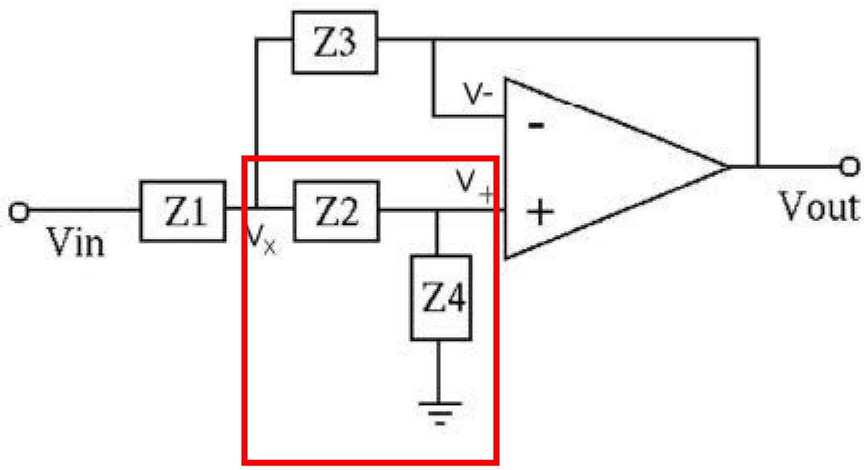
Grader: Terry Kim

Problem1

a) Consider the circuit below.



from the negative feedback, we know $V_+ = V_- = V_{out}$
KCL at V_x node

$$\frac{V_{in}-V_x}{Z_1} = \frac{V_x - V_{out}}{Z_3} + \frac{V_x - V_{out}}{Z_2}$$ (5pts)

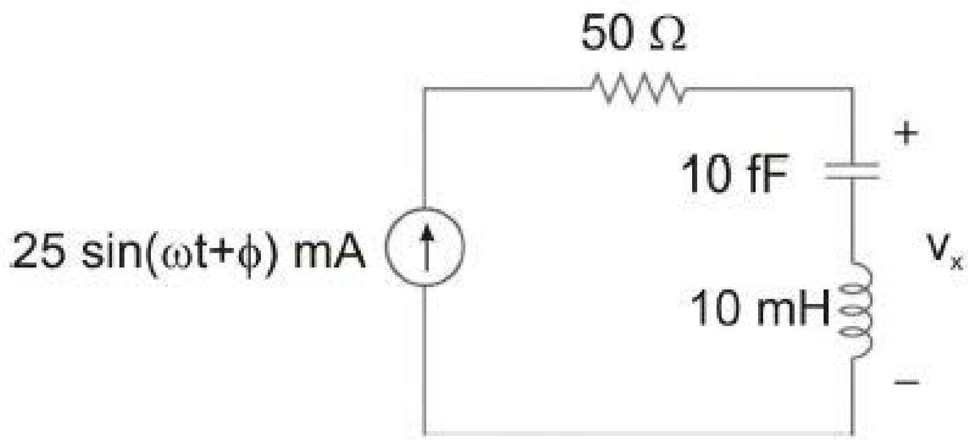
Voltage divider in the red box above with Z_2 & Z_4

$$V_{out} = \left(\frac{Z_4}{Z_2 + Z_4}\right) V_x$$

$$V_x = \frac{Z_2 + Z_4}{Z_4} V_{out} \text{ or}$$

$$V_x = (1 + \frac{Z_2}{Z_4}) V_{out}$$ (5pts)

(b) Find ω such that $v_x = 0$ for this circuit at $t = \infty$. *(5 points)*



In order to have

$V_x = 0$, we need to have impedence across V_x should be zero.

$$Z_x(\omega) = Z_c(\omega) + Z_L(\omega) = 0$$

$$Z_x(\omega) = \frac{1}{j\omega C} + j\omega L = 0$$

$$Z_x(\omega) = \frac{-j}{\omega C} + j\omega L = 0$$

$$Z_x(\omega) = j\left(\frac{-1}{\omega C} + \omega L\right) = 0$$

$$Z_x(\omega) = 0 \text{ when } \left(\frac{-1}{\omega C} + \omega L\right) = 0$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 10^{-15} \times 10 \times 10^{-3}}} = \frac{1}{\sqrt{1 \times 10^{-16}}}$$

$$\omega = 1 \times 10^{8} \frac{rad}{s}$$

(5pts)

a) What is the value of V_c at $t = 0^+$? **Write It in the BOX BELOW.** *(5 points)*

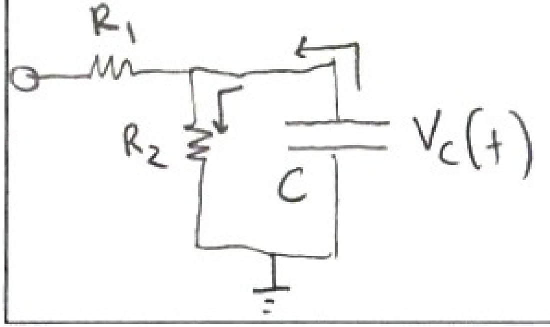
$$V_c(0^-) = \boxed{\frac{R_2}{R_1+R_2} V_S = V_c(0^+)}$$

since V_c cannot change instantenously.

b) Using whatever method you like, provide a ***symbolic expression*** for the voltage $V_C(t)$ for $t > 0$ in the **BOX BELOW.** *(15 points)*

Open, no current can flow

Equivalent circuit is:



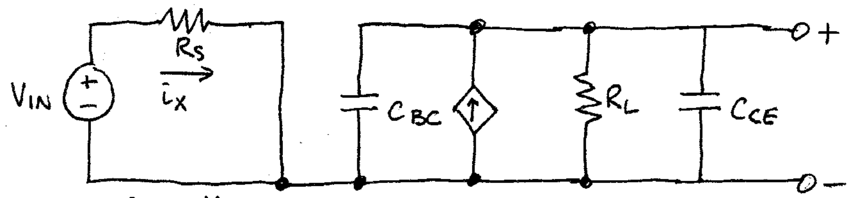
$$\tau = R_2 C$$

$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right] \cdot e^{-t/\tau}$$

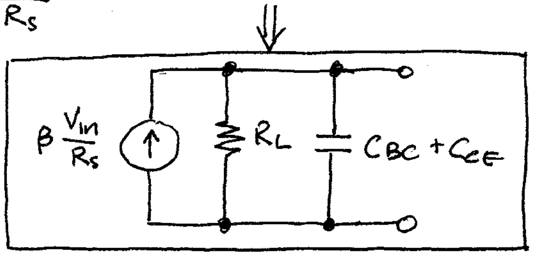
$$V_c(\infty) = 0 \Rightarrow \boxed{V_c(t) = \frac{R_2}{R_1 + R_2} \cdot V_s \cdot e^{-t/R_2 C}}$$

MT#2 Problem 3

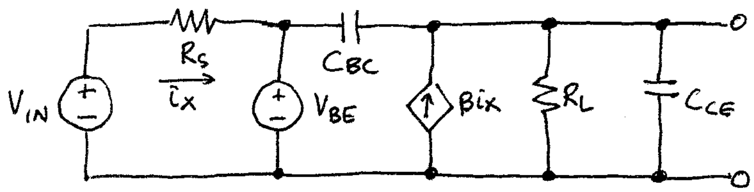
(a) assuming V_{BE} is a DC voltage: → it becomes a short circuit



$$i_X = \frac{V_{in}}{R_S}$$

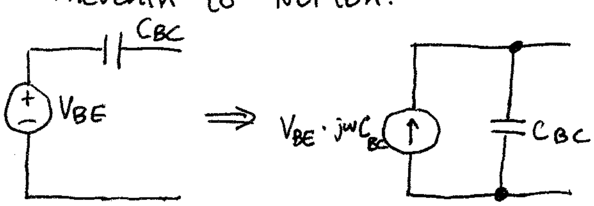


assuming V_{BE} is an AC voltage:

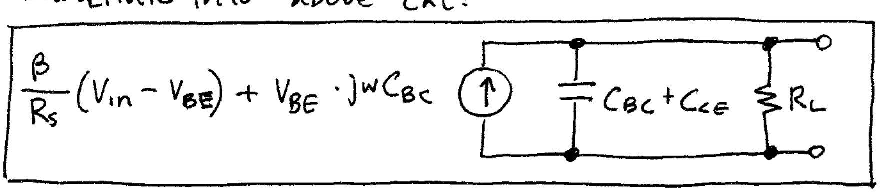


$$i_X = \frac{V_{IN} - V_{BE}}{R_S}$$

Thevenin to Norton:



Substitute into above ckt:



(b) V_{BE} is DC

$$V_{out} = \frac{\beta}{R_S} \cdot V_{in} \cdot \left(R_L \,\|\, \frac{1}{j\omega(C_{BC} + C_{CE})} \right) \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{\beta}{R_S} \left[\frac{R_L}{1 + j\omega(C_{BC} + C_{CE}) R_L} \right]}$$

V_{BE} is AC

$$V_{out} = \left[\frac{\beta}{R_S} \cdot V_{in} - \frac{\beta}{R_S} \cdot V_{BE} + V_{BE} \cdot j\omega C_{BC} \right] \cdot \left[\frac{R_L}{1 + j\omega(C_{BC} + C_{CE}) R_L} \right]$$

In this case, the output voltage is a function of both V_{in} and V_{BE}. By superposition, the total output is:

$$V_{out} = H_1(\omega) \cdot V_{in} + H_2(\omega) \cdot V_{BE} \quad \text{where} \quad \begin{cases} H_1(\omega) \triangleq \left. \frac{V_{out}}{V_{in}} \right|_{V_{BE} = 0} \\ H_2(\omega) \triangleq \left. \frac{V_{out}}{V_{BE}} \right|_{V_{in} = 0} \end{cases}$$

→

MT #2 Problem 3

(b) continued...

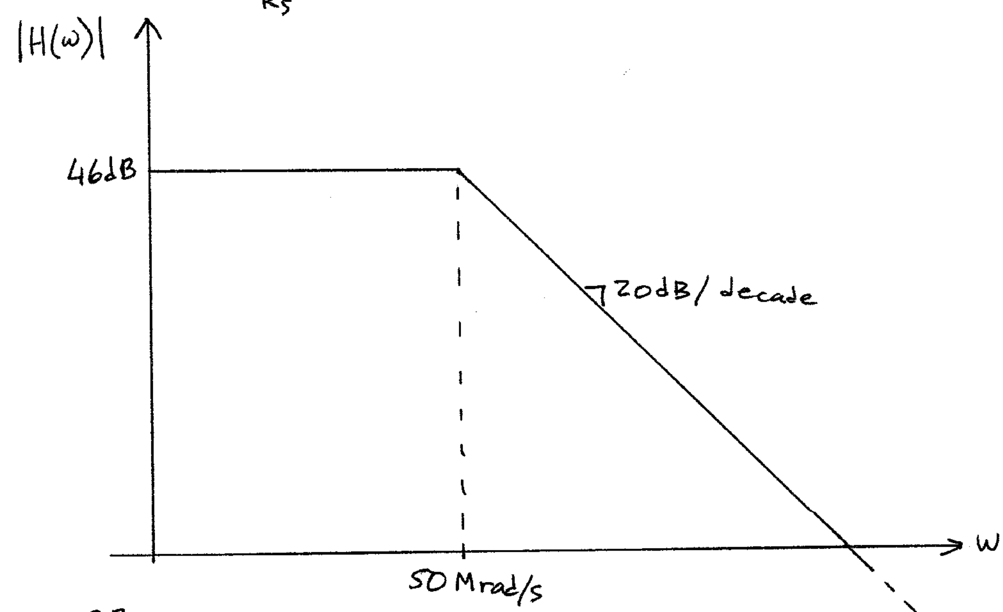
The question asked for $\frac{V_{out}}{V_{in}}$, which is $H_1(\omega)$. So, we only need to write that transfer function:

$$V_{out}\Big|_{V_{BE}=0} = \left[\frac{\beta}{R_S}\cdot V_{in}\right]\cdot\left[\frac{R_L}{1+j\omega(C_{BC}+C_{CE})R_L}\right]$$

$$\boxed{H_1(\omega) = \frac{V_{out}}{V_{in}}\Big|_{V_{BE}=0} = \left[\frac{\beta}{R_S}\right]\left[\frac{R_L}{1+j\omega(C_{BC}+C_{CE})R_L}\right]}$$

(this is the same as the answer in the DC case above)

(c) The transfer function is a single-pole lowpass with DC gain equal to $\frac{\beta R_L}{R_S}$ and the pole located at $\omega = \frac{1}{(C_{BC}+C_{CE})R_L}$



$$\frac{\beta R_L}{R_S} = 200$$

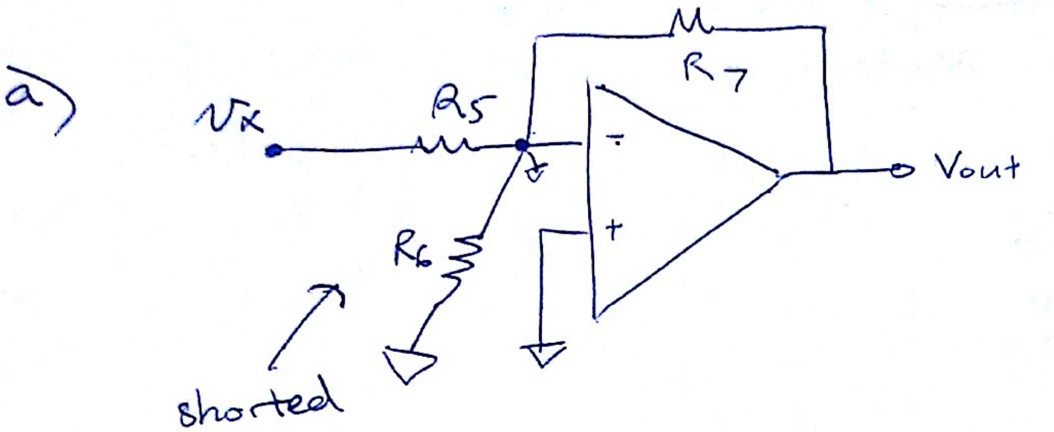
$$20\log(200) = 46\text{ dB}$$

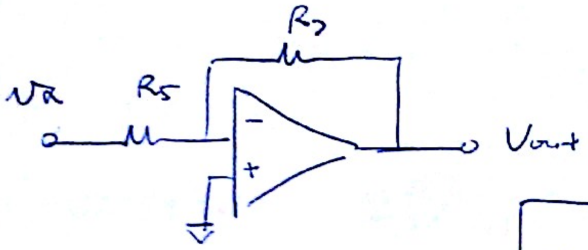
$$\frac{1}{(C_{BC}+C_{CE})R_L} = 50\text{ Mrad/s} = 8\text{ MHz}$$

Grader = Andre

MT #2 Problem 4

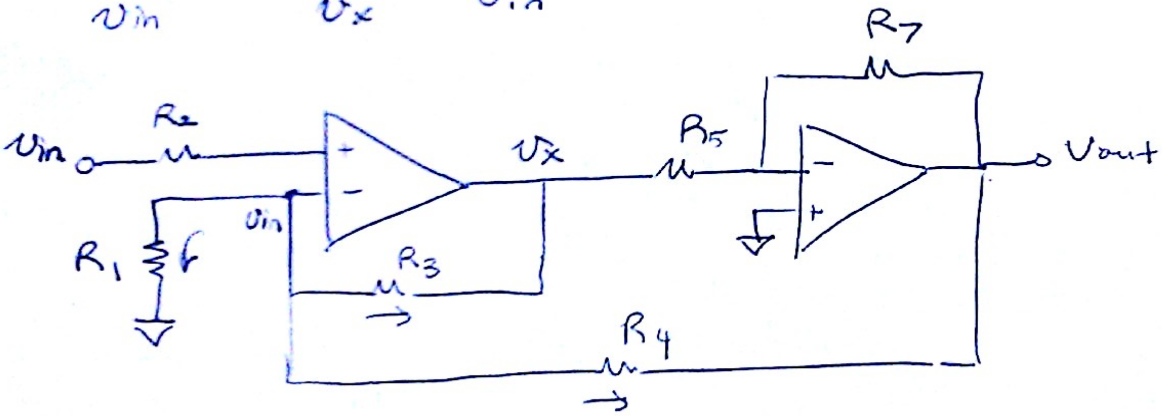
a)





$\Rightarrow$ inverting amp $\Rightarrow$ $\boxed{\dfrac{v_{out}}{v_x} = -\dfrac{R_7}{R_5}}$

b) $\dfrac{v_{out}}{v_{in}} = \dfrac{v_{out}}{v_x} \cdot \dfrac{v_x}{v_{in}}$



$$\frac{v_{in}}{R_1} + \frac{v_{in} - v_x}{R_3} + \frac{v_{in} - v_{out}}{R_4} = 0$$

$$\frac{v_{in}}{R_1} + \frac{v_{in}}{R_3} + \frac{v_{in}}{R_4} = \frac{v_{out}}{R_4} + \frac{v_x}{R_3} = \frac{v_{out}}{R_4} - \frac{R_5}{R_7 R_3} v_{out}$$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{1/R_1 + 1/R_3 + 1/R_4}{1/R_4 - \frac{R_5}{R_3 R_7}} = \frac{R_3R_4R_7 + R_1R_4R_7 + R_1R_3R_7}{R_1R_3R_7 - R_1R_4R_5}}$$

Rubric for # 3:

a) +2 Correct definition of i_x
   +1 For trying something
   +5 for correct V_{th} or I_N
   +5 for correct R_{eq}
   +2 for drawing the 2-element equivalent circuit

b) +3 Nodal/Mesh equations
   +4 Correct Answer (+2 if very close but wrong)
   +1 If answer is in proper form
   +2 If answer does not contain V_{BE}

c) +10 for Low Pass Filter shape and proof (+5 for shape without proof)
   +2 for $|H(w)|$
   +3 for w_c
   -2 for missing labels on axis

Rubric for #4:

a) +5 for recognizing equivalent ckt or for solving one nodal equation at V_- (right amp)
   +2.5 for shorting R_6 or equations do not contain R_6
   +5 For correct answer
   -1 for missing sign
   Full credit given for correct answer.

b) +2.5 for utilizing result from part a.
   +5 for proper setup for KCL at V_- (left amp)
   +5 for correct answer
   -2.5 for algebraic error or answer is close but wrong
   -1 for wrong sign