EE 40: Introduction to Microelectronic Circuits
# Spring 2008: Midterm 2

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Total Time Allotted : 50 min
Total Points: 50

1. This is a closed book exam. However, you are allowed to bring two pages (8.5 x 11) notes, with writing on both sides.

2. No electronic devices, i.e. calculators, cell phones, computers, etc.

3. Show the steps used to arrive at your answer, where necessary. Partial credit will be given if you have the proper steps but an incorrect answer. A correct answer for a problem involving multiple steps where it is not clear how you arrived at the answer may not be given full credit.

4. Write your final answers into the boxes.

5. Remember to put down units.

Last (Family) Name:	
First Name:	
Student ID:	Lab Section:
Signature:	

		Max	Score
1	(a-g)	10	
1	(h-k)	10	
2		6	
3	(a)	4	
3	(b)	4	
4		10	
5		6	
Total		50	
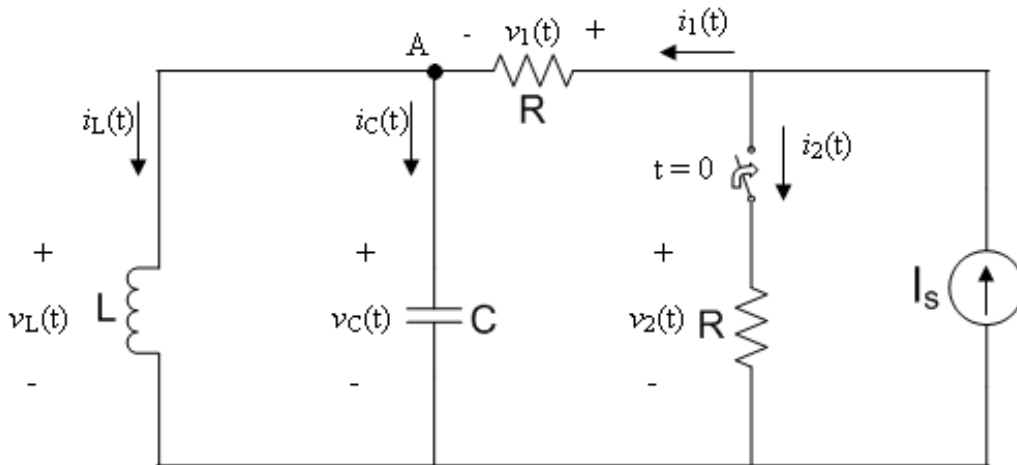
In the circuit shown above the current source is a DC source. The switch is open prior to time 0, and the current has reached steady state prior to time 0.

At time 0 the switch closes.

The reference configuration to use for each circuit element in solving the problem is shown in the figure. Please provide all answers in terms of I_s, R, L, and C.

(a) What is $i_L(0_-)$? (2 pts)

$$i_L(0_-) = I_s$$
(because cap is open circuit in DC steady state)

(b) What is $v_C(0_-)$? (2 pts)

$$v_C(0_-) = 0$$
(because inductor is short circuit in DC steady state)

(c) What is $i_L(0_+)$? (1 pt)

$$i_L(0_+) = i_L(0_-) = I_s$$

(d) What is $v_C(0_+)$? (1 pt)

$$v_C(0_+) = v_C(0_-) = 0$$

(e) What is $i_1(0_+)$? (2 pts)

Using KVL,

$$v_2(0_+) = v_C(0_+) + v_1(0_+) = v_1(0_+)$$

$$i_2(0_+) = i_1(0_+).$$

Combining this with

$$i_2(0_+) + i_1(0_+) = I_s,$$
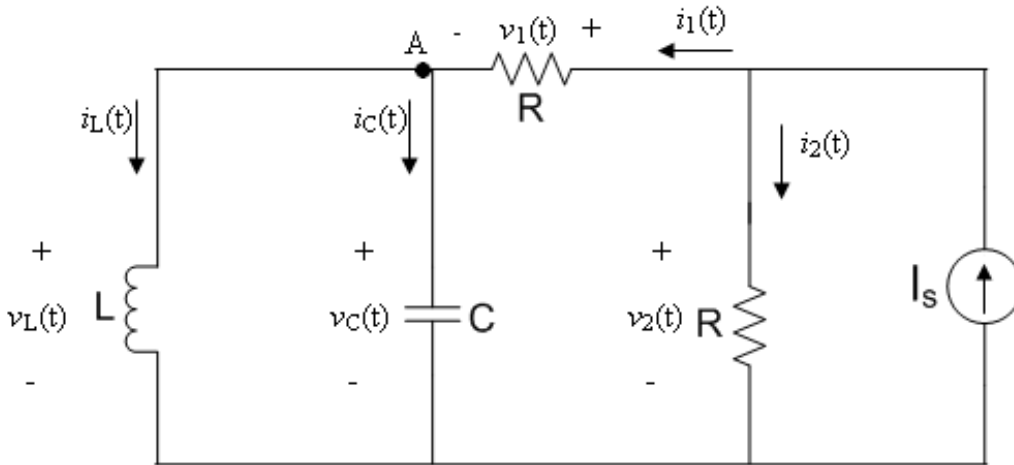
we get:

$$i_1(0_+) = I_s/2$$

(f) What is $i_C(0_+)$? (1 pt)

$$i_C(0_+) = -I_s/2$$
(because $i_C(0_+) + i_L(0+) = i_1(0_+)$)

(g) What is $\frac{dv_C}{dt}(0_+)$? (1 pt)

$$\frac{dv_C}{dt}(0_+) = \frac{1}{C}i_C(0_+) = \frac{-I_s}{2C}$$

For your convenience the circuit after the switch closes at time $t = 0$ is redrawn below. Use the same reference configurations as in the original figure.

The rest of the problem refers to the situation after time 0.

(h) Writing an appropriate KVL equation, express $i_1(t)$ in terms of $v_C(t)$ (and the parameters I_s, R, L, and C if needed) for $t \geq 0_+$. (2 pts)

$$Ri_1(t) + v_C(t) = Ri_2(t)$$
$$Ri_1(t) + v_C(t) = R(I_s - i_1(t))$$
$$2Ri_1(t) = -v_C(t) + RI_s$$

$$\boxed{i_1(t) = -v_C(t)/2R + I_s/2}$$

(i) Using KCL at node A, express $i_L(t)$ in terms of $v_C(t)$ and the parameters for $t \geq 0_+$. (2 pts)

$$i_L(t) + i_C(t) = i_1(t)$$

$$i_L(t) = -\frac{1}{C}\frac{dv_C}{dt}(t) + i_1(t)$$

$$\boxed{i_L(t) = -C\frac{dv_C}{dt}(t) - \frac{v_C(t)}{2R} + \frac{I_s}{2}}$$

(j) Write down a differential equation that $v_C(t)$ must satisfy for $t \geq 0$. (4 pts)

$$v_c(t) = v_L(t) = L\frac{di_L(t)}{dt} = L\frac{d}{dt}\left[-C\frac{dv_C}{dt}(t) - \frac{v_C(t)}{2R} + \frac{I_s}{2}\right]$$

$$v_c(t) = -LC\frac{d^2v_C}{dt^2}(t) - \frac{L}{2R}\frac{dv_C}{dt}(t) + \frac{L}{2}\frac{dI_s}{dt}$$

(note that the undriven circuit has two resistors in series, so is a parallel RLC circuit with an effective resistance of $2R$, and this is reflected in the homogenous equation for $v_C(t)$)
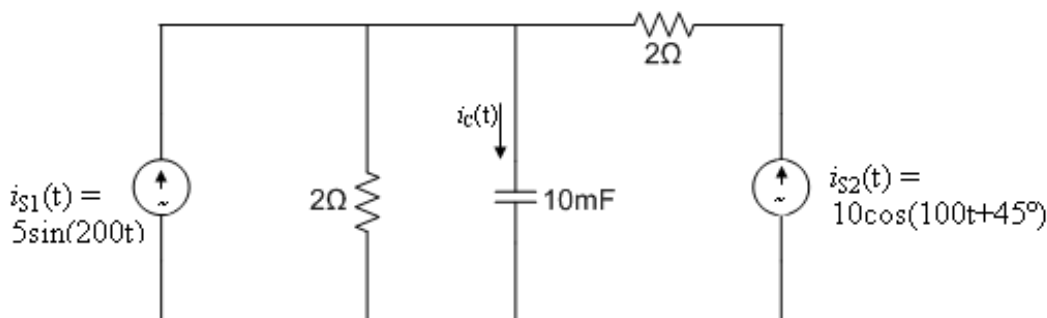
$$\boxed{\frac{d^2v_C}{dt^2}(t) + \frac{1}{2RC}\frac{dv_C}{dt}(t) + \frac{v_C(t)}{LC} = \frac{1}{2C}\frac{dI_s}{dt} = 0}$$

(k) For what combination of parameter values is the circuit critically damped for $t \geq 0_+$? (Note: if your answer depends on I_s, you have done something wrong). (2 pts)

$$\alpha = \omega_0$$

$$\frac{1}{4RC} = \frac{1}{\sqrt{LC}}$$

4

$$4R = \sqrt{\frac{L}{C}}$$

2. (6 pts) In the circuit above the two sources are at different frequencies $\omega_1 = 200\,rad/sec$ and $\omega_2 = 100\,rad/sec$ respectively.

Use superposition and phasor analysis to determine $i_C(t)$.

$$\arctan 2 = 63.435°$$
$$\arctan 4 = 75.96°$$

First consider just S_1. We replace the other current source by an OPEN circuit.

Observe that $\omega C = 200 * 10 * 10^{-}3 = 2$, so

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2} = -j/2.$$

Using current divider, we find that

$$\mathbf{I}_{c1} = \frac{\mathbf{Z}_R}{\mathbf{Z}_R + \mathbf{Z}_C}\mathbf{I_{s1}} = \frac{2\Omega}{2\Omega - \frac{j}{2}\Omega}\mathbf{I_{s1}} = \frac{4}{4-j}\mathbf{I_{s1}} = \frac{4}{\sqrt{17}\angle\arctan\frac{-1}{4}}5\angle -90° = \frac{20}{\sqrt{17}}\angle(-90° + \arctan\frac{1}{4})$$

Alternatively, using the other form of current divider (or just multiplying top and bottom of the above expression by j), we find that

$$\mathbf{I}_{c1} = \frac{\frac{1}{\mathbf{Z}_c}}{\frac{1}{\mathbf{Z}_R} + \frac{1}{\mathbf{Z}_C}}\mathbf{I_{s1}} = \frac{j2}{\frac{1}{2} + j2}\mathbf{I_{s1}} = \frac{4j}{1+4j}\mathbf{I_{s1}} = \frac{4\angle 90°}{\sqrt{17}\angle\arctan 4}5\angle -90° = \frac{20}{\sqrt{17}}\angle(-\arctan 4) = \frac{20}{\sqrt{17}}\angle -75.96°$$

Now consider just S_2. We replace the other current source by an OPEN circuit.

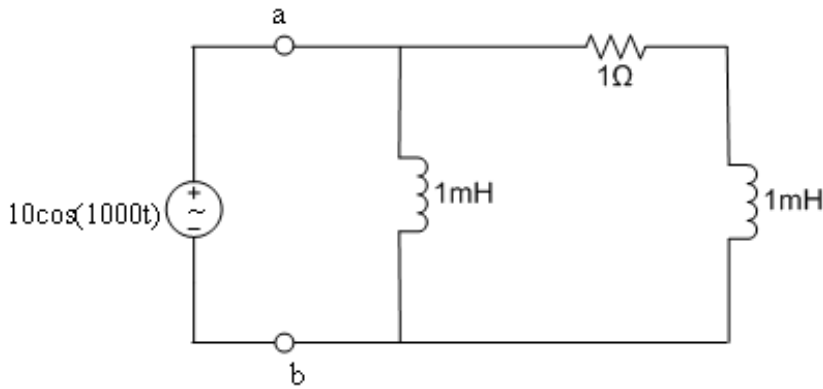Observe that $\omega C = 100 * 10 * 10^{-}3 = 1$, so $\omega C = 1$.

This time, we find that

$$\mathbf{I}_{c1} = \frac{\frac{1}{\mathbf{Z}_c}}{\frac{1}{\mathbf{Z}_R} + \frac{1}{\mathbf{Z}_C}}\mathbf{I_{s1}} = \frac{j}{\frac{1}{2} + j}\mathbf{I_{s1}} = \frac{2j}{1+2j}\mathbf{I_{s1}} = \frac{2\angle 90°}{\sqrt{5}\angle\arctan 2}10\angle 45° = \frac{20}{\sqrt{5}}\angle(135° - \arctan 2) = \frac{20}{\sqrt{5}}\angle(135° - 63.435°)$$

To find the total solution, we add the previous solutions:

$$i_C(t) = i_{C1}(t) + i_{C2}(t) = \frac{20}{\sqrt{17}}\cos(200t - 75.96°) + \frac{20}{\sqrt{5}}\cos(100t + 135° - 63.435°)$$

Rubric: 1 pt for correct $\mathbf{I}_{C1} = \frac{2\Omega}{2\Omega - \frac{j}{2}\Omega}\mathbf{I_{s1}}$. 1 pt for $i_{C1}(t) = \frac{20}{\sqrt{17}}\cos(200t - 75.96°)$. Likewise 2 pts for i_{C2}. 2 more points for adding them.

3. (8 pts total) Consider the circuit above operating in AC steady state at frequency $\omega = 1000$rads/sec.

(a) What is the average power dissipated in the resistance? (4 pts)

There are several ways to do this. The easiest is to work directly with the resistor. Using voltage divider, we obtain the voltage across the resistor as:

$$\mathbf{V}_R = \mathbf{V}_S \frac{R}{j\omega L + R}.$$

Note that $\mathbf{V}_S = 10V$, $j\omega L = 1\Omega$, and $R = 1\Omega$, so that plugging these values in, we have

$$\mathbf{V}_R = 10 \frac{1}{j+1} = \frac{10}{\sqrt{2}\angle 45°}.$$

Hence, $V_R(t) = \frac{10}{\sqrt{2}} \cos(1000t - 45°)$.

To find the average power dissipated, we note that across a resistor, $P_{ave} = \frac{|\mathbf{V}_R|^2}{2R} = (10/2)^2 = 25$.

It was a bit more algebraically messy to do this problem by instead finding the average power supplied across the current source (and using the fact that inductors do not dissipate average power to equate this to the power dissipated by the resistor). One could do this as follows:

Observe that we would like to find $P_{ave} = Re\{\mathbf{I}_s \mathbf{V}_s\}/2$.

Recall that $\mathbf{I}_s = \mathbf{V}_s/\mathbf{Z}_{eq}$, so we need to find $1/\mathbf{Z}_{eq}$.

Using the formula for parallel elements, we see that $\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{(j\omega L)} + \frac{1}{(R+j\omega L)} = -j + \frac{1}{1+j} = -j + (1-j)/2 = (1 - j3)/2$.

Hence $P_{ave} = Re\, \mathbf{I}_s \mathbf{V}_s/2 = Re\, \frac{\mathbf{I}_s^2}{2\mathbf{Z}_{eq}} = Re(\frac{100}{2}\frac{1+3j}{2}) = \frac{100}{4} = 25$.

This agrees with our answer using the other method.

(you could also find $\mathbf{I}_s$ in polar notation, and then draw a triangle to find $\cos(\theta)$, where θ is the angle between current and voltage, and then use $P_{ave} = V_{rms} I_{rms} \cos(\theta)$).

$$\boxed{P_{avg} = 25}$$

(b) We now want to put a capacitor in parallel with the load across the terminals a and b, resulting in the circuit below.
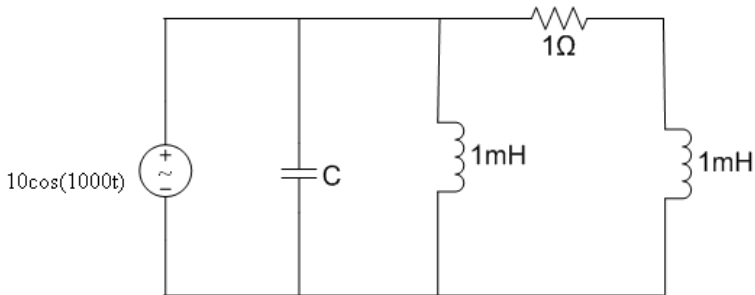
What should be the value of C such that no reactive power is drawn from the source? (4 pts)

We write
$$\frac{1}{Z_{tot}} = \frac{1}{1/j\omega C} + \frac{1}{Z_a}$$
where Z_a is the impedance of the inductor and resistor combination found in part (a).
Using the value from part (a), $\frac{1}{Z_{tot}} = j\omega C + \frac{1-3j}{2}$.
We want Z_{tot} to be entirely real, which is equivalent to requiring its inverse to be real. Therefore we can set the imaginary component to 0, getting
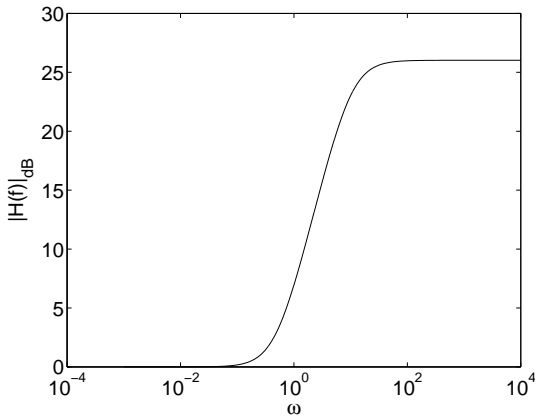$$j\omega C - 3j/2 = 0,$$
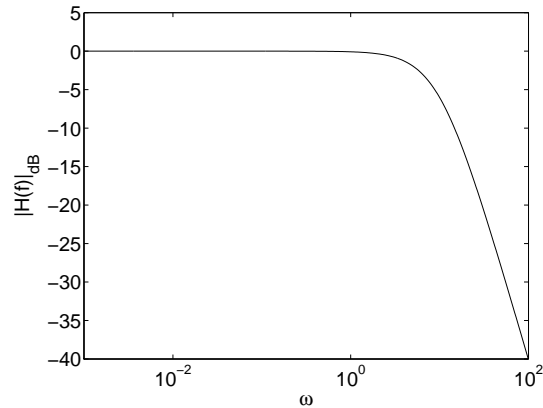or $C = 3/2\omega = 1.5mF$.

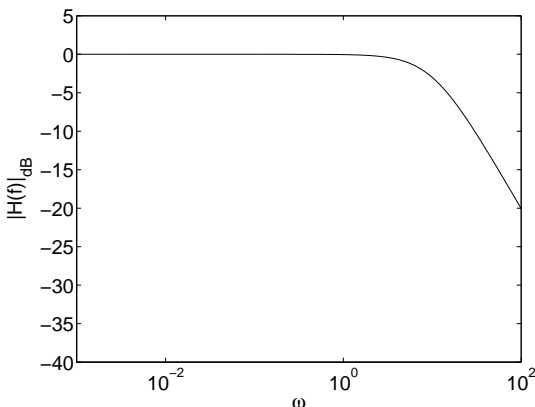$$\boxed{C = 1.5mF}$$
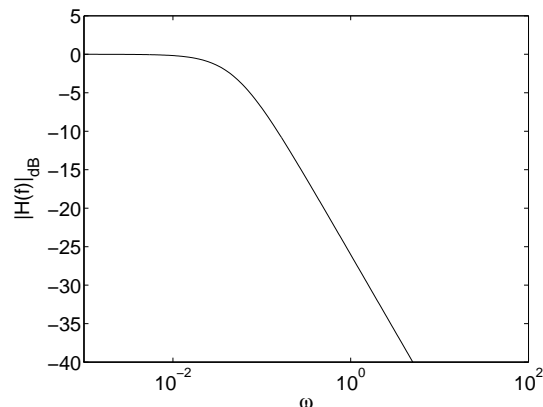
4. (10 pts total)

Bode plots:

(a) Plot A

(b) Plot B

(c) Plot C

(d) Plot D

Match the magnitude Bode plots (above) with the corresponding transfer functions (below) (4 pts).

Transfer function	$H_1(\omega) = \dfrac{1}{1+j20\omega}$	$H_2(\omega) = \dfrac{10+j20\omega}{10-j\omega}$	$H_3(\omega) = \dfrac{10}{10+j\omega}$	$H_4(\omega) = \dfrac{10}{(10+j\omega)^2}$
Corresponding Bode plot (a,b,c, or d?):	d	a	c	b

Explain your reasoning (6 pts): Many explanations were possible.

plot (a) has one pole and one zero. It is the only plot with two frequencies at which things happen. Hence the transfer function is $H_2(\omega)$

plot (c) is a simple low pass filter with break frequency $\omega_B = 10$, and thus will have transfer function $H_c(\omega) = \frac{1}{1+j\omega/10}$.

plot (b) is the same as plot (c) but stretched vertically by a factor of 2. Therefore we square $H_c(\omega)$, to get $H_d(\omega) = \frac{1}{(1+j\omega/10)^2}$.

plot (d) the only plot with a break frequency at $\omega = \frac{1}{20}$.

For those who prefer to discuss things in terms of poles and zeros, (a) has one pole and one zero. (b) has two poles. (c) has a pole at $\omega = 10$. (d) has a pole at $\omega = 1/20$.
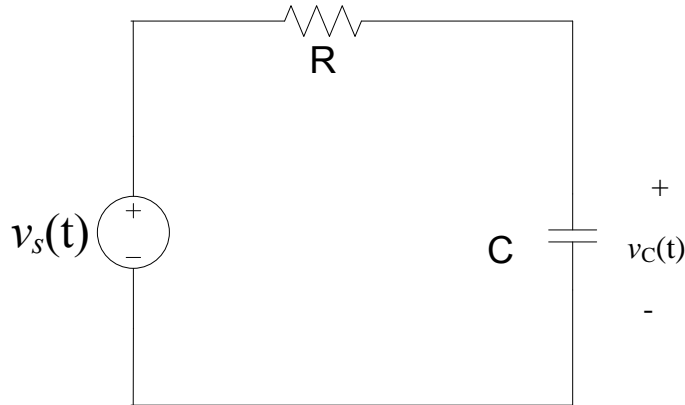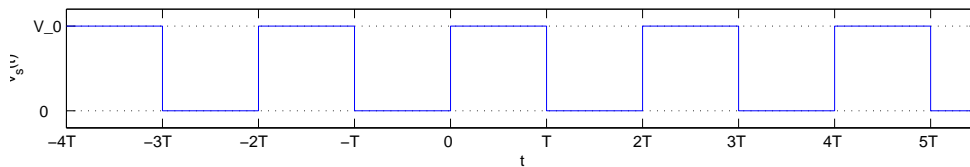
5. (6 pts)

Figure 1: Circuit

Figure 2: Voltage source

In the circuit above (Fig. 1) the voltage source is periodic with period $2T$ and alternates between a value of V_0 for T seconds, followed by a value of 0 for T seconds as shown (Fig. 2).

Sketch the steady state response $v_C(t)$, labeling the graph completely, i.e. $v_C(t)$ should be determined exactly. You need not explain the details by which you arrived at $v_C(t)$. However, you should explain why your sketched $v_C(t)$ is correct.

(Hint: Try to think about the values that the steady state $v_C(t)$ should have when t lies in an interval that is an even multiple of T and the value it should have when t lies in an interval that is an odd multiple of T. Write two equations, relating these two values, using your understanding of RC circuits.)

Without writing anything down, we can observe that when the source is on, the voltage across the capacitor should increase exponentially (i.e. quickly at first, then more slowly) towards the source voltage. When the source voltage is turned to 0, then the capacitor should discharge exponentially towards 0 (more quickly at first, then more slowly). This gives rise to a saw tooth pattern (2 pts for getting this far).

Note that $V_C(t)$ should be continuous unless you anticipate infinite current!

Further, note that it will not quite achieve the asymptotic values of 0 and V_0, because it is given finite time (and you are given no information indicating that the time constant is smaller than your period T) (2 more points for getting this far).

For full points, we can quantify things:

Recall the formula

$$v(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}})e^{-t/\tau}.$$

Applying it to our situation, we find that

On odd intervals $[Tn, T(n+1)]$, we have

$$V_c(t) = V_0 + (V_c(0) - V_0)e^{-(t-Tn)/RC}$$

On even intervals $[T(n+1), T(n+2)]$, we have

$$V_c(t) = 0 + V_c(T)e^{-[t-T(n+1)]/RC}.$$

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To find the precise values $v_c(0)$ and $v_c(T)$, we enforce continuity at the boundary points:

$$V_{\text{odd}}(\text{end}) = V_0 + (V_c(0) - V_0)e^{-T/RC} = V_c(T)e^0 = V_{\text{even}}(\text{start})$$

and

$$V_{\text{odd}}(\text{start}) = V_c(0) = V_c(T)e^{-T/RC} = V_{\text{even}}(\text{end})$$

We find

$$V_0 - V_0 e^{-T/RC} = V_c(T)e^0 - V_c(0)e^{-T/RC} = V_c(T)e^0 - V_c(T)e^{-2T/RC} = V_c(T)(1 - e^{-2T/RC})$$

Therefore

$$V_c(T) = V_0(1 - e^{-T/RC})/(1 - e^{-2T/RC}),$$

and

$$V_c(0) = V_0(e^{-T/RC} - e^{-2T/RC})/(1 - e^{-2T/RC}).$$

(note that we had better get that the amount it rises in an odd interval plus the amount it falls in an even interval sum to 0, so $(V_c(0) - V_0) + V_c(T) = 0$. Indeed, we can check that this holds.)

The thick green lines on the sketch indicate the actual capacitor voltage, while the thin ones show how it would continue if the source did not change value.
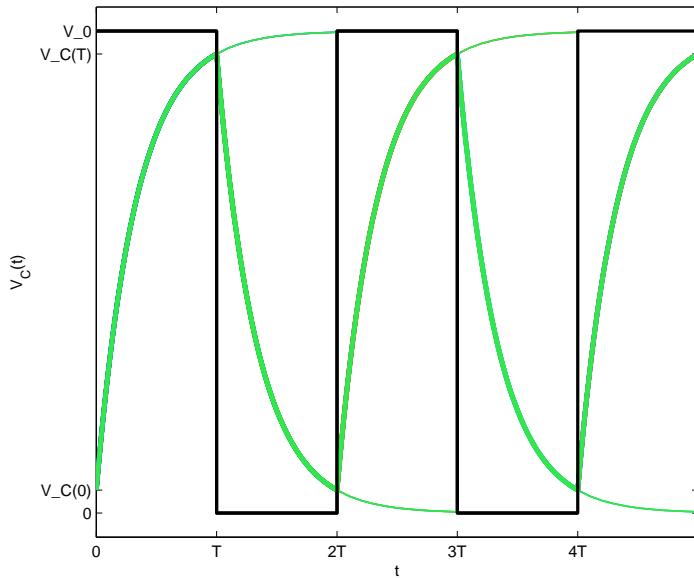

Figure 3: Capacitor Voltage

Rubric: 2 pts for realizing it has an exponential (with the right concavities) sawtooth pattern. 2 more for realizing it does not quite reach the extreme points 0 and V_0. 2 final points for the precise values it reaches at the ends of each interval.