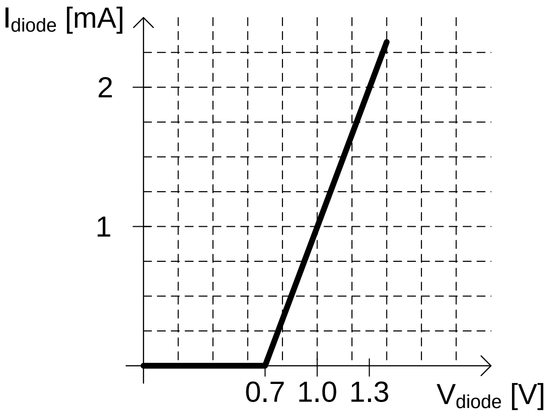


Figure 2: I-V characteristics of diodes in circuit shown in figure 1.

a) Derive an expression for V_{diode} [V] as a function of I_{diode} [A] for the "on-region" of the diode and give an equivalent circuit that has the same I-V characteristic in that region (10 pts)

$V_{diode} = 0.7 + 300 * I_{diode}$

Equivalent circuit: A 0.7 V voltage source in series with a 300 Ohm resistor

b) Calculate V_G, the DC voltage at the gate of the MOS transistor (15 pts).

We basically have a resistive voltage divider between 5 V and 3 * 0.7 V, with resistors of 2000 Ohm and 3 x 300 Ohm

➔ $V_G = 2.1 + (5-2.1)*900/(2000 + 900) = 2.1 + 2.9/2900 * 900 = 3$ V

Alternatively once can solve the following equation for the current through the 3 diodes in series

$I_{diodes} = (5 – 3 * V_{diode})/2000 = (5 – 3*(0.7 + 300\ I_{diodes}))/2000$ to get $I_{diodes} = 1$ mA

➔ $V_G = 5 – 2000*0.001 = 3$ V

c) Express V_{GS}, the DC voltage between the gate and the source of the transistor, as a function of I_{DS}. (4 pts)

$V_{GS} = V_G – V_{diode} = 3 – 0.7 – 300 * I_{DS} = 2.3 – 300 * I_{DS}$

d) Assume that for a similar circuit you found out that

$$V_{GS} = 2.4 - I_{DS} * 200.$$

Using the I-V characteristic for the transistor in saturation (given below) and Load-Line analysis find **V_{GSQ} and I_{DSQ}** (12 pts)

(Note that the given V_{GS} vs. I_{DS} relation is not necessarily equal to what you found in part c). This is a new starting point for the problem in case you encountered some problems in parts a) – c))

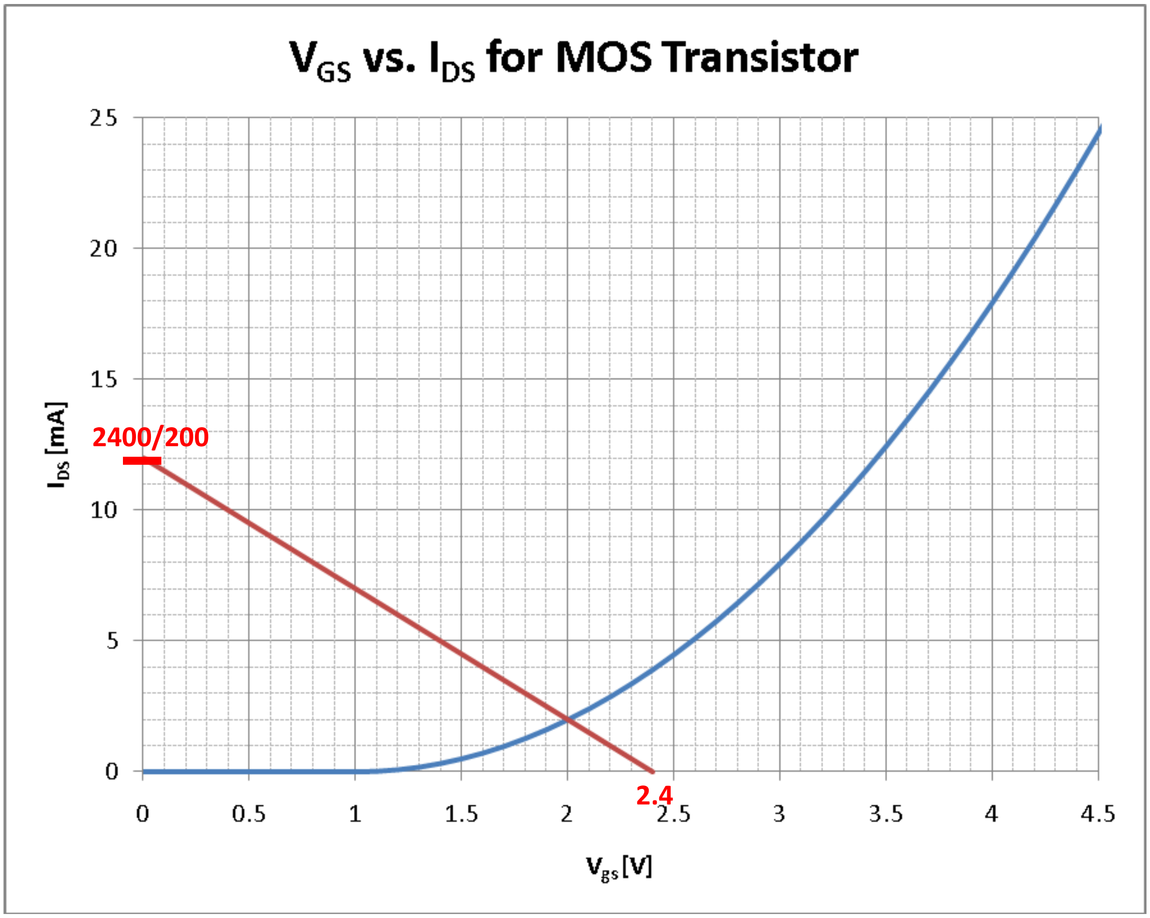


Figure 3: I_{DS} vs. V_{GS} for the transistor used in circuit in Figure 1 (in saturation)

From the load-line analysis above we found I_{DS} = 2mA and V_{GS} = 2 V

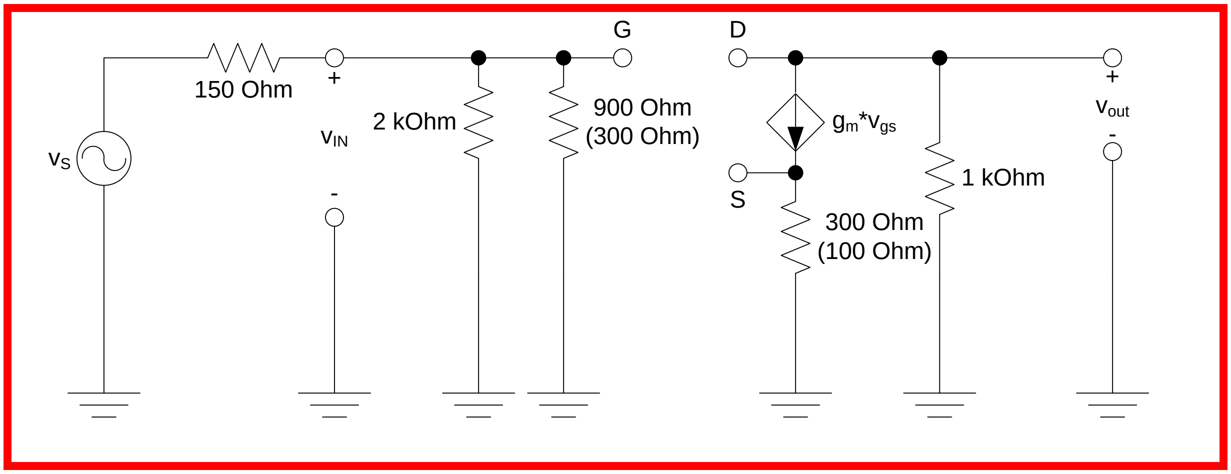
e) For the values found in d) and given in the problem, in Figure 1 and the I-V characteristic of the diode: Is the transistor in saturation? Why or why not? (12 pts)

If I_{DS} is 2mA the voltage drop across 1kOhm resistor is 2 V. From the diode characteristics the voltage drop across the diode is 1.3 V. That leaves 5 – 3.3 V for V_{DS} which is 1.7 V. We have to check whether $V_{DS} > V_{GS} – V_T$ and since $1.7 > 2 – 1$ V the transistor is in saturation.
If one argues that V_G is 3 V and since V_{GS} is equal to 2 V the voltage drop across the diode has to be 1 V it is ok as well. In that case V_{DS} would be 2V and the transistor still in saturation.

f) Based on the values you got in part d), calculate g_m (K of transistor is $2mA/V^2$) (assume the transistor is in saturation regardless of your answer in part e)) (4 pts)

$g_m = 2 \text{ sqrt } (K * I_{DS}) = 4$ mS

g) Draw the small signal equivalent circuit for the circuit given in Figure 1. You can neglect r_d of the transistor. If you don't know how to deal with the diodes simply replace them with 100 Ω resistors in the small signal circuit (for partial credit). If you include the full diode model, explain how you do it. (27 pts if model includes diodes, 24 pts if you choose to replace diodes with 100 Ω resistors)



Diodes can be simply replaced by their equivalent resistance in the on region. The DC voltage sources of the diode model are replaced by a short.

Values in parenthesis are values if 100 Ohm resistor used in stead of diodes

h) For this part of the problem assume a g_m of 10 mS (again this might be different from the g_m you found in part f))

Using small signal analysis and the circuit you drew in g), find the small signal voltage gain **$A_v=v_{out} / v_{in}$ and the input impedance** of the circuit <u>without</u> the AC source and the 150 Ω resistor connected to V_{in} (you can leave the numerical results in fractions). (24 pts)

$v_{out} = - g_m * v_{GS} * 1$ kOhm

$v_{GS} = v_{in} - g_m * v_{GS} * 300 \rightarrow v_{gs} = v_{in} / (1 + 300*g_m)$

- ➔ $V_{out} = - g_m * 1 \text{ kOhm} * v_{in} / (1+300*g_m)$
- ➔ $A_V = 10/(1+3) = -2.5$ (-5 if 100 Ohm resistors are used instead of diodes)

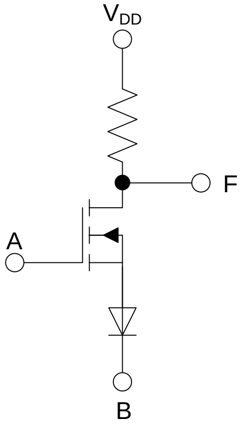
$R_{in} = v_{in}/i_{in}$ = 2kOhm//900kOhm is approx. 600 Ohm (=1800000/2900)
(6000000/2300 or approx 300 Ohm if 100 Ohm resistor used instead of diodes)

i) Now connect the AC source and the 150 Ω resistance to the circuit again (as shown in figure 1) (8 pts)
Calculate the AC output voltage v_{out} for a source voltage amplitude v_s of 0.1 V.

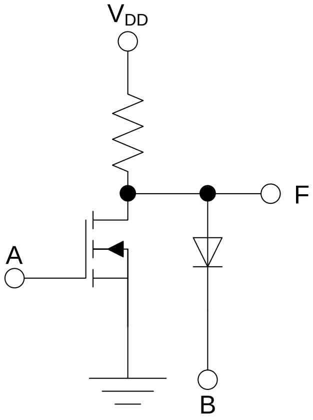
$V_{out} = v_s * R_{in} / (150 + R_{in}) * A_V = 0.1 * 4/5 * 2.5 = 0.2$ V (0.33 V if 100 Ohm resistor is used)

Problem 2: Logic Circuits (36 pts)

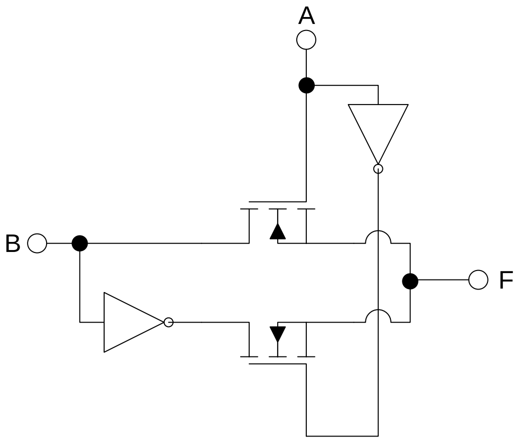
a) Complete the truth tables for the following circuits. Assume $V_{DD} = 5$ V, diodes are ideal ($V_{on} = 0$ V), the transistors (all NMOS) have a V_{to} of 1 V, a logic 0 at the input is 0 V, a logic 1 at the input is V_{DD} and any voltage above (below) V_{DD} / 2 at the output is considered a logic 1 (logic 0). (24 pts)



A	B	F
0	0	1
0	1	1
1	0	0
1	1	1



A	B	F
0	0	0
0	1	1
1	0	0
1	1	0



A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

b) For this part assume any diode you use has an on-voltage of 0.6 V, but is otherwise ideal. Further assume that any transistor you use has a non-zero on-resistance but an infinite off-resistance. V_{t0} of the transistors are 1 V and V_{DD} is 5 V

Design a **2-input OR** gate with an **output high level equal to V_{DD}** and an **output low level being equal to GND** using any device (out of R, L, C, Diodes, NMOS, PMOS Transistors) and as many devices you want. Explain why your circuit fulfills the requirements. (12 pts)

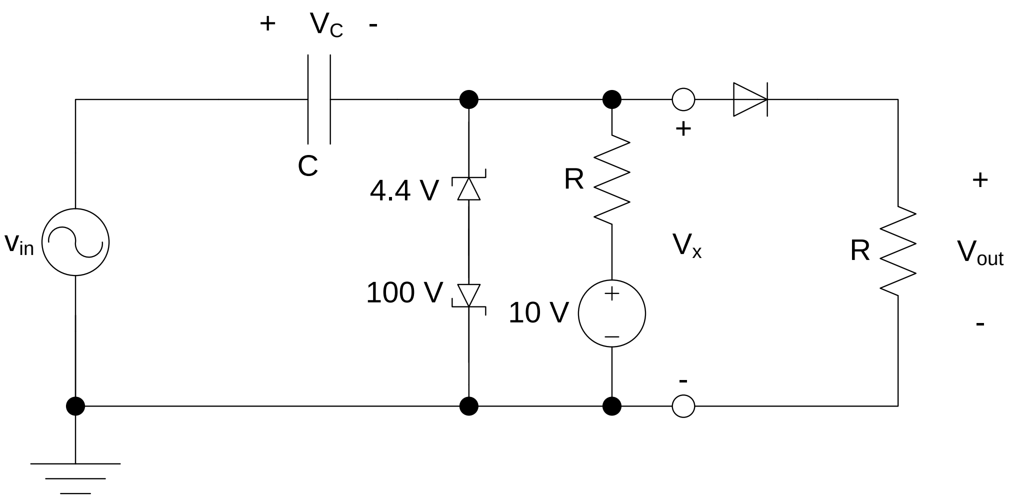
There are a couple of possible solutions:

e.g. implementing a NOR (CMOS or transistor-resistor logic) gate and then connecting a CMOS inverter to its output.
Or an OR gate in diode logic and then connecting 2 inverters in series to its output to make sure the output levels are 0 and VDD for low and high respectively

Problem 3: Diode Circuits (60 pts)

In this problem you are to analyze the circuit below. Assume a forward bias turn-on threshold voltage for each diode (Zener as well as regular diode) of 0.6 V. Further assume that the resistors R are big enough such that the capacitor C does not charge nor discharge noticeably through them within a period of V_{in}.

If you don't know how to solve the problem with both Zener diodes, you can replace the 100 V Zener diode with a regular diode (for partial credit). Clearly state that you do so though!



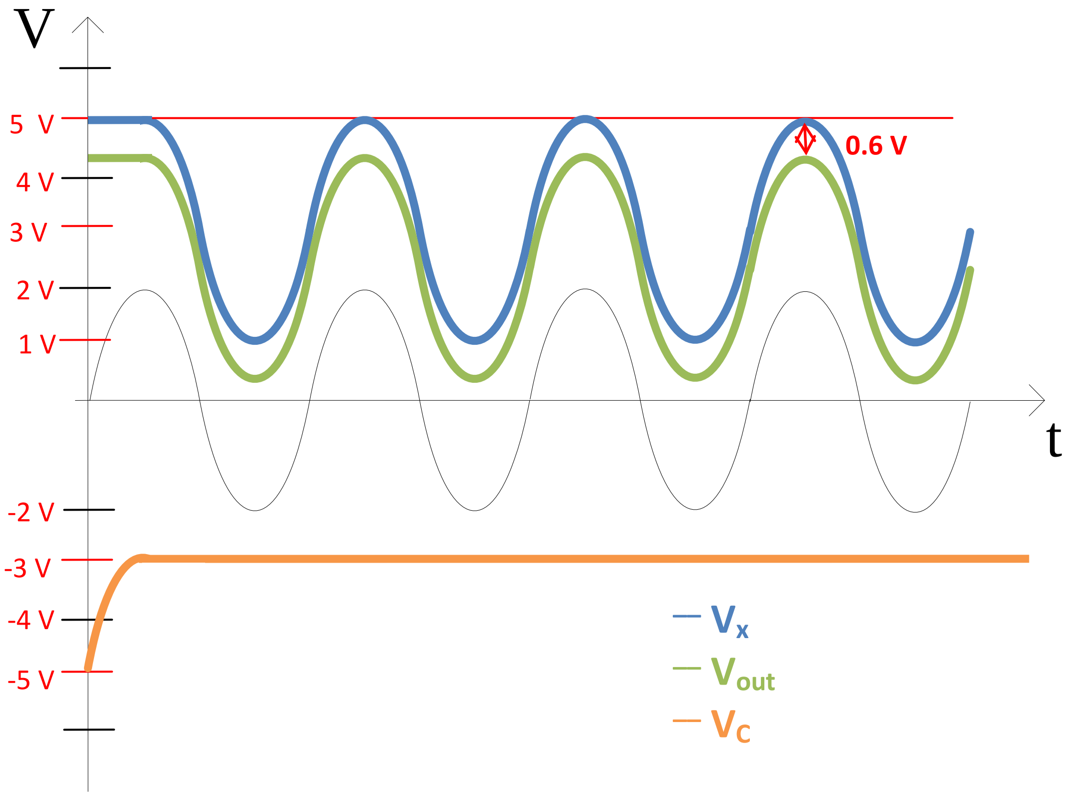
Using the 100 V Zener diode or a regular diode does not make a difference here since we never reach 100 V here

a) What is V_x and V_{out} if V_{in} = 0 V? (6 pts; (4 pts if regular instead of 100 V Zener diode is used)

V_X = 4.4 + 0.6 V = 5 V (4.4 V Zener diode in reverse breakdown, 100 V Zener diode forward biased)

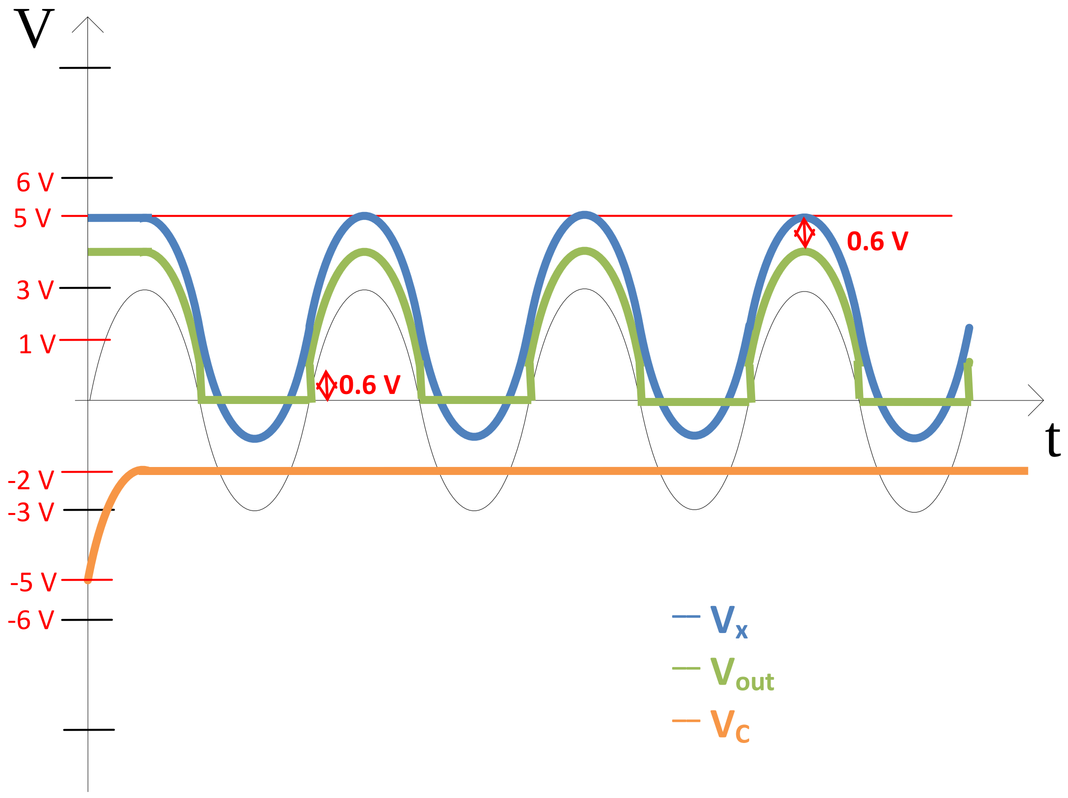
V_{out} = V_x – 0.6 V = 4.4 V

b) Now assume that V_{in} has been 0 for a while before it gets turned on at t=0 and has an amplitude of 2 V. V_{in} is given in the graph below. Complete the graph with V_C, V_x, and V_{out}. Make sure you label your graph! (Derive V_C at t=0 from part a)) (24 pts)



V_C = -5 V at the beginning but charges to -3 V within the first quarter of T (since V_X is clamped to 5 V and V_{in} raises to + 2V → Capacitor charges). From there on, since V_x is never larger than 5 V again, the zener diode branch is basically always off and the voltage across the capacitor cannot change anymore (no charge/discharge path) and $V_x = V_{in} - V_C$ for t > T/4. V_{out} is always V_x – 0.6 V due to the voltage drop across the diode.

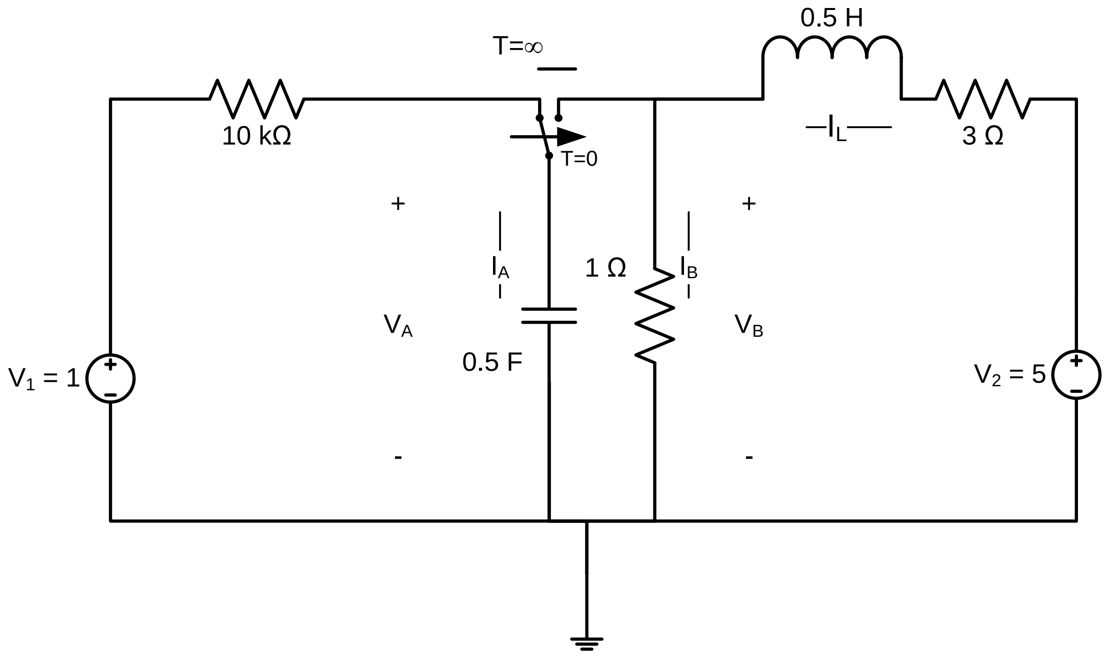
c) Repeat part b), but now with an V_{in} = 3 V (28 pts)



V_C = -5 V at the beginning but charges to -2 V within the first quarter of T (since V_X is clamped to 5 V and V_{in} raises to + 3V → Capacitor charges). From there on, since V_x is never larger than 5 V again, the zener diode branch is basically always off and the voltage across the capacitor cannot change anymore (no charge/discharge path) and $V_x = V_{in} - V_C$ for t > T/4. V_{out} is V_x – 0.6 V due to the voltage drop across the diode if Vx > 0.6 V and zero otherwise (after all it is a simple half-wave rectifier)

Problem 4: Transient Analysis (90 pts)

The following circuit is given



The switch has been closed (connected to the left) for a long time but at T=0, the switch is flipped

a. What are V_A, V_B, I_A, I_B, and I_L at time $T=0^-$ (10 pts)

$V_A = 1$ V (capacitor is an open circuit at DC)

$V_B = 5 * ¼ = 1.25$ V (voltage divider between 1 & 3 Ω resistor, inductor is a short at DC)

$I_A = 0$ A (capacitor is an open circuit at DC)

$I_B = 1.25 / 1 = 1.25$ A (Ohm's Law)

$I_L = I_B = 1.25$ A (KCL)

b. Immediately after the switch has been flipped:
What are V_A, V_B, I_A, I_B and I_L the current through the inductor at time $T=0^+$ (after the switch has been flipped) (10 pts)

V_A = 1 V (voltage at capacitor cannot change instantaneously)
V_B = 1 V (V_A and V_B are the same now)
$I_A = I_L – I_B$ = 0.25 A (current through inductor cannot change instantaneously)
I_B = 1 / 1 = 1 A (Ohm's Law)
I_L = 1.25 A (current through inductor cannot change instantaneously)

c. What are V_A, V_B, I_A, I_B after a long time ($T = \infty$) (8 pts)

V_A = 1.25 V (capacitor is an open circuit at DC)
V_B = 5 * ¼ = 1.25 V (voltage divider between 1 & 3 Ω resistor, inductor is a short at DC, capacitor an open circuit at DC)
I_A = 0 A (capacitor is an open circuit at DC)
I_B = 1.25 / 1 = 1.25 A (Ohm's Law)

d. Write the differential equation to find V_A (30 pts)

$V_2 = I_{total} * 3 + V_L + V_A = 3 * I_{total} + L\ dI_{total}/dt + V_A = 5 = 3 * I_{total} + 0.5\ dI_{total}/dt + V_A$
$I_{total} = I_A + I_B = C * dV_A / dt + V_A / 1 = 0.5 * dV_A / dt + V_A$
Plugging in I_{total} in first equation

➔ $5 = 3 * V_A + 1.5\ dV_A/dt + 0.25 * d^2V_A/d^2t + 0.5\ dV_A/dt + V_A$
$20 = d^2V_A/d^2t + 8\ dV_A/dt + 16\ V_A$

e. Solving for V_A

i. Give the forced response for V_A (4 pts)

V_A = 1.25 V (DC solution)

ii. Find the natural response for V_A (9 pts)

damping ratio can be found from differential equation found in part f

- ➔ $\zeta=1$
- ➔ Critical damping
- ➔ Natural response: $V_A(t) = K_1 e^{-4t} + K_2 * t * e^{-4t}$

iii. Using the above and the initial conditions given, find the complete solution for V_A (9 pts)

$V_A(t) = K_1 e^{-4t} + K_2 * t * e^{-4t} + 1.25$

$V_A(t=0) = 1\text{ V} = K_1 + 1.25 \rightarrow K_1 = -0.25\text{ V}$

$I_A(t=0) = 0.25 = C * dV_A / dt = 0.5 * (-4*K_1 + K_2) = 0.5 + 0.5K_2 \rightarrow K_2 = -0.5\text{ V}$

$V_A(t) = -0.25 e^{-4t} + -0.5\text{ V} * t * e^{-4t} + 1.25$

f. After a long time we flip the switch back

i. What happens to the capacitor and the voltage source V_1? (5 pts)

Right after the switch gets flipped back V_C stays at 1.25 V and the capacitor starts getting discharged through the 10 kOhm resistor to 1 V. During that time current flows from the capacitor to the voltage source. It basically gets “charged”

ii. What happens to the inductor and the voltage source V_2? (5 pts)

Nothing! Removing the charged capacitor does not change anything to the circuit on the right, since the capacitor was not drawing any current anymore anyway