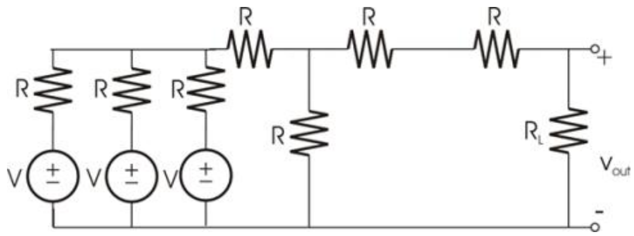
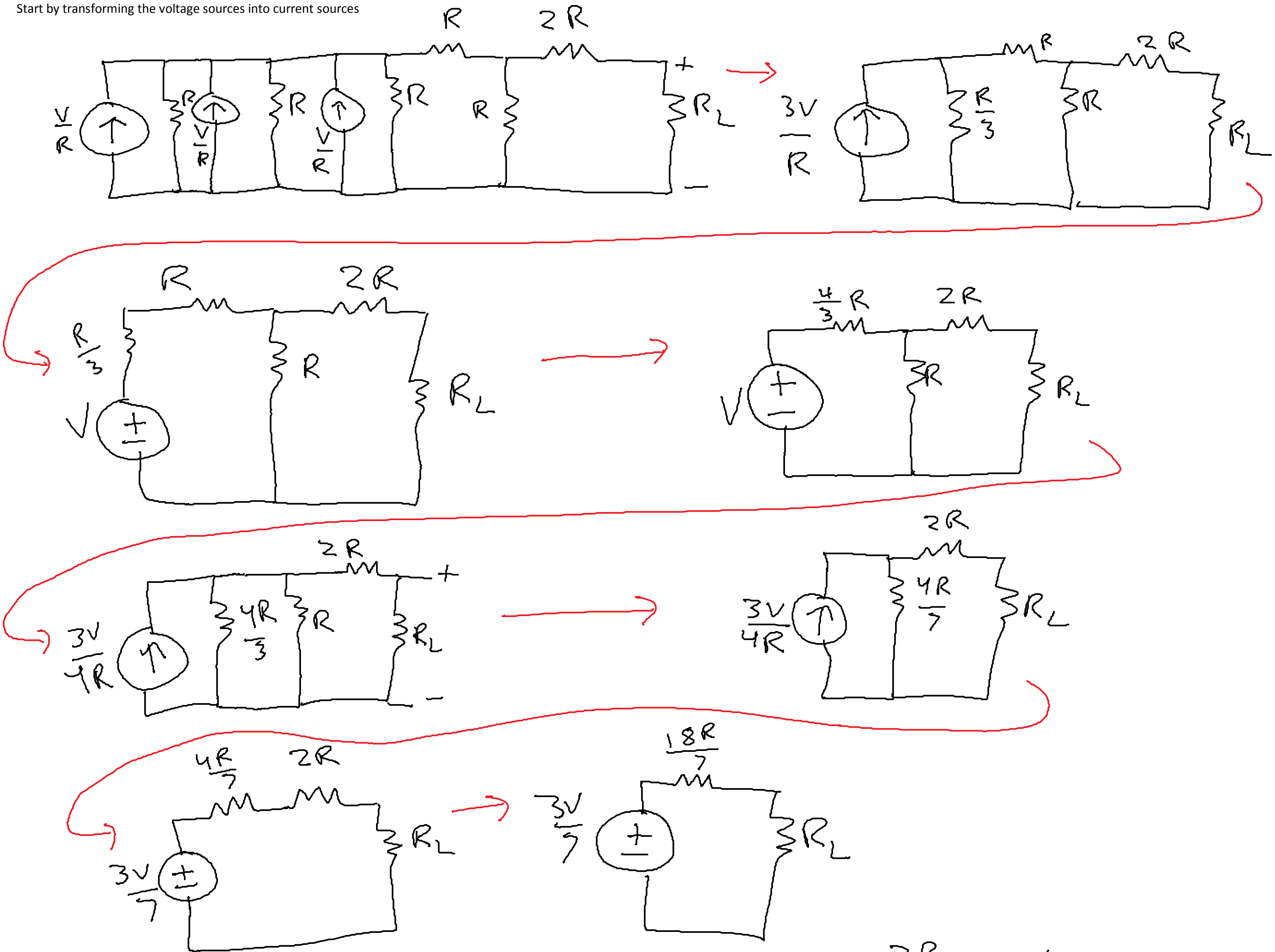
Problem 1



Start by transforming the voltage sources into current sources



$$V_{out} = \frac{R_L}{R_L + \frac{18R}{7}} \cdot \frac{3V}{7} = \frac{3R_L}{18R+7R_L} V$$

Notes:

- The above solution is the "intended" solution, but this circuit is pretty simple so any of the circuit solving techniques you've learned could have been used. Doing the source transforms to simplify the circuit makes it much easier though, which is why this solution is preferred. I'm not going to draw out all the possible methods of solving it as there are at least 3 main ways to solve it with little variations in simplifications you can make at each step.
- You don't actually need to perform the first set of source transforms, if you see that the nodes on the positive side of all the voltage sources are in fact at the same voltage because there are fixed voltage supplies connected to those nodes, so they are all in fact one node, and then you can just combine the 3 resistors in parallel.

Rubric:

- If you followed the above strategy (generally simplified the circuit to reduce it to a voltage divider):
  - 5 pts for simplifying the left side by combining the voltage sources and resistors
  - 5 pts for solving the right side of the circuit using KVL/KCL or simply combining more resistors or doing more source transforms
  - 2.5 pts for the correct expression (Incidentally, a lot of you wrote some very funky complex fractions which I simplified to check if they were correct. In the future, please simplify your complex fractions).
  - For the two 5 pt parts, I awarded 2.5 pts for partially solving that part
- If you used another method (eg. Write out all the nodal or mesh equations and solve):
  - 5 pts for proper setup (eg. Correct KCL/KVL equations or other valid simplifications)
  - 7.5 pts for correct follow through and answer (I broke this down a little into 2.5 pt increments in some cases where the work was clear and the error was simple)
  - If there was a partially correct set up (eg. Started setting up nodal analysis but forgot a node or skipped a node) I awarded 2.5 pts
  - If there was no valid set up at all, I unfortunately awarded 0 pts. Examples of this include combining resistors in ways that don't make sense, assuming that the voltage drop across the resistors on the left is the full voltage of the power supply V, etc).

Problem 3:

$I_X = I_5 - I_4$

Mesh 1: $i_1R_3 + (i_1 - i_3)R_4 = 0$

+4 for writing correct equation; +2 for partial correct answer

Mesh 2+3(Supermesh): $i_2R_1 + (i_3 - i_1)R_4 + (i_3 - i_5)R_5=0$

+4 for writing correct equation; +2 for partial correct answer

Mesh2+3 (Supermesh): $i_2 - i_3 = \alpha i_x = \alpha(I_5 - i_4)$

+5 for correct supermesh

+4 for writing correct equation; +2 for partial correct answer

Mesh 4: $i_4 = -i_y$

+4 for writing correct equation; +2 for partial correct answer

Mesh 5: $(i_5 - i_3)R_5 + (i_5 - i_4)R_6 + i_5R_2 = 0$

+4 for writing correct equation; +2 for partial correct answer

Simplify:

+0.5 for correct arithmetic for each mesh (total +2.5)

$i_1(R_3 + R_4) - i_3R_4 = 0$

$-i_1R_4 + I_2R_1 + i_3(R_4 + R_5) -i_5R_5=0$

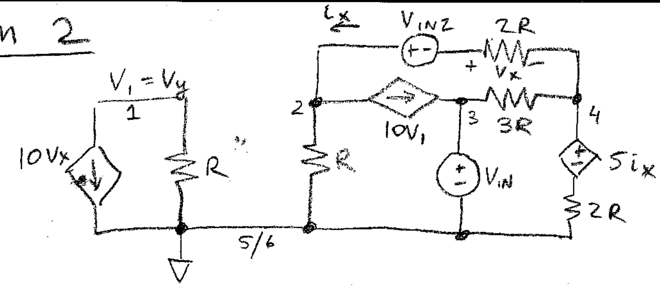
$i_2 - i_3 + \alpha i_4 - \alpha i_5 = 0$

$i_4 = -i_y$

$- i_3R_5 - i_4R_6 + i_5(R_2 + R_5 + R_6) = 0$

-2 for redundant equations such as equation involving β for loop 4 (You don't know the voltage across an ideal current source so you cant mesh that loop)

PROBLEM 2



$$i_x = -\frac{V_x}{2R} = \frac{V_4 + V_{IN2} - V_2}{2R}$$ 2 pts

$$V_x = V_2 - V_{IN2} - V_4$$ 2 pts

Choose ground: 2.5 pts
Identify 5&6 as same node: 2.5 pts

node 3: $\boxed{V_3 = V_{in}}$ 5 pts node 5 (6): $\boxed{V_5 = V_6 = 0}$

KCL, node 1:

$$10V_x + V_1/R = 0$$ 2 pts

$$10(V_2 - V_4 - V_{IN2}) + V_1/R = 0$$

$$\boxed{\frac{1}{R}V_1 + 10V_2 - 10V_4 = 10V_{in2}}$$ 5 pts

KCL, node 2:

$$\frac{V_2}{R} + 10V_1 - i_x = 0$$ 2 pts

$$\boxed{10V_1 + V_2\left(\frac{1}{R} + \frac{1}{2R}\right) - \frac{1}{2R}V_4 = \frac{V_{in2}}{2R}}$$ 5 pts

Note:
Node 5/6 is the best choice for ground because it has the largest # of connections. If you picked a different node to make ground, your answer may still be correct, but it will be significantly different from this solution.

KCL, node 4: $i_x + \frac{V_4 - V_3}{3R} + \frac{V_4 - 5i_x}{2R} = 0$ 2 pts

$$i_x\left(1 - \frac{5}{2R}\right) - \frac{V_3}{3R} + V_4\left(\frac{1}{2R} + \frac{1}{3R}\right) = 0$$

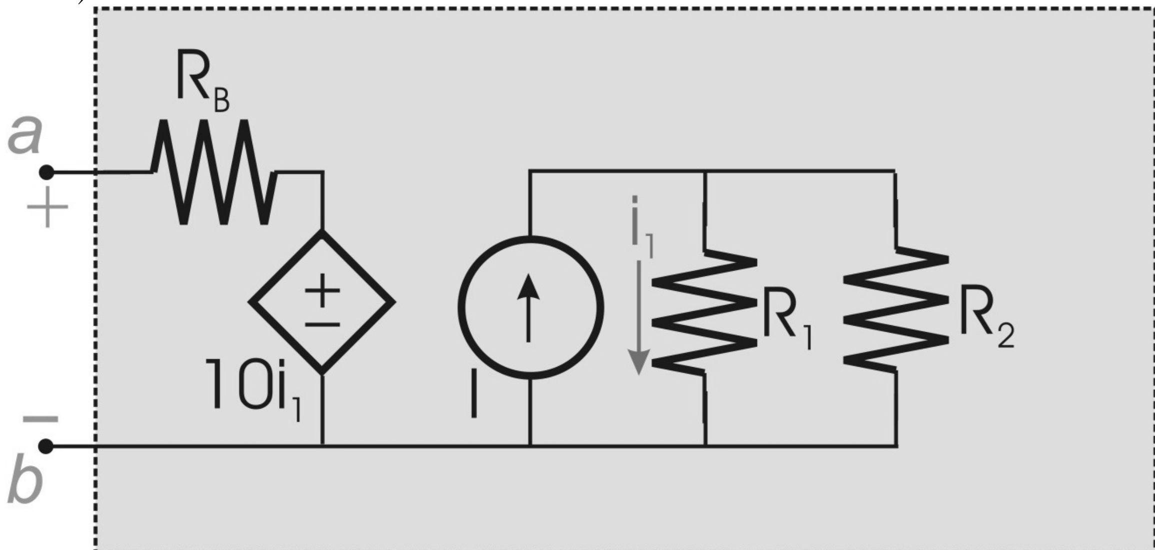
$$\left(\frac{V_4 + V_{in2} - V_2}{2R}\right)\left(1 - \frac{5}{2R}\right) = V_4\left(\frac{1}{2R} - \frac{5}{4R^2}\right) - V_2\left(\frac{1}{2R} - \frac{5}{4R^2}\right) + V_{in2}\left(\frac{1}{2R} - \frac{5}{4R^2}\right)$$

$$\boxed{V_2\left[-\frac{1}{2R} + \frac{5}{4R^2}\right] + V_3\left[-\frac{1}{3R}\right] + V_4\left[\frac{1}{2R} + \frac{1}{3R} + \frac{1}{2R} - \frac{5}{4R^2}\right] = -V_{in2}\left[\frac{1}{2R} - \frac{5}{4R^2}\right]}$$

5 pts

Problem 4 *Equivalent circuits* (12.5 points)

a) Provide the simplest equivalent circuit for the grey box (measured from terminals a and b).



Solution:

Find i_1 (+2.5 pts):

$$i_1 = \frac{R_2}{R_1 + R_2} \cdot I$$ (Current Divider)

Find Vth = Voc across a and b (+2.5 pts):

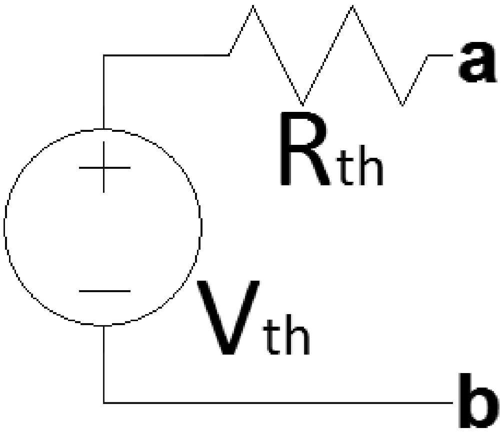
$$V_{th} = V_{oc} = 10i_1 = \frac{10R_2}{R_1 + R_2} \cdot I$$

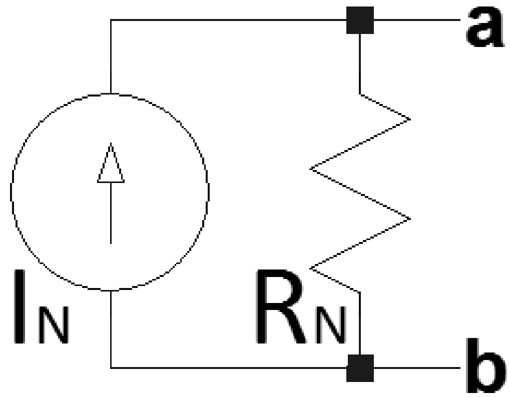
Find Rth (+2.5 pts):

$$I_{sc} = \frac{10i_1}{R_B}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{10i_1}{\frac{10i_1}{R_B}} = R_B$$

Draw a Thevenin or Norton Equivalent Circuit (+2.5 pts)
Correct Answer (+2.5 pts):





$$V_{th} = \frac{10R_2 \cdot I}{R_1 + R_2}$$

$$R_N = R_B$$

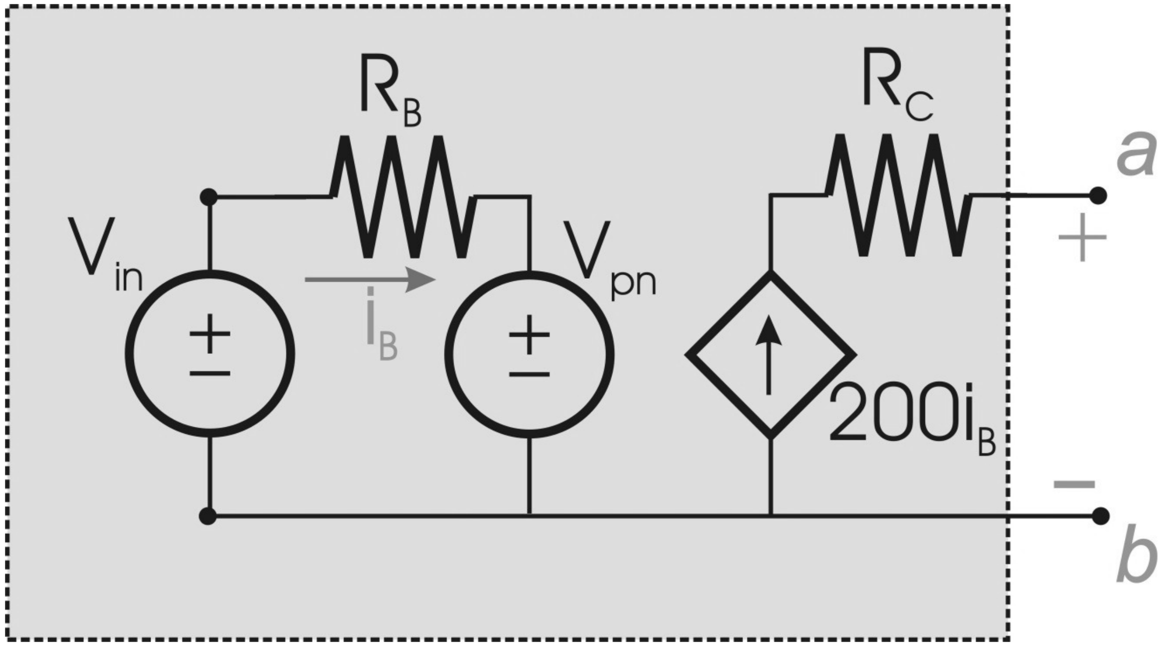
OR

$$I_N = \frac{10R_2 \cdot I}{R_B(R_1 + R_2)}$$

$$R_N = R_B$$

Note: This is a long way to find the equivalent circuit. If you followed your intuition and skipped some steps to get to the correct answer, you still got the full credit. ☺

b) Provide the simplest equivalent circuit for the grey box (measured from terminals a and b). (12.5 points)



Solution:

Find i_B (+2.5 pts):

$$i_B = \frac{V_{in} - V_{pn}}{R_B}$$

Find Isc (+2.5 pts):

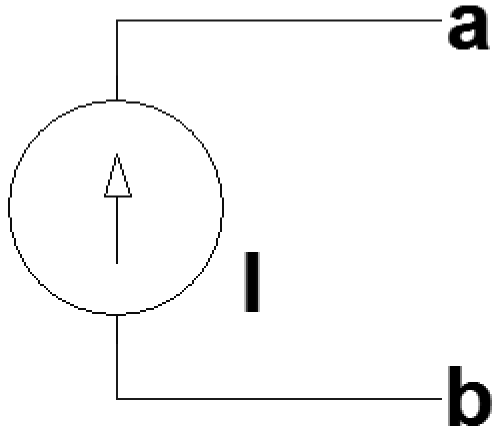
$$I_{sc} = 200 \cdot i_B = \frac{200(V_{in} - V_{pn})}{R_B}$$

Find Voc and R_N (+2.5 pts): (open-circuit resistance is infinity)

$$V_{oc} = IR_{oc} = 200 i_B \cdot \infty = \infty$$

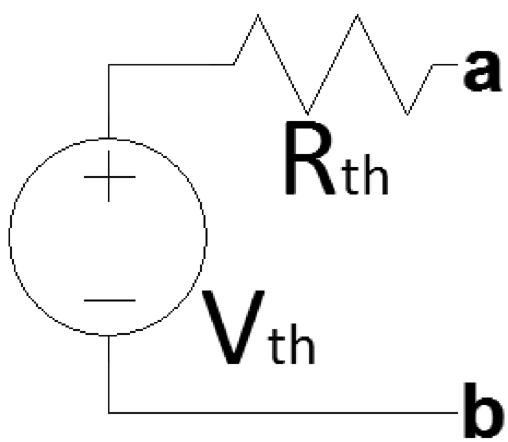
$$R_N = \frac{V_{oc}}{I_{sc}} = \infty$$

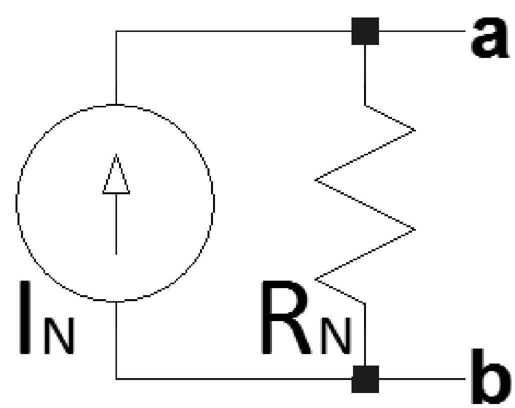
Draw an equivalent circuit with a current source (+2.5 pts):
Correct answer (+2.5 pts):



$$I = \frac{200(V_{in} - V_{pn})}{R_B}$$

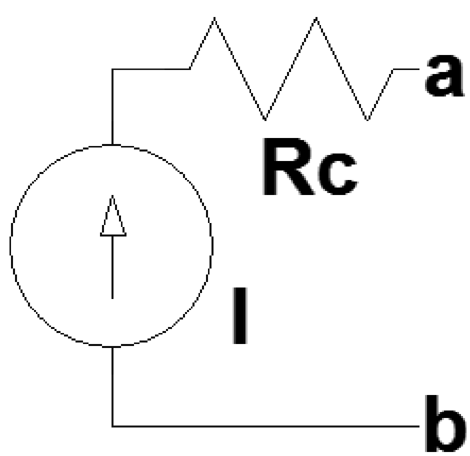
Some common incorrect solutions:





(LEFT): Maximum of 5 pts.

(RIGHT): Maximum of 7.5 pts.



This one was close but deserves 10 pts.

One Final Note: If you drew a BJT,
You got an automatic +**0** for either problem.
(The point of finding an equivalent circuit is to simplify common devices like BJTs into something solvable using a simple circuit analysis.)

Solution by Yun Jae Cho