Final
EE40
Spring 2015

NAME:______________________________ **SSID:**______________________________

Instructions

Read all of the instructions and all of the questions before beginning the exam.

There are 6 problems in this exam. The total score is 150 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

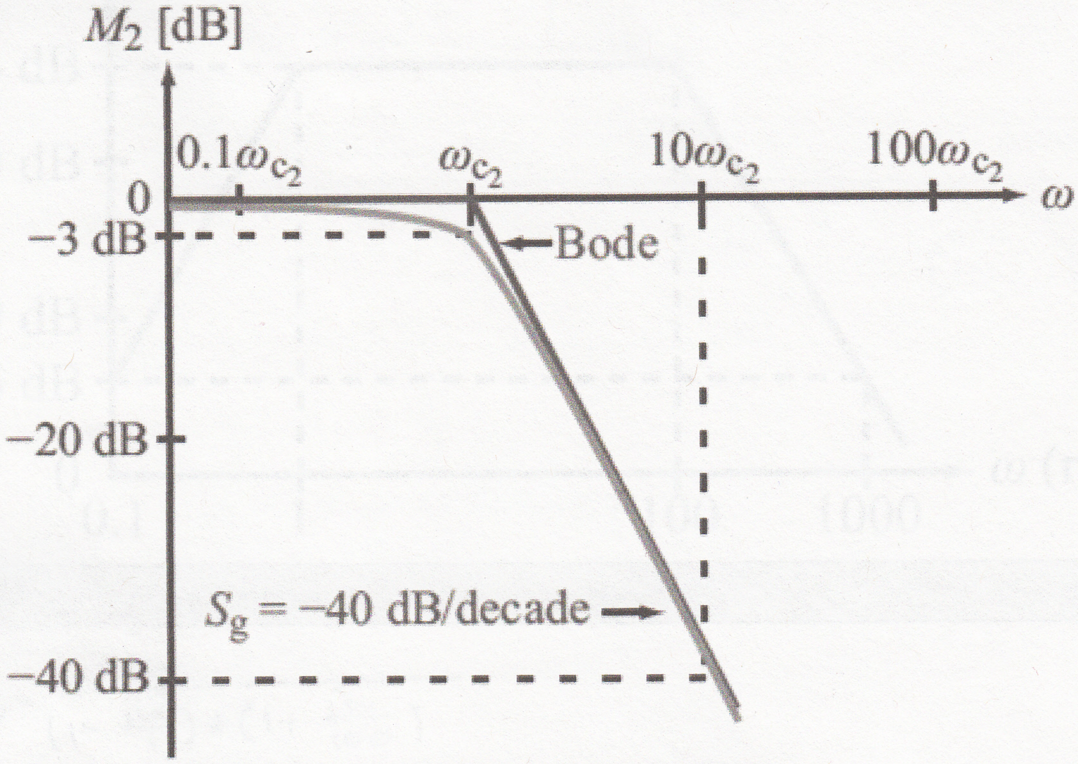
Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam.

Be sure to provide units where necessary.

GOOD LUCK!

PROBLEM	POINTS	MAX
1		?
2		?
3		?
4		?
5		?
6		?

Problem 1 *Warm up (n points each, m points total)*
Consider the following magnitude Bode plot:



a) Provide a transfer function that can generate this magnitude Bode plot:

Solution:
$H(\omega) = \dfrac{1}{\left(1 + \frac{j\omega}{\omega_{c2}}\right)^2}$

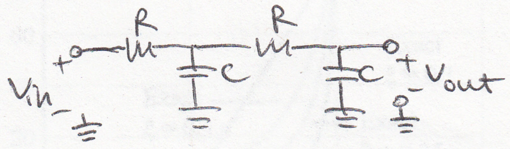
two poles at w_{c2}.

$$H = \frac{1}{\left(1 + \frac{s}{w_{c2}}\right)^2}$$

$\log|H(\omega=0)| = \log(1) = 0$

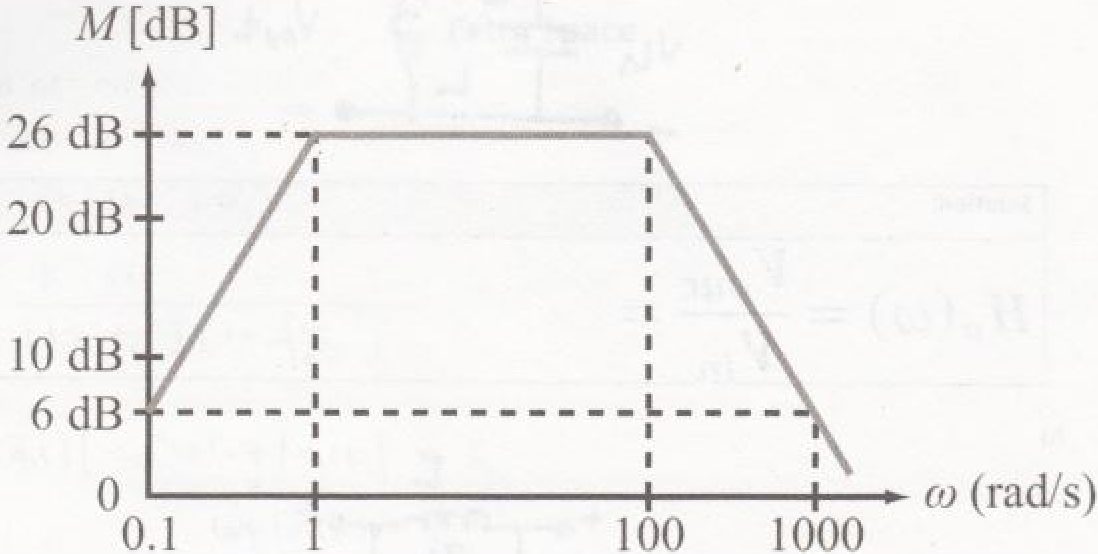
clearly label. V_{in} and V_{out}

b) Provide a circuit which has the Bode magnitude plot above: There's no need to provide numberical values for the components

Solution:
 $\dfrac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega)$

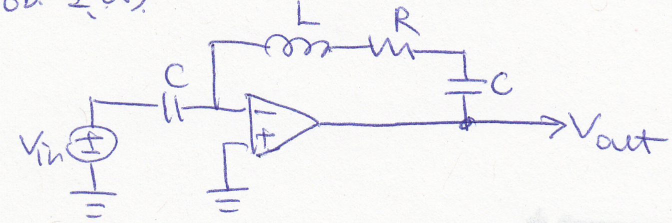
hint: $20\log 2 = 6$.

c) Provide a transfer function that produces the Bode plot below:



Solution:
$H(\omega) = \dfrac{j20\omega}{\left(1+\frac{j\omega}{1}\right)\cdot\left(1+\frac{j\omega}{100}\right)}$

Prob. 1, d)



$$-\frac{V_{out}}{V_{in}} = s^2LC + sRC + 1$$

Extra Space

c). zero at origin

one pole at 1 rad/s

one pole at 100 rad/s

$$H(\omega) = \frac{K \cdot j\omega}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{100}\right)}$$

$$20\log|H(\omega=0.1)| = 20\log|0.1K| = 6$$

$$\log|0.1K| = \frac{6}{20}$$

$$0.1K = 2$$

$$K = 20.$$

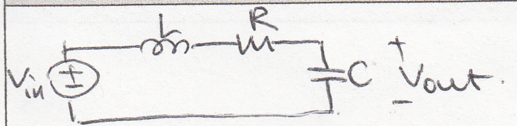
Problem 2 *(n points)*

We want to design the *simplest* ~~a~~ voltage filter with:

- a center frequency, ω_{center} = 1 ~~GHz~~ rad/s.
- a voltage transfer function magnitude at the center frequency, $|\hat{H}_V(\omega_{center})|$ = 100 dB
- which uses only linear passive components

a) Please draw the circuit below inside the box: (clearly label Vin and Vout)

Solution:



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{sC}}{sL+R+\frac{1}{sC}} = \frac{1}{1+sRC+s^2LC}$$

at $w_0 = \frac{1}{\sqrt{LC}} = 10^9$

$20\log 10^5 = 100\text{dB}$

$$\left|\frac{V_{out}}{V_{in}}(w_0)\right| = \frac{1}{\frac{RC}{\sqrt{LC}}} = \frac{\sqrt{L}}{R\sqrt{C}} = 10^5$$

b) Please list any relevant component values below:

Solution:

any R, L, C combination that fulfills the following:

$$\begin{cases} \frac{1}{\sqrt{LC}} = 10^9 \\ \frac{\sqrt{L}}{R\sqrt{C}} = 10^5 \end{cases}$$

works.

e.g. $\frac{1}{LC} = 10^{18} \Rightarrow C = 1p = 10^{-12} \quad L = 10^{-6} = 1\mu H$

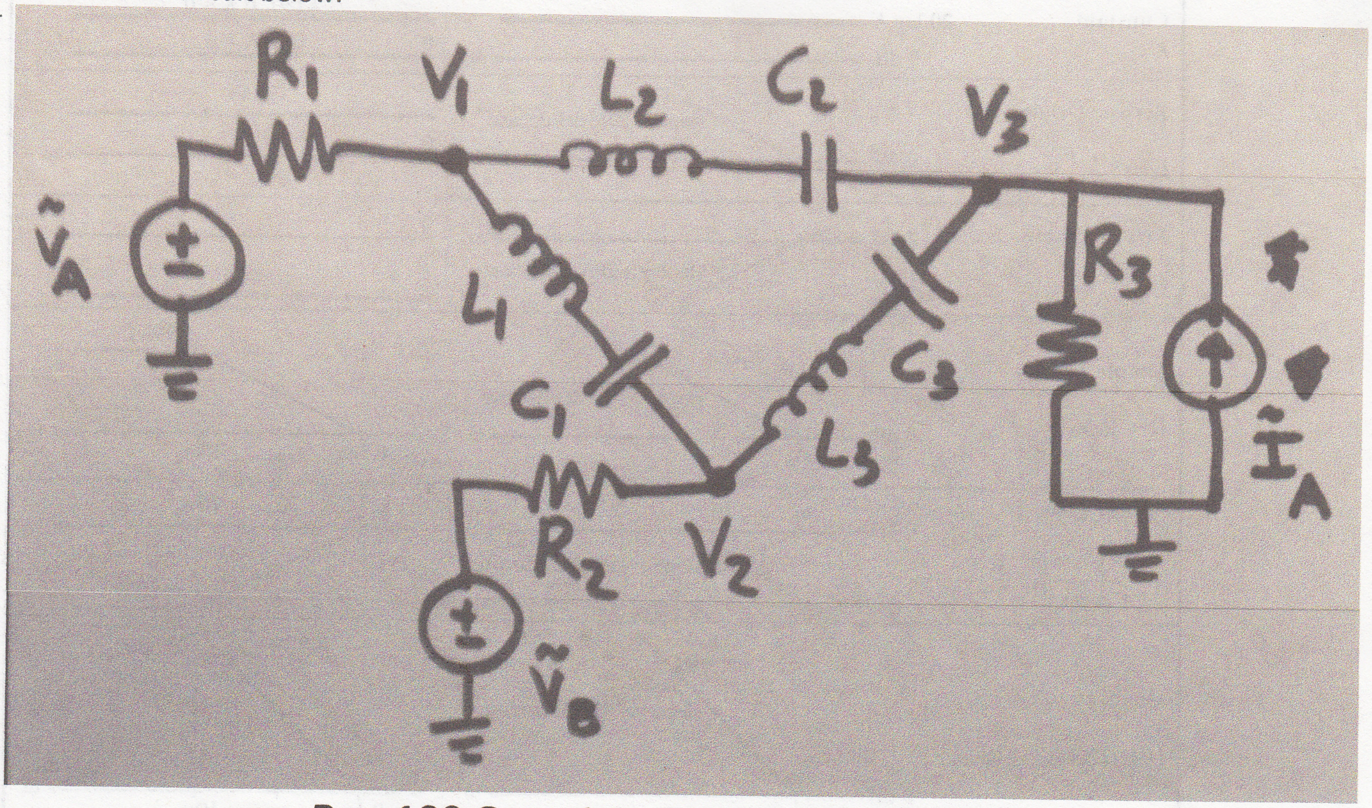
$$\frac{1}{R}\sqrt{\frac{10^{-6}}{10^{-12}}} = \frac{1}{R}10^3 = 10^5 \Rightarrow R = 10m\Omega$$

are sinusoidal.

c) Is the circuit resonant? If so, what is the Q? If not, why not?

Solution:
It resonants at ω_{center} $Q = \frac{\omega_0 L}{R} = \frac{10^9 \cdot 10^{-6}}{10^{-2}} = 10^5$

Problem 3 Nodal *(n points)*
Consider the circuit below.



$R_1 = 100\ \Omega$ $L_1 = 1\ \mu H$ $C_1 = 1\ \mu F$
$R_2 = 100\ \Omega$ $L_2 = 5\ \mu H$ $C_2 = 200\ nF$
$R_3 = 100\ \Omega$ $L_3 = 10\ \mu H$ $C_3 = 100\ nF$

The three sources are provided as phasors:
$\tilde{V}_A = 1\ V, 0°$ at $\omega = 10^6$ rad/s
$\tilde{V}_{\cancel{A}\,B} = 1\ V, 0°$ at $\omega = 10^6$ rad/s
$\tilde{I}_A = 1\ A, 0°$ at $\omega = 10^6$ rad/s

Solution:

$V_1 = V_2 = V_3(t) = 34\cos(10^6 t)$

$$\frac{V_A - V_1}{R_1} = \frac{V_1 - V_3}{sL_2 + \frac{1}{sC_2}} + \frac{V_1 - V_2}{sL_1 + \frac{1}{sC_1}}$$

$$\frac{V_1 - V_3}{sL_2 + \frac{1}{sC_2}} + \frac{V_2 - V_3}{sL_3 + \frac{1}{sC_3}} + I_A = \frac{V_3}{R_3}$$

$$\frac{V_1 - V_2}{sL_1 + \frac{1}{sC_1}} + \frac{V_B - V_2}{R_2} = \frac{V_2 - V_3}{sL_3 + \frac{1}{sC_3}}$$

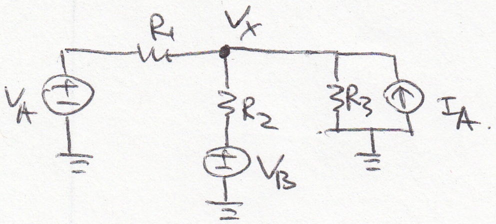
$$sL_1 + \frac{1}{sC_1} = j10^6\,10^{-6} + \frac{1}{j10^6 \cdot 10^{-6}} = j - \frac{j}{j} = 0.$$

$$sL_2 + \frac{1}{sC_2} = j10^6 \cdot 5\,10^{-6} + \frac{1}{j10^6\,200\,10^{-9}}$$

$$= 5j + \frac{5\,1000}{j\,200} = 0$$

$$sL_3 + \frac{1}{sC_3} = j10^6\,10 \cdot 10^{-6} + \frac{1}{j10^6 \cdot 100\,10^{-9}} = j10 + \frac{1000}{j\,1} = 0$$

equivalent ckt:



$$V_1 = V_2 = V_3 = V_x.$$

$$\frac{V_A - V_x}{R_1} + \frac{V_B - V_x}{R_2} + I_A = \frac{V_x}{R_3}$$

$$\frac{\frac{V_A}{R_1} + \frac{V_B}{R_2} + I_A}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = V_x$$

$$V_x = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_A + I_A}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$V_x = \frac{\frac{2}{100} V_A + I_A}{\frac{3}{100}}$$

$$= \frac{2V_A + 100 I_A}{3}$$

$$= \frac{2}{3} V_A + \frac{100}{3} I_A$$

$$= \cancel{\frac{2}{3}}$$

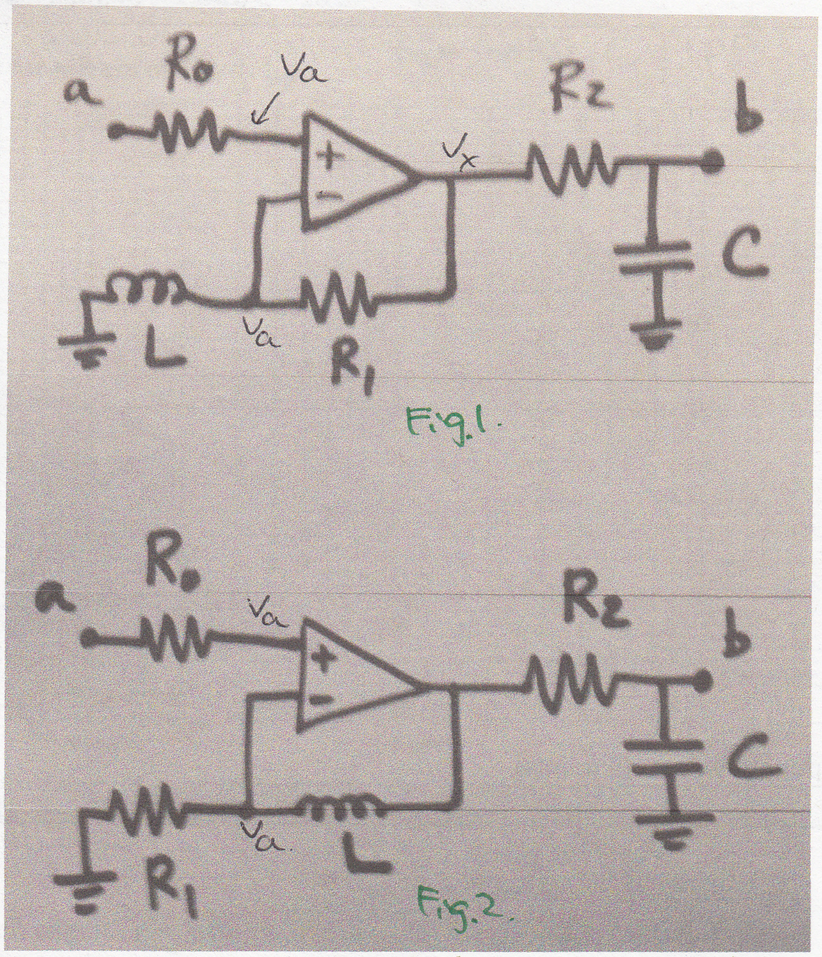
$$V_x(t) = \frac{2}{3} \cos(\omega_0 t) + \frac{100}{3} \cos(\omega_0 t)$$

$$= 34 \cos(10^6 t)$$

Problem 4 *(n points)*

Consider the two circuits below.

$$\begin{cases} \frac{V_a}{SL} = \frac{V_x - V_a}{R_1} \\ \frac{V_x - V_b}{R_2} = V_b SC \end{cases}$$



$$\begin{cases} \frac{V_a}{R_1} = \frac{V_x - V_a}{SL} \\ \frac{V_x - V_b}{R_2} = V_b SC \end{cases}$$

a) Provide **SYMBOLIC** transfer functions for BOTH circuits. (a as input port and b as output port).

Transfer function for the left circuit: (Fig.1)	**Transfer function for the right circuit:** (Fig.2)
$\cancel{\frac{V_a}{V_b} = \frac{\frac{j\omega L}{R_1}(1+j\omega R_2 C)}{1+\frac{j\omega L}{R_1}}}$ $\frac{V_b}{V_a} = \frac{1+\frac{SL}{R_1}}{SLR_1(1+SR_2C)}$ $\frac{V_b}{V_a} = \frac{10^3(1+\frac{S}{10^9})}{S(1+\frac{S}{10^7})}$	$\frac{V_b}{V_a} = \frac{1+\frac{SL}{R_1}}{1+SR_2C}$

$\cancel{\frac{V_a}{V_b} = \frac{j\omega 10^{-9}(1+j\omega/50k)}{1+j\omega/10^9}}$

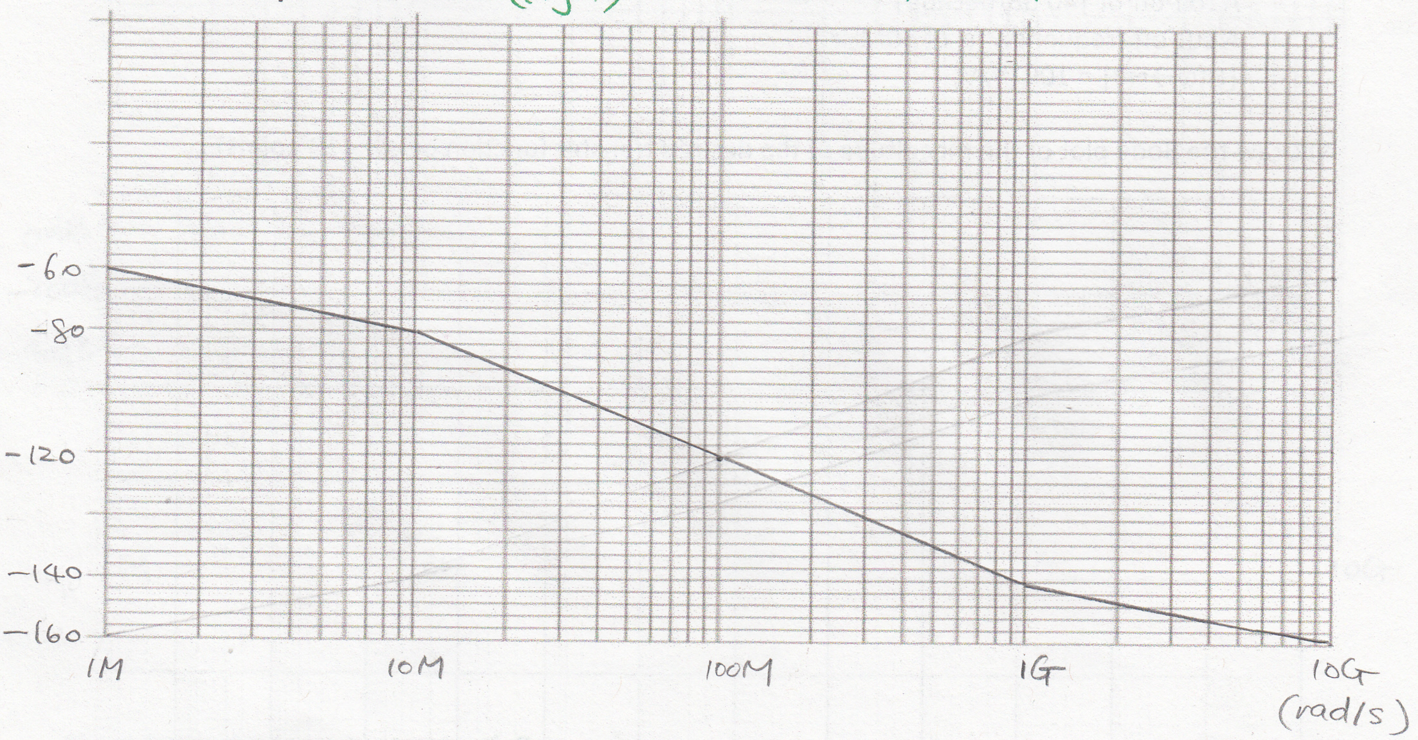
$\frac{V_b}{V_a} = \frac{1+\frac{j\omega}{10^9}}{1+\frac{j\omega}{10^7}}$

$20\log|H(\omega=1M)| = 20\log 10^{-3} = -60dB$

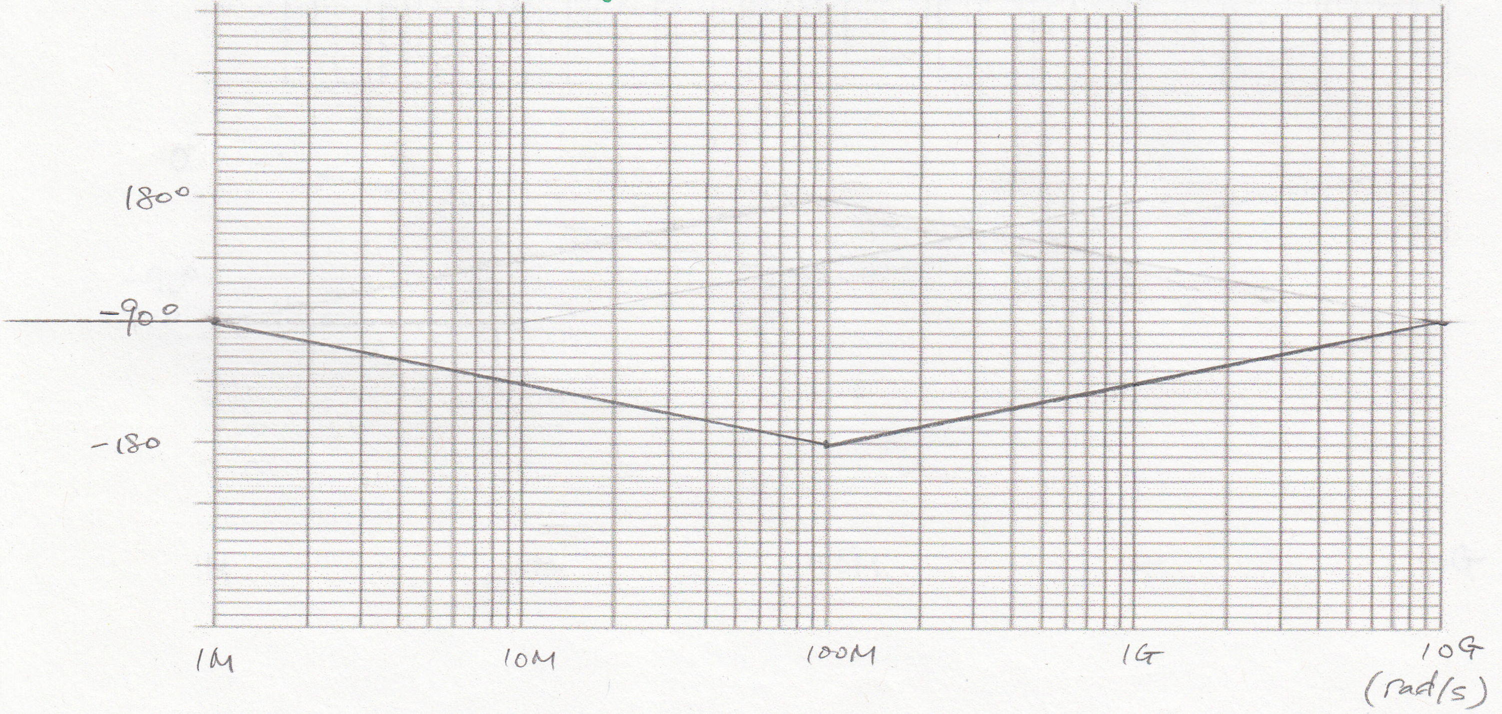
b) Provide Bode magnitude and phase plots for both circuits: (n points)

$R_0 = 100\ \Omega$, $R_1 = 1\ k\Omega$, $L = 1\ \mu H$, ~~$R_2 = 1\ M\Omega$, $C = 20\ pF$~~ $R_2 = 100\ k\Omega$, $C = 1\ pF$

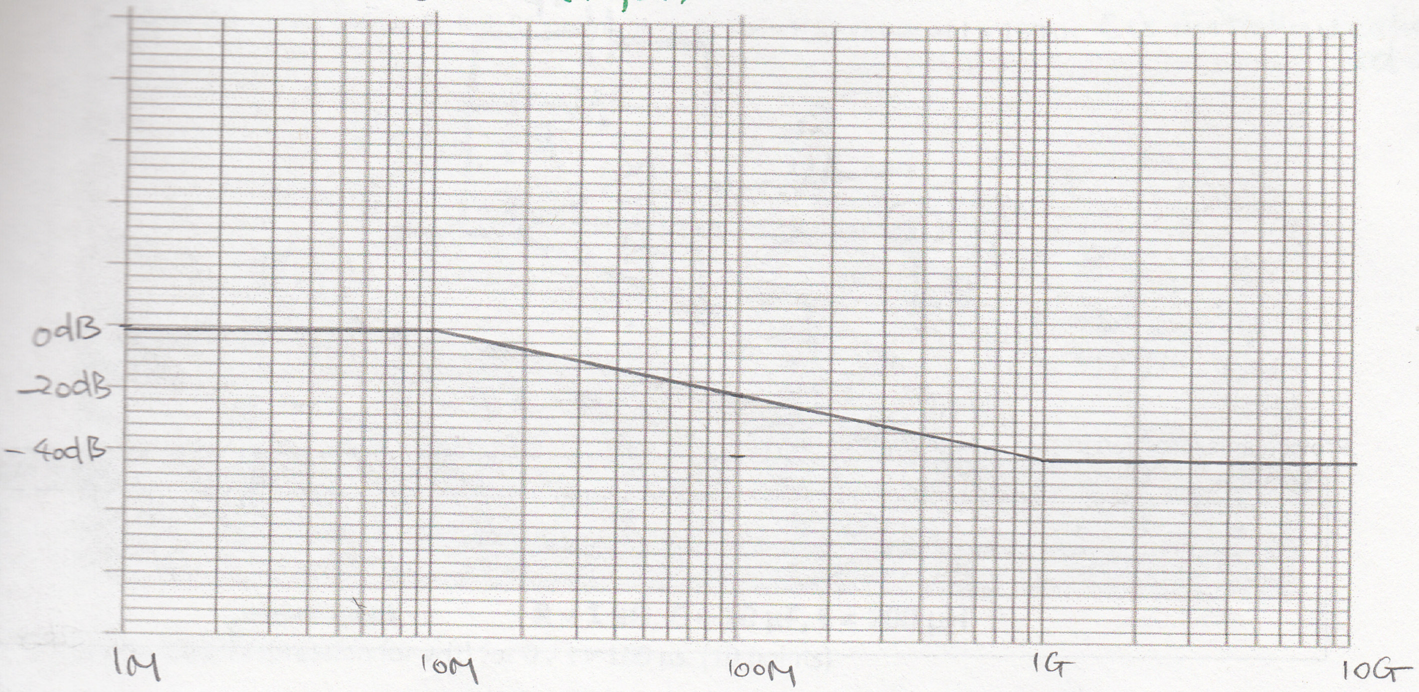
Magnitude Bode plot for ~~left circuit~~ (keep) ✓ (Fig. 1.)



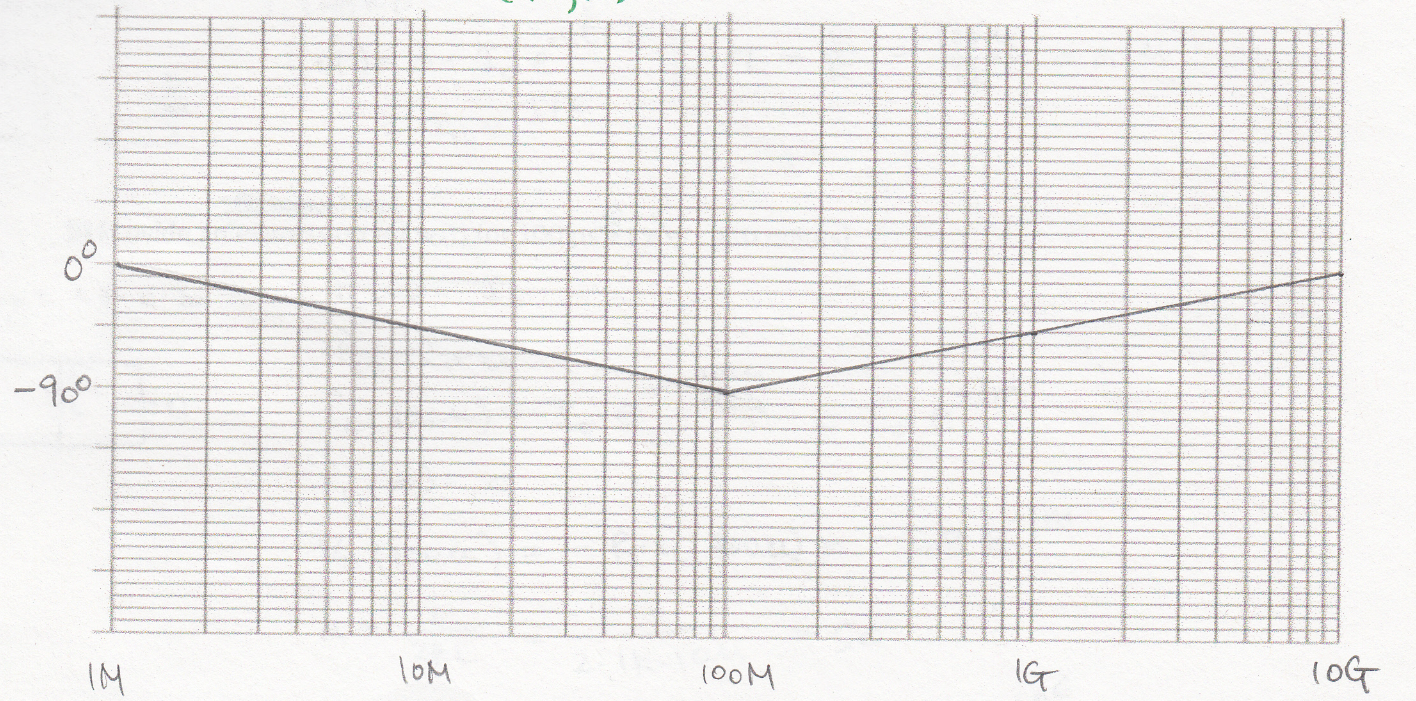
Phase Bode plot for left circuit (Fig. 1)



Magnitude Bode plot for right circuit (Fig.2.)



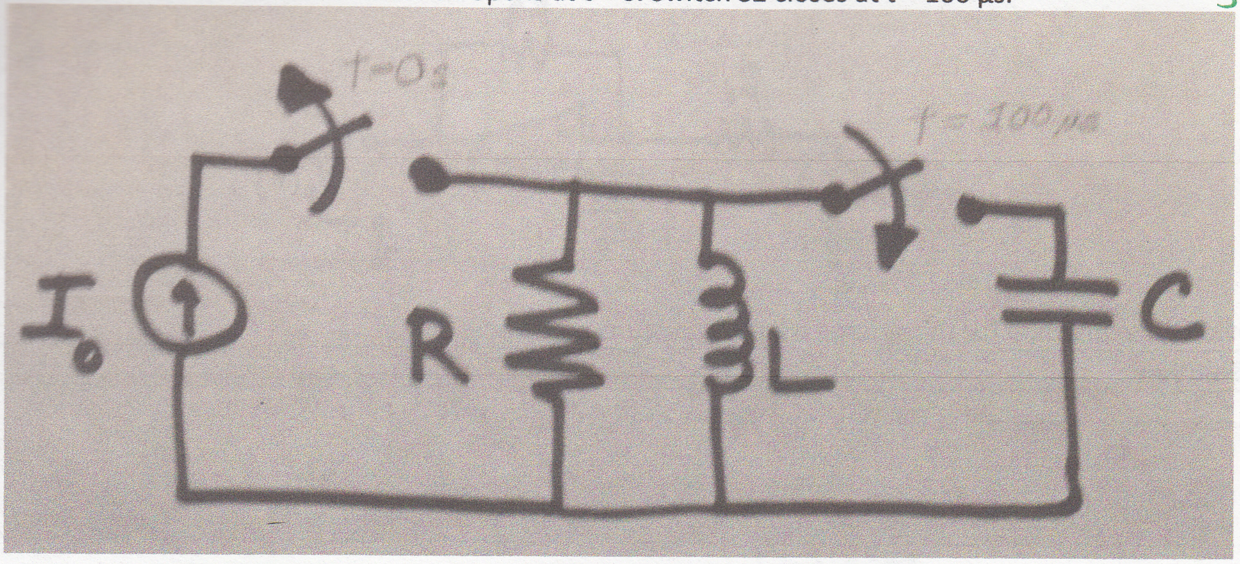
Phase Bode plot for right circuit (Fig.2)



Problem 5 *(25 points)*

Io is given.

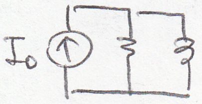
Consider the circuit below. Switch S1 opens at t = 0. Switch S2 closes at t = 100 µs. C is initially unchanged.



R = 1 kΩ, C = 10 µF, L = 100 µH

a) Provide an ~~numerical~~ expression for $v_C(t)$ for 0 > t > 100 µs. (10 points)

t<0

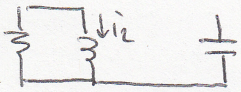


$$\boxed{V_c(t) = 0 \ V}$$

$i_L(0) = I_0$

0<t<100us

$i_L(\infty) = 0$



$i_L(t) = 0 + I_0 e^{-t/\tau}$, $\tau = \frac{L}{R} = \frac{100u}{1k} = 0.1u$

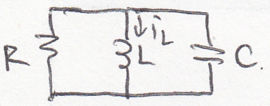
$= I_0 e^{-t/\tau}$

b) Provide an ~~numerical~~ expression for $v_C(t)$ for 100 µs ~~>~~ < t ~~>~~ < ∞ (10 points)

100us < t < ∞

~~$i_L(0) = I_0$~~

~~$i_L(\infty) = 0$~~



$i_L(100u) = I_0 e^{-\frac{100u}{0.1u}} = I_0 e^{-1m}$

$i_L(\infty) = 0$

$V_L(100u) = -R \cdot i_L(100u) = -I_0 R e^{-1m}$

$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 1k \cdot 10u} = 50$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{sqrt(100u \cdot 10u)} = \frac{10^6}{\sqrt{1000}}$

$50^2 < \frac{(10^6)^2}{1000}$

$\therefore \alpha < \omega_0 \Rightarrow$ underdamped.

Extra Space

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$ known

$$D_1 = i_L(100u) - i_L(\infty) = I_0 e^{-1m}$$ known

$$D_2 = \frac{\frac{1}{L} V_L(100u) + \alpha\left[i_L(100u) - i_L(\infty)\right]}{\omega_d}$$ Known.

$$\cancel{\therefore i_L(t) = e^{-\alpha t}(D_1 \cos \omega_d t + D_2 \sin \omega_d t)}$$

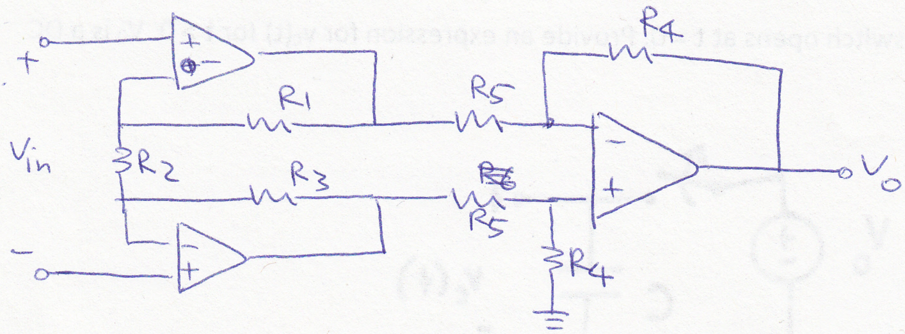
$$\cancel{V_L(t) = L \frac{di_L}{dt} = L(-\alpha)\, e^{-\alpha t}(D_1 \cos \omega_d t + D_2 \sin \omega_d}$$

$$i_L(t) = e^{-\alpha(t-100u)}\left(D_1 \cos \omega_d(t-100u) + D_2 \sin \omega_d(t-100u)\right)$$

$$\cdot\ V_L(t) = L \frac{di_L}{dt} = L \cdot e^{100u\alpha} \cdot (-\alpha)\, e^{-\alpha(t)}\left(D_1 \cos \omega_d(t-100u) + D_2 \sin \omega_d(t-100u)\right)$$

$$+ L e^{100u\alpha} \cdot e^{-\alpha t} \cdot \left(-D_1 \omega_d \sin \omega_d(t-100u) + D_2 \omega_d \cdot \cos \omega_d(t-100u)\right)$$

Prob 6. a)

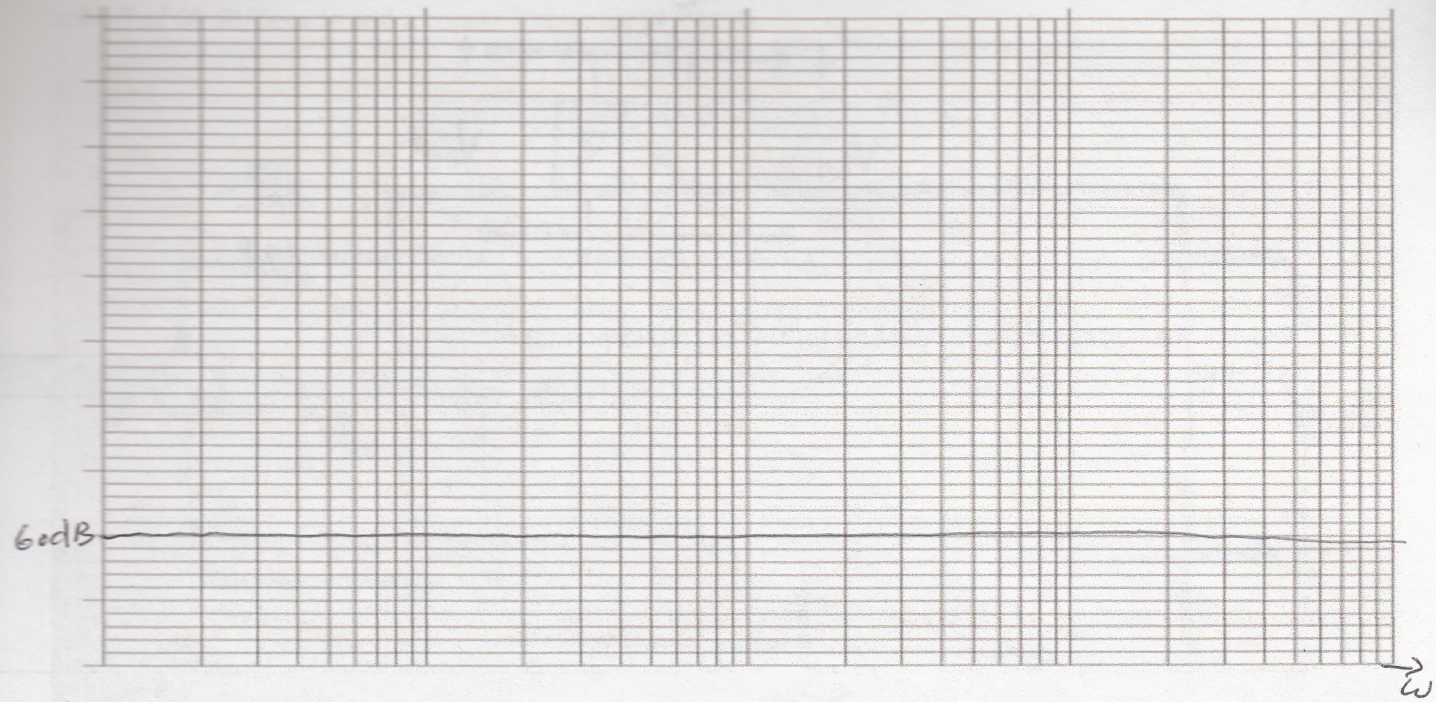


$$\frac{V_o}{V_{in}} = \left(\frac{R_4}{R_5}\right)\left(\frac{R_1+R_2+R_3}{R_2}\right) = 1000$$

e.g. $R_4 = \frac{100}{3}k\Omega$, $R_5 = 100\Omega$

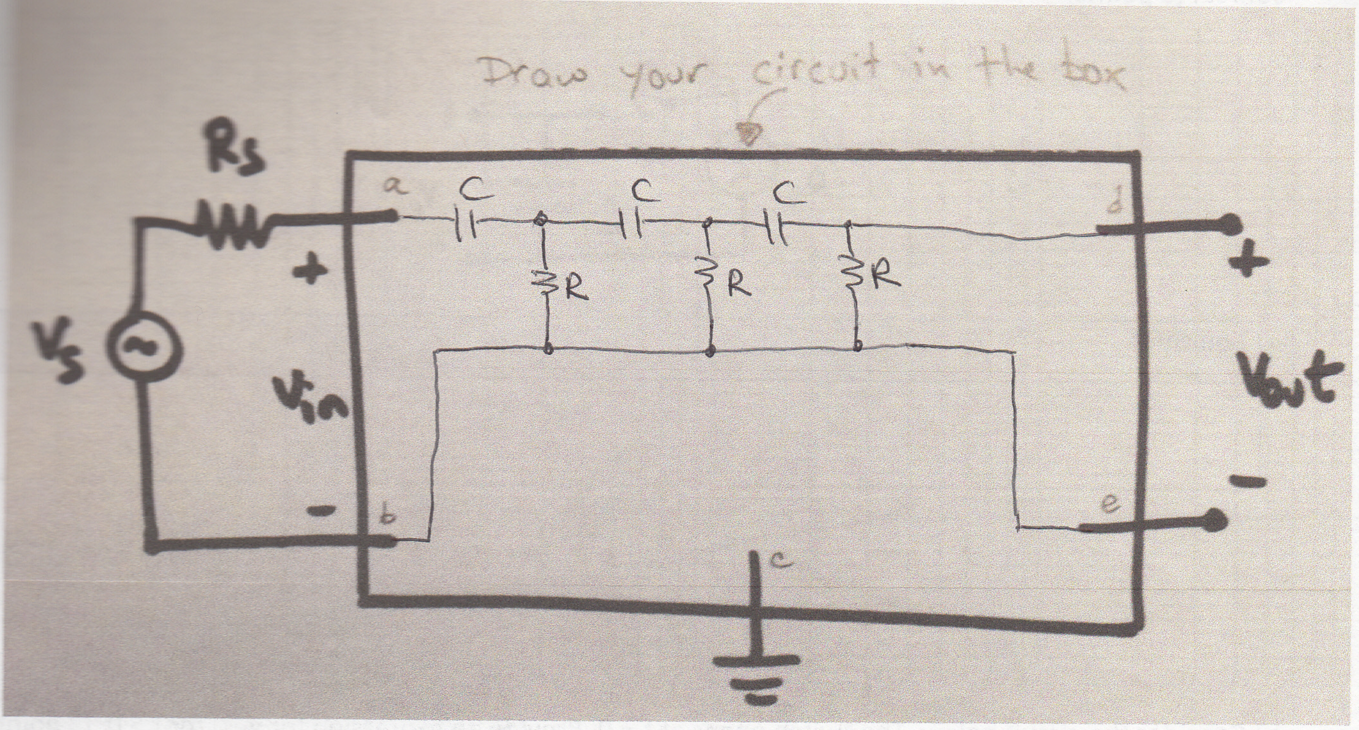
$R_1 = R_2 = R_3 = 100\Omega$

b) Provide the magnitude Bode plot of $|\hat{H}(\omega)|$.



$$20\log|H(\omega)| = 20\log 10^3 = 60\ \text{dB}$$

c) Instead of amplifying, we want to filter out 60 Hz noise. Design a passive non-resonant highpass filter with a roll-off of 60 dB/decade and a corner frequency of 300 Hz.



$$H(\omega) = \left(\frac{R}{R+\frac{1}{sC}}\right)^3 = \left(\frac{sRC}{1+sRC}\right)^3$$

$$RC = \frac{1}{300 \times 2\pi}$$

~~20log (H(ω~~

$$\cancel{20\log|H(\omega=1)| = 60\log RC = 60\log\frac{1}{300\times 2\pi}}$$

$$20\log|H(\omega=2\pi)| = 60\log\frac{1}{300}$$

d) Provide the Magnitude Bode plot of $|\hat{H}(\omega)|$.